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Application of Selected Geometrical 3D Object Description Algorithms to the Problem of Model Retrieval

***Abstract**. In the paper the comparison of three geometrical approaches to the representation of 3D shapes is presented. Two of them are well-known and popular, i.e. Extended Gaussian Image and Shape Histograms. They are compared with a method based on the transformation of points from Cartesian to spherical co-ordinates. For the purpose of the experiments the „Princeton Shape Benchmark" database was applied, which became popular in the task of experimental evaluation of 3D shape descriptors. The current rapid development of computer hardware makes the processing of 3D scenes faster. Hence, the description of objects in image processing, recognition and indexing is possible. Amongst many existing applications of 3D shape descriptors their usage in CAD systems, biometrics, entertainment, virtual reality and image retrieval are especially tempting. In the paper the last listed problem is analysed.*

Streszczenie.** W artykule opisano porównanie trzech podejść geometrycznych do zagadnienia reprezentacji kształtu 3D. Dwa spośród nich to dobrze znane i popularne techniki, tj. Extended Gaussian Image oraz Shape Histograms. Zostały one porównane z metodą opartą na przekształceniu punktów z kartezjańskiego do sferycznego układu współrzędnych. Na potrzeby eksperymentu wykorzystano bazę „Princeton Shape Benchmark", która stała się popularna w eksperymentalnej ocenie deskryptorów kształtu 3D. Obecny szybki rozwój sprzętu komputerowego przyspiesza przetwarzanie scen trójwymiarowych. Dzięki temu opis obiektów na potrzeby przetwarzania obrazów, rozpoznawania oraz indeksowania stał się możliwy. Pośród wielu istniejących zastosowań deskryptorów kształtu 3D ich użycie w systemach CAD, biometrii, rozrywce, wirtualnej rzeczywistości oraz indeksowaniu obrazów jest szczególnie pożądane. W niniejszym artykule ostatnie z wymienionych zastosowań jest analizowane. **Porównanie trzech podejść geometrycznych do zagadnienia reprezentacji kształtu 3D

Keywords: 3D shape description, geometrical descriptors, spherical co-ordinates
Słowa kluczowe: opis kształtu 3D, deskryptory geometryczne, współrzędne sferyczne

Introduction

Lately, application of three dimensional shape models in various areas of multimedia has become very popular and successful. It comes from significant technological achievements in graphics hardware and software development. The usage of 3D shape descriptors can significantly improve the proper scene analysis. This can be helpful for example in 3D computer-aided design, when great number of small shapes can appear within a project. In this case the fast approaches are especially needed. Similarly, the same algorithms can be applied in the virtual reality, e.g. for the development of artificial worlds. However, specialised applications arise more recently, e.g. in biometrics [1] and forestry [2]. Three properties of efficient three-dimensional shape descriptor can be enumerated [3]: "1. Descriptive power (the similarity measure based on the descriptor should deliver a similarity ordering that is close to the application driven notion of resemblance); 2. Conciseness and ease of indexing (the descriptor should be compact in order to minimise the storage requirements and accelerate the search by reducing the dimensionality of the problem. Very importantly, it should provide some means of indexing and thereby structuring the database in order to further accelerate the search process); 3. Invariance under transformations (the computed descriptor values have to be invariant under an application dependent set of transformations. Usually, these are the similarity transformations, however, some applications like e.g. retrieval of articulated objects may additionally demand invariance under certain deformations, etc.)". The problem of the invariance under affine transforms is often indicated as the most important, not only for 3D objects, but also for planar shapes as well [4].

In the paper an approach based on the usage of transformation from Cartesian to spherical co-ordinates is experimentally compared with two other well-known and the most widely used algorithms, namely the Extended Gaussian Image (EGI) and the Shape Histogram. Their outcomes were compared experimentally by means of the „Princeton Shape Benchmark" database [5], which is a very popular benchmark for the evaluation of 3D model representation techniques.

The rest of the paper is organised as follows. The second section describes briefly the 3D shape representation algorithms. The third one provides the detailed description of the descriptors selected for the experimental comparison with the proposed algorithm, which is presented in the fourth section. The fifth section is devoted to the experimental results, and the last section concludes the paper.

A brief review of 3D model description algorithms

The algorithms for the representation of three-dimensional shapes can be divided based on the most important characteristic properties into four main groups: geometrical, symmetrical, structural and local [6].

The first group covers methods that use the basic geometrical features of an object. Amongst them, the Extended Gaussian Image (EGI, [7]) is one of the most popular and widely used algorithms. It uses the Gaussian image, which is obtained by mapping the surface normals of an object into the unitary (Gaussian) sphere. An improvement of the above-mentioned is the Complex Extended Gaussian Image, CEGI [8]. The difference is that the second one additionally uses the imaginary part of the complex number containing the distance between the face and the pre-assumed origin.

Another approach is the Shape Distribution [9]. It is based on the selection of the shape function, that is invariant to particular transformations of a model. Originally, several functions were proposed [9], which are applied for the derivation of shape distributions. Taking them into account the histogram is calculated, which is the final description for a model.

The fourth geometrical approach is the Shape Histogram [10]. In this algorithm the space is divided into regular subparts and the histogram for those subspaces is calculated. Another exemplary geometrical approaches are three-dimensional moments (e.g. [3]) and spherical harmonics [11].

An exemplary symmetrical approach is the Reflective Symmetry Descriptor [12], which is a 2D function describing symmetries for each plane going through the centre of gravity of a 3D model.

The Multiresolutional Reeb Graph (MRG, [13]) is a method belonging to the structural descriptors. The first step of this algorithm is the construction of a function that is later applied to the process of partitioning a model into sub-areas. Those sub-areas are represented in the Reeb graph, and are connected with other neighbouring nodes. The MRG graph is constructed using many Reeb graphs, calculated for the increasing numbers of areas.

A novel approach was proposed in [14], where the covariance matrices representing descriptors were applied instead of the descriptors themselves.

All the above 3D shape descriptors were global. An example of the local methods was presented in [15]. It was based on canonical geometric scale-space analysis and encoded the local shape information within the inherent support region of each feature.

There are many other approaches for the three-dimensional model representation. However their exhaustive description exceeds the main goals of this paper (it can be found for example in [16], [17], [18]). Only the most important approaches in historical manner, according to the four general categories were mentioned here.

Description of the methods selected for the comparison

The Extended Gaussian Image (EGI) is the first method selected for the experiments. Its description, provided in this section, is based on [7].

The Gaussian image for a 3D model is obtained by associating with each point on its surface the point on the Gaussian sphere that has the same surface orientation.

The more precise definition of the algorithm starts with the definition of the Gaussian curvature. For a small patch *δO* on the object, each point belonging to it corresponds to a point on the Gaussian sphere *δS*. For curved surface of the object the normals of its points will point into various directions. On the other hand, for planar surface the surface normals will be planar and therefore map into a single point. This suggests the definition of the Gaussian curvature as to be equal to the limit of the ratio of the two areas as they tend to zero [7]:

(1) $$K = \lim_{\delta O \to 0} \frac{\delta S}{\delta O} = \frac{dS}{dO}.$$

From the above one can obtain integrals [7]:

(2) $$\iint_O K dO = \iint_S dS = S,$$

where S is the area of the corresponding patch on the Gaussian sphere. The formulation on the left is called the integral curvature. This relationship allows dealing with surfaces that have discontinuities in surface normal [7]. We can rewrite the above relationship using the following formula [7]:

(3) $$\iint_S 1/K \;\; dS = \iint_O dO = O,$$

where *O* denotes the area of the corresponding patch on the object.

Taking the described relationship into account one can indicate a mapping associating the inverse of the Gaussian curvature at a surface point of the object with the corresponding point on the Gaussian sphere. Let us assume that *u* and *v* denote the parameters used for the identification of points on the original surface, while *ζ* and *η* on the Gaussian sphere. In that case the Extended Gaussian Image can be formulated as [7]:

(4) $$G(\xi, \eta) = \frac{1}{K(u, v)},$$

where point *(ζ, η)* lies on the Gaussian sphere and has the same normal as point *(u, v)* from the original surface.

For the needs of the experiments the Shape Histogram was selected as the second compared method. Its description provided below is based on the definition given in [10]. In general, the 3D shape histograms are constructed using the partitioning of the space with the model. The obtained cells are assigned to particular bins in histogram. Many histograms can be formulated since there are many possible decomposition methods. The authors of the approach proposed three exemplary models – a shell model, a sector model, and a spider-web model, which is a combination of the former two [10]. The first one applies the concentric shells around the central point, what gives the invariance to rotation. The sector model uses sectors emerging from the centre of an object. The last model is a combination of the both previously presented ones. In all cases the obtained histograms describe the three-dimensional models.

Description of the algorithm based on the transformation into spherical co-ordinates

The proposed method uses the approach known from the planar shape descriptors based on the usage of transformation from Cartesian into polar co-ordinate systems. However, for three dimensions each point is represented using three co-ordinates, both in Cartesian and resultant systems. The algorithm starts with the calculation of the centre of gravity of an object:

(5) $$L = (L_x, L_y, L_z) = (\frac{1}{n}\sum_{i=1}^{n} x_i, \frac{1}{n}\sum_{i=1}^{n} y_i, \frac{1}{n}\sum_{i=1}^{n} z_i),$$

where: $P_i = (x_i, y_i, z_i)$ – co-ordinates of *i*-th vertex; *n* – number of vertices of a 3D model.

Later, all co-ordinates are modified in order to place the centre of gravity *L* in the centre of the co-ordinate system. This process is performed by subtraction of the L_i from the co-ordinates of particular points belonging to an object. In result, the representation becomes invariant to translation.

After the above-mentioned process transformation to the mathematical spherical co-ordinates is realised. For a point P_i on a model the following co-ordinates are obtained:

- *r* – the radial distance from the origin of the co-ordinates system *O*;
- *θ* – polar angle;
- *φ* – azimuth angle.

The transformation is performed using the formulas:

(6) $$r = \sqrt{x^2 + y^2 + z^2},$$

(7) $$\theta = \arctan(\frac{\sqrt{x^2 + y^2}}{z}) = \arccos(\frac{z}{\sqrt{x^2 + y^2 + z^2}}),$$

(8) $$\varphi = \arctan(\frac{y}{x}).$$

Later the radial distances are normalised through dividing all their values by the largest. In result, the obtained values belong to the interval <0,1>, what gives scale invariance. In the proposed approach for the description of a 3D object the radial distances were chosen, since they turned out to be the most efficient from the particular values and their combinations, what was experimentally proven in [19].

Experimental results and discussion

For the evaluation of three selected algorithms models taken from the "Princeton Shape Benchmark" [5] were applied. This database contains three-dimensional polygonal models collected from the World Wide Web (see Fig.1).

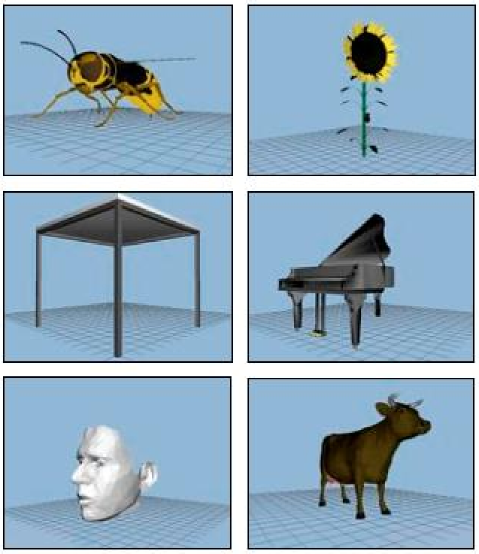

Fig. 1. Examples of 3D objects from the Princeton Shape Benchmark [5].

For the experiments 312 models, belonging to 13 classes were taken. The retrieval was considered successful if the Euclidean distance between the represented testing and template object was the smallest for objects belonging to the same class. It can be considered as the distance-based recognition, where the closest object in the feature space is found and selected as the result of the retrieval. No threshold was applied for the rejection of the objects that were too dissimilar.

The results obtained by Extended Gaussian Image are provided in Table 1., and for the Shape Histogram in Table 2. The three-dimensional shell model was used in the latter one. Table 3. contains the results achieved using the described in the previous section method based on the usage of spherical co-ordinates.

Taking the three provided tables into account one can conclude the following remarks. The EGI algorithm gave better results than the Shape Histogram. It is clearly visible on the accomplished overall retrieval results. The EGI achieved above 60% efficiency, while the second algorithm only almost 37%. Nevertheless, the described in the paper algorithm based on the transformation from Cartesian into spherical co-ordinates performed much better than the two mentioned 3D model descriptors. It obtained more than 71% accuracy.

Table 1. Experimental results for retrieval of 3D models using EGI.

Class no.	Properly retrieved objects	Wrongly retrieved objects	Percentage of properly retrieved objects [%]	Percentage of wrongly retrieved objects [%]
1	41	30	57,75	42,25
2	23	12	65,71	34,29
3	10	9	52,63	47,37
4	17	15	53,13	46,87
5	8	2	80,00	20,00
6	18	18	50,00	50,00
7	4	2	66,67	33,33
8	2	1	66,67	33,33
9	28	15	65,12	34,88
10	7	3	70,00	30,00
11	20	13	60,61	39,39
12	4	4	50,00	50,00
13	6	0	100,00	0,00
AVG	**188**	**124**	**60,26**	**39,74**

Table 2. Experimental results for retrieval of 3D models using Shape Histogram.

Class no.	Properly retrieved objects	Wrongly retrieved objects	Percentage of properly retrieved objects [%]	Percentage of wrongly retrieved objects [%]
1	21	50	29,58	70,42
2	17	18	48,57	51,43
3	4	15	21,05	78,95
4	18	14	56,25	43,75
5	2	8	20,00	80,00
6	16	20	44,44	55,56
7	2	4	33,33	66,67
8	0	3	0,00	100,00
9	29	14	67,44	32,56
10	1	9	10,00	90,00
11	3	30	9,09	90,91
12	1	7	12,50	87,50
13	1	5	16,67	83,33
AVG	**115**	**197**	**36,86**	**63,14**

Table 3. Experimental results for retrieval of 3D models using spherical co-ordinates.

Class no.	Properly retrieved objects	Wrongly retrieved objects	Percentage of properly retrieved objects [%]	Percentage of wrongly retrieved objects [%]
1	46	25	64,79	35,21
2	23	12	65,71	34,29
3	13	6	68,42	31,58
4	29	3	90,63	9,38
5	7	3	70	30
6	32	4	88,89	11,11
7	3	3	50	50
8	2	1	66,67	33,33
9	35	8	81,4	18,6
10	6	4	60	40
11	21	12	63,64	36,36
12	3	5	37,5	62,5
13	4	2	66,67	33,33
AVG	**224**	**88**	**71,79**	**28,21**

Conclusions

The problem of three-dimensional object representation was a topic of this paper. It became popular nowadays, since it is applied to many areas of multimedia, e.g. Computer Aided Design, entertainment, virtual reality, biometrics, retrieval, and so on. Here, the last from the enumerated was explored. For this purpose two well-known algorithms (Extended Gaussian Image and Shape Histogram) and one new approach, based on the usage of spherical co-ordinates, were experimentally compared. For this purpose 312 objects, belonging to 13 classes, taken from Princeton Shape Benchmark database [5] were used,

Amongst the analysed 3D model representation methods the best result was obtained using the described in the paper approach applying the transformation of points belonging to an object from Cartesian to spherical co-ordinates. The algorithm gave more than 71% retrieval efficiency. The comparison of the other two investigated algorithms indicated the better efficiency of the Extended Gaussian Image, which achieved the retrieval rate above 60%. The Shape Histogram was the worst. It obtained only 37% of the efficiency.

The problem of 3D shape retrieval is considered very difficult and challenging, especially if working with large multimedia databases. Many models belonging to different classes are similar to each other. On the other hand, objects inside one class can significantly vary in appearance. Hence, there is a need of developing efficient methods for invariant representation of three-dimensional objects.

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