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Application of the "stretching" function for solving electrical circuits with differential-algebraic relations

Abstract. *In the article the problem of an accurately and effectively solving complex electrical circuits was considered. Application of the Kirchhoff's laws enables us to model such circuits by both differential and algebraic equations. The presented approach is based on the direct shooting method. To preserve the continuity of the state variables and consistency of the initial conditions, in the nonlinear optimization problem the "stretching" function was applied. The presented method can be used to carry out the circuits simulation in a computationally efficient manner.*

Streszczenie. *W artykule poruszono problem dokładnego i skutecznego rozwiązywania złożonych obwodów elektrycznych. Zastosowanie praw Kirchoffa umożliwia modelowanie takich obwodów zarówno przez równania różniczkowe jak i algebraiczne. Zaprezentowane podejście bazuje na bezpośredniej metodzie strzałów. Aby zapewnić ciągłość zmiennych stanu oraz spójne warunki początkowe, w zadaniu programowanie nieliniowego zastosowano funkcję "rozciągającą". Zaprezentowana metoda może być wykorzystana w symulowaniu obwodów w sposób wydajny obliczeniowo.* ***(Zastosowanie funkcji "rozciągających" do rozwiązywania obwodów elektrycznych z relacjami różniczkowo-algebraicznymi)***

Keywords: electrical circuit simulation, differential-algebraic systems, stretching function, inconsistent initial conditions.
Słowa kluczowe: symulacja obwodów elektrycznych, równania różniczkowo-algebraiczne, funkcja rozciągająca, niespójne warunki początkowe.

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1. Introduction

Nowadays, numerical simulation of electrical circuits plays a key role in the design of electrical networks, in which elements like resistors, capacitors, inductors, transistors and sources are interconnected [9]. Similar networks can be encountered in chemical engineering and biotechnology, where chemical reactors and bioreactors are interconnected to enhance the overall productivity of reactor networks. Therefore, circuit simulators can be used to analyze the dynamical behaviour of other systems on the analogy of electrical systems and various complex networks and technological systems [8].

Solving nonlinear electrical system described by ordinary differential equations is still an important task [12, 14]. Our investigations were focused on computational methods for nonlinear differential-algebraic equations connected with such systems.

The modelling of electrical circuits generally leads to systems of differential-algebraic equations (DAEs). After some elimination processes, DAE systems can be rewritten in the form of ordinary differential equations (ODEs), which cannot present the basic the process in the same manner like DAEs. A few advantages of a DAE formulation are the following:

- the variables keep their original physical interpretation,
- it is easier to vary design parameters in an implicit model,
- the algebraic equations typically describe conservation laws or explicit equality constraints and they should be kept invariant,
- the implicit model does not require the modelling simplifications often necessary to get an ODE,
- it may be difficult to reformulate the problem as an ODE when nonlinearities are present,
- the system structure can be exploited by problem-specific solvers,
- less specialized mathematical expertise is required on the part of the designer [1].

To solve both the system of stiff ordinary differential equations and differential-algebraic equations, the direct shooting method coupled with existing DAE and ODE *solvers* was proposed. In this way, the problem of solving complex DAE system was transformed into a nonlinear programming (NLP) problem. To solve the NLP problem, the existing NLP methods can be used [3, 16].

In this paper, one auxiliary function technique, known as the "stretching" function technique, was applied. The "stretching" function was constructed on the obtained local optimal solution. The motivation behind this approach was to utilize the previous local minima, which is the best solution found so far. This is used to create the starting points to increase the amount of time exploring more promising regions of the search space [18].

The presented algorithm was applied to solve the ring modulator, which is the highly nonlinear differential-algebraic system.

In the Section 2 a short review of the differential-algebraic equations is given and the basic modeling principles are described. The direct shooting approach for solving differential-algebraic electrical systems is presented in the Section 3. The "stretching" function in solving the optimization problems is outlined in the Section 4. In the Section 5 the application of the presented algorithm in the ring modulator is presented and discussed. The considerations are concluded in the Section 6.

The same notation as in [7] was used throughout the whole article.

2. Differential-algebraic electrical systems

The idealized basic both linear and nonlinear elements, like resistors, capacitors and nonlinear semiconductor devices, are used to model the electrical circuits [11]. The waveforms in the time domain describe the electrical behaviour of a selected parameter in the circuit, e.g. the potentials at each node between two or more adjacent elements or the branch currents.

The circuits are characterized by the type of the elements, their electrical value and by the topology of the system. One of the assumption is, that the nodes between elements are electrically ideal. Generally, two main kinds of relations are distinguished.

A. Characteristic element equations

These equations describe the relations between the voltage drop between the nodes of an element and the current in the branch. Nonlinear models for semiconductor devices, like these obtained for diodes and transistors, are used to describe physical reality to a great extent. The main modelling principles are as follows:

- to model the electrical circuit, the five basic elements can be used: resistor, capacitor, inductor, voltage source and current source,
- the nonlinear controlled voltage or current sources and resistors exhibit the time independent behaviour,
- the dynamic behaviour can be observed, when the nonlinear controlled capacitors or inductors are present.

B. Kirchhoff's laws

The voltages and branch currents observed in the electrical circuit fulfill Kirchhoff's laws and depend on the topology of the system

- Kirchhoff's current law (KCL), which states, that the algebraic sum of currents traversing each cutset of the network must be equal to zero at every instant of time.
- Kirchhoff's voltage law (KVL) indicates, that the algebraic sum of voltages around each loop of the network must be equal to zero at every instant of time.

The another important fact, which holds true for every circuit, is the conservation of charge over time.

The mentioned assumptions and relations, which come from the Kirchhoff's laws, lead us to the electrical systems, which can be described by system of differential-algebraic equations in fully-implicit form

$$F\big(x(t), z(t), p, t\big) = 0, \tag{1}$$

where $x(t) \in \mathcal{R}^{n_x}$ is a differential state, $z(t) \in \mathcal{R}^{n_z}$ is an algebraic state and $p \in \mathcal{R}^{n_p}$ is a vector of global parameters constant in the time. Then the vector-valued nonlinear function $F : \mathcal{R}^{n_x} \times \mathcal{R}^{n_z} \times \mathcal{R}^{n_p} \times \mathcal{R} \to \mathcal{R}^{n_F}$ is considered.

The *fully-implicit* form presented in eq. (1) is very general and can be difficult to analyze in all possible situations, in which essentially different properties can be observed.

On the other hand, when only dynamical features of the systems are under considerations, the description using ordinary differential equation is enough

$$\dot{x}(t) = G\big(x(t), p, t\big), \tag{2}$$

but in such manner some interesting relations between variables and their physical interpretations can be lost.

In some practical applications, like the discussed in this article, the obtained dynamical system can be stiff, and computationally difficult to solve and to simulate.

C. Singularly perturbed problem

In the real-life electrical circuits, multiscale dynamical systems, known as *singularly perturbed problem*, has to be considered

$$\begin{aligned} \dot{x}(t) =& \ \mathcal{F}_1\big(x(t), z(t), p, t\big) \\ \varepsilon\dot{z}(t) =& \ \mathcal{F}_2\big(x(t), z(t), p, t\big) \end{aligned} \tag{3}$$

with $\varepsilon \ll 1$. Approximate solution of the stiff ODE system (3) can be obtained by solving the following DAE system

$$\begin{aligned} \dot{x}(t) =& \ \mathcal{F}_1\big(x(t), z(t), p, t\big) \\ 0 =& \ \mathcal{F}_2\big(x(t), z(t), p, t\big) \end{aligned} \tag{4}$$

with consistent initial conditions. The appropriate numerical methods for solving the DAE system (4) were thoroughly studied in [5, 13, 19].

3. Direct shooting approach

In the previous section, the problem of solving the system of stiff ordinary differential equations was transformed into the other task - solving the differential-algebraic equations in the computationally efficient manner. Let us to consider the electrical system, which were described by the index-one differential-algebraic equations (DAEs)

$$\begin{aligned} \mathcal{B}(\cdot)\dot{x}(t) =& \ \mathcal{F}_1\big(x(t), z(t), p, t\big) \\ 0 =& \ \mathcal{F}_2\big(x(t), z(t), p, t\big), \end{aligned} \tag{5}$$

and which in many applications could have some highly nonlinear components.

The initial values of the differential and algebraic states and values for the system parameters are prescribed as follows

$$x(t_0) = x_0, \tag{6}$$

$$z(t_0) = z_0, \tag{7}$$

$$p(t_0) = p_0. \tag{8}$$

This description (5)-(8) is valid only for single-stage systems. There is a quite different situation, when the direct shooting method was applied, because each stage can be described by different set of the nonlinear differential-algebraic equations.

Let us assume, that to solve the electrical circuit, the multiple shooting method was applied and the number of shots N were prescribed.

A suitable partitioning of the time horizon $[t_0, t_f]$ into N subintervals $[t_i, t_{i+1}]$ with

$$t_0 < t_1 < \cdots < t_N = t_f, \tag{9}$$

was used.

By the multiple shooting method, the DAE model is parametrized in some sense too. The solution of the DAE system is decoupled on the N intervals $[t_l, t_{l+1}]$. In this way the initial values s_x^l and s_z^l of the differential and algebraic states at times t_i are introduced as the additional decision variables. The trajectories $x(t)$ and $z(t)$ are obtained as a set of trajectories $x^l(t)$ and $z^l(t)$ on each time interval $[t_{l-1}, t_l]$. The mentioned trajectories $x^l(t)$ and $z^l(t)$ are the solutions of an initial value problem

(10)

$$\begin{aligned} \mathcal{B}^l(\cdot)\dot{x}(t) =& \ \mathcal{F}_1^l\big(x^l(t), z^l(t), p\big) \\ 0 =& \ \mathcal{F}_2^l\big(x^l(t), z^l(t), p\big) + \alpha^l(t_l) g^l\big(s_x^l, s_z^l, p\big) \\ t \in& \ [t_{l-1}, t_l], \qquad l = 1, \cdots, N. \end{aligned}$$

The relaxation parameter $\alpha^l(t_l)$ was introduced to allow an efficient DAE solution for the initial values of state trajectories s_x^l, s_z^l, that may temporarily violate the consistency conditions [4]. In this way, the trajectories $x^l(t)$ and $z^l(t)$ on the interval $[t_{l-1}, t_l]$ are the functions of the initial values and parameters s_x^l, s_z^l and p.

The multiple shooting method is often known as the *parallel shooting method*. It means, that DAE system can be solved parallel for each time interval $[t_{l-1}, t_l]$.

The parametrization of the problem of solving differential-algebraic electrical circuit as the multistage

DAE system using the multiple shooting approach leads us to the following nonlinear programming problem (11)-(18)

$$\sum_{l=2}^{N} \|c_{cont}^{l}\|_2 + \sum_{l=1}^{N} \|c_{cons}^{l}\|_2 = \tag{11}$$
$$= \Psi(s_x^l, s_z^l, p) = \Psi(\chi) \to \min$$

where c_{cont}^{l} denotes violation of the continuity conditions

$$c_{cont}^{l} = s_x^l - x^{l-1}(t_{l-1}) = 0, \qquad l = 2, \cdots, N, \tag{12}$$

and c_{cons}^{l} denotes violation of the consistency conditions

$$c_{cons}^{l} = g^l(s_x^l, s_z^l, p) = 0, \qquad l = 1, \cdots, N. \tag{13}$$

The lower and upper bounds on the decision variables are present

$$\chi_L \leq \chi \leq \chi_U, \tag{14}$$

$$\chi = [s_x^1, ..., s_x^N, s_z^1, ..., s_z^N, p]^T, \tag{15}$$

$$\chi_L = [s_{x,L}^1, ..., s_{x,L}^N, s_{z,L}^1, ..., s_{z,L}^N, p_L]^T, \tag{16}$$

$$\chi_U = [s_{x,U}^1, ..., s_{x,U}^N, s_{z,U}^1, ..., s_{z,U}^N, p_U]^T \tag{17}$$

and the DAE system in each interval

(18)
$$\begin{aligned} \mathcal{B}^l(\cdot)\dot{x}(t) = {} & \mathcal{F}_1^l\big(x^l(t), z^l(t), p\big) \\ 0 = {} & \mathcal{F}_2^l\big(x^l(t), z^l(t), p\big) + \alpha^l(t_l) g^l(s_x^l, s_z^l, p), \\ t \in {} & [t_{l-1}, t_l], \qquad l = 1, \cdots, N. \end{aligned}$$

The presented approach, which is valid for the multistage systems with differential-algebraic constraints, can be applied for solving the interconnected electrical networks [2, 10]. It means, that the structure of the systems can be used for solving the system in an effective manner [15].

4. The "stretching" function

The problem of solving the electrical circuits with differential-algebraic relations can be transformed into the nonlinear optimization problem with only lower and upper bounds (11)-(18)

$$\Psi(\chi) \to \min . \tag{19}$$

$$\chi_L \leq \chi \leq \chi_U. \tag{20}$$

To avoid the stopping in the local solutions, which do not provide a better value of the objective function (11) than the currently obtained one, the "stretching" method [17] was applied in the proposed algorithm.

The "stretching" technique is one of the recently methods proposed the first time in [18]. "Stretching" relies of transforming the objective function in a such way, that the knowledge of previously detected local minimum is incorporated in its new form.

This technique consists of a two-phase transformation of the objective function, which was extended in the proposed approach to the three-step algorithm using the projection method.

First step

The first phase of the transformation makes all the local minima with values higher than the value of the obtained local minimizer χ^* disappear by "stretching" the objective function $\Psi(\chi)$ upwards.

Second step

The second stage stretches the neighborhood of χ^* and changes the detected minimum to a maximum.

Third step

In the last step, the projection $\mathcal{P}(\chi_i, \chi_{L_i}, \chi_{U_i})$ of the solution into a feasible region is applied

(21)
$$\mathcal{P}(\chi_i, \chi_{L_i}, \chi_{U_i}) = \begin{cases} \chi_{L_i} & \text{if} \quad \chi_i \leq \chi_{L_i}, \\ \chi_i & \text{if} \quad \chi_i \in (\chi_{L_i}, \chi_{U_i}), \\ \chi_{U_i} & \text{if} \quad \chi_i \geq \chi_{U_i}. \end{cases}$$

Assume χ^* to be already obtained minimizer so far of the objective function $\Psi(\chi)$. The "stretching" technique was defined as the following function transformation

(22)
$$\mathcal{G}(\chi) = \Psi(\chi) + \gamma_1 \|\chi - \chi^*\| \Big(\text{sign}(\Psi(\chi) - \Psi(\chi^*)) + 1\Big),$$

$$\mathcal{H}(\chi) = \mathcal{G}(\chi) + \frac{\gamma_2 \Big(\text{sign}(\Psi(\chi) - \Psi(\chi^*)) + 1\Big)}{2 \tanh \Big(\mu(\mathcal{G}(\chi) - \mathcal{G}(\chi^*))\Big)}, \tag{23}$$

where γ_1, γ_2 and μ are arbitrary chosen positive parameters, and sign$(\cdot)$ is the well known three-valued *sign* function.

The parameter γ_1 controls the upward stretching of the objective function through the transformation $\mathcal{G}(\chi)$ in eq. (22). The parameters γ_2 and μ determine the range of the effect and magnitude of the elevation, respectively.

The function $\mathcal{H}(\chi)$ in eq. (23) is called as the "stretching" function of the objective function $\Psi(\chi)$.

The main steps of the new hybrid algorithm for solving the electrical circuits with differential-algebraic constraints are the following:

The new hybrid algorithm

Step 1. Choose a start point χ_0. Set $k = 0$.
Step 2. Local search. Start from χ_k and use the quasi-Newton algorithm to search for local minimizer $\chi_{k^\star}$.
Step 3. Construct the function $\mathcal{H}(\chi)$ defined in (19), with $\chi^* = \chi_{k^\star}$, $\gamma_1 = 1e + 4$, $\gamma_2 = 1$ and $\mu = 1e - 8$.
Step 4. Global search. Use $\chi_{k^\star}$ as an initial point and execute simulated annealing algorithm on the function $\mathcal{H}(\chi)$ until a point $\chi^*_{(k+1)}$ is obtained.

The phase of the local search provides the locally optimal solution, which do not necessary satisfies all the constraints. Using this solution as the start point in global search, one can obtain such a solution, which has a lower value of the objective function $\Psi(\chi)$. It means, that the constraints can be slightly violated. When the quasi-Newton algorithm, starting from the point given by global search procedure, converges to the previously obtained solution, then it provides a basis for stopping the algorithm.

5. Application in the ring modulator

The algorithm proposed in the previous section was applied for solving the highly nonlinear system of the ring modulator, which in detail was presented in [13]. In the presented example, the computations were performed without any other transformations, like presented in [20].

The simulations were executed in Matlab environment using Wrocław Centre for Networking and Supercomputing.

The ring modulator mixes a low frequency signal $e_1(t)$ with a high frequency signal $e_2(t)$. The modulated signal is then used as an input for an amplifier.

The system of differential equations (obtained by the potentional function method) is given by

(24) $$C\dot{U}_1 = I_1 - I_3 \cdot 0.5 + I_4 \cdot 0.5 + I_7 - U_1/R,$$

(25) $$C\dot{U}_2 = I_2 - I_5 \cdot 0.5 + I_6 \cdot 0.5 + I_8 - U_2/R,$$

(26) $$C_S\dot{U}_3 = I_3 - \mathcal{D}(U_{D_1}) + \mathcal{D}(U_{D_4}),$$

(27) $$C_S\dot{U}_4 = -I_4 + \mathcal{D}(U_{D_2}) - \mathcal{D}(U_{D_3}),$$

(28) $$C_S\dot{U}_5 = I_5 + \mathcal{D}(U_{D_1}) - \mathcal{D}(U_{D_3}),$$

(29) $$C_S\dot{U}_6 = -I_6 - \mathcal{D}(U_{D_2}) + \mathcal{D}(U_{D_4}),$$

(30) $$C_P\dot{U}_7 = -U_7/R_i + \mathcal{D}(U_{D_1}) + \mathcal{D}(U_{D_2}) - \mathcal{D}(U_{D_3}) - \mathcal{D}(U_{D_4}),$$

(31) $$L_h\dot{I}_1 = -U_1,$$

(32) $$L_h\dot{I}_2 = -U_2,$$

(33) $$L_{S2}\dot{I}_3 = U_1 \cdot 0.5 - U_3 - R_{g2} \cdot I_3,$$

(34) $$L_{S3}\dot{I}_4 = -U_1 \cdot 0.5 + U_4 - R_{g3} \cdot I_4,$$

(35) $$L_{S2}\dot{I}_5 = U_2 \cdot 0.5 - U_5 - R_{g2} \cdot I_5,$$

(36) $$L_{S3}\dot{I}_6 = -U_2 \cdot 0.5 + U_6 - R_{g3} \cdot I_6,$$

(37) $$L_{S1}\dot{I}_7 = -U_1 + e_1(t) - (R_0 + R_{g1}) \cdot I_7,$$

(38) $$L_{S1}\dot{I}_8 = -U_2 - (R_a + R_{g1}) \cdot I_8.$$

The characteristic of the diodes is fitted by

(39) $$\mathcal{D}(U_D) = 40.67286402 \cdot 10^{-9}(e^{17.7493332 \cdot U_D} - 1),$$

the voltage at the different diodes are

(40) $$U_{D_1} = U_3 - U_5 - U_7 - e_2(t),$$

(41) $$U_{D_2} = -U_4 + U_6 - U_7 - e_2(t),$$

(42) $$U_{D_3} = U_4 + U_5 + U_7 + e_2(t),$$

(43) $$U_{D_4} = -U_3 - U_6 + U_7 + e_2(t),$$

the technical parameter are given by

(44)
$$\begin{array}{ll} R_{g1} = 36.3[\Omega], & R_{g2} = R_{g3} = 17.3[\Omega], \\ R_0 = R_i = 50[\Omega], & R_a = 600[\Omega] \\ R = 25000[\Omega], & \\ C = 16 \cdot 10^{-9}[F], & C_P = 10 \cdot 10^{-9}[F], \\ L_h = 4.45[H], & L_{S1} = 0.002[H], \\ L_{S2} = L_{S3} = 0.0005[H], & \end{array}$$

and the two entry signals are

(45) $$e_1(t) = 0.5 \cdot \sin(2\pi 10^3 t)[V],$$

(46) $$e_2(t) = 2 \cdot \sin(2\pi 10^4 t)[V].$$

Initial values for the problem are

(47) $$U_i(0) = 0[V], \qquad i = 1, \cdots, 7,$$

(48) $$I_i(0) = 0[A], \qquad i = 1, \cdots, 8.$$

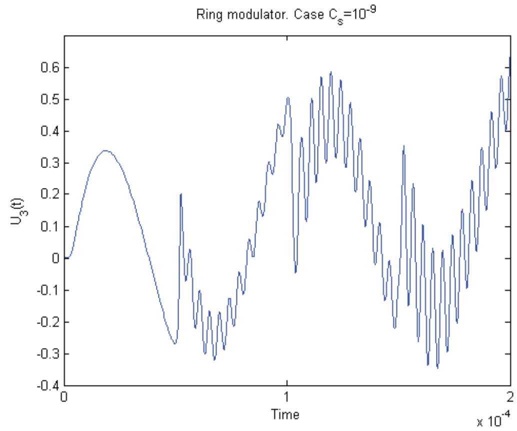

Fig. 1. Trajectory of $U_3(t)$. Case $C_S = 10^{-9}$.

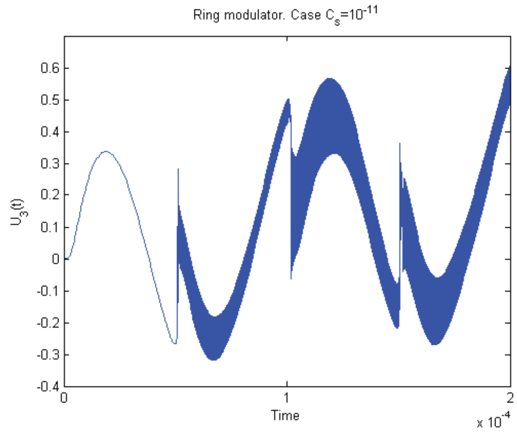

Fig. 2. Trajectory of $U_3(t)$. Case $C_S = 10^{-11}$.

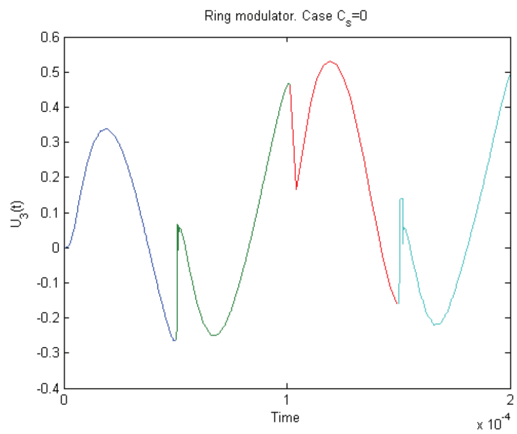

Fig. 3. Trajectory of $U_3(t)$. Case $C_S = 0$.

Depending on the value of the capacity C_S, problem of solving ordinary differential equations or differential-algebraic systems is to be considered. In some practical applications, the value of C_S is equal to 10^{-9} or smaller. One of the approach in solving systems of stiff ordinary differential equations, is to consider the systems of differential-algebraic equations (DAEs). The DAE systems can be solved effectively using existing numerical procedures.

Let us to consider the case, when $C_S = 10^{-9}$. The trajectory of $U_3(t)$ over the time interval $[0, 2 \cdot 10^{-4}]$ was plotted on the Fig. 1. The trajectory of $U_3(t)$, when $C_S = 10^{-11}$, was presented on the Fig. 2. This function shows well the numerical difficulties of the problem. One can observe high oscillations, whose frequency increases when value of the C_S decreases. In the differential-algebraic case, $C_S = 0$, the oscillations disappear.

The direct shooting approach was used to transform optimal control problem to the nonlinear optimization problem with the both continuous and pointwise constraints. The resulted finite-dimensional optimization problem was solved using presented hybrid technique with the "stretching" function. The simulations were executed with initial values of all decision variables equal to zero.

The duration of the process t_f was divided into 4 equidistant intervals. There are the time instants, when the DAE solver notified the critical points and could not solve the system forward in the time.

To solve the problem, the multistage representation of the ring modulator was considered. As a result, there are two vectors $|c_{cont}|$ and $|c_{cons}|$, which denotes violation of the continuity conditions and violation of the consistency conditions, respectively.

The trajectory of $U_3(t)$ with $C_S = 0$ can be observed on the Fig. 3.

The new hybrid algorithm enables us to progress toward the optimal global solution, because it uses the main feature of the simulated annealing approach - escaping from the local minima. The "stretching" function ensures vanishing of the other local optimum, while it preserves the global one.

6. Conclusion

In the article the problem of solving complex electrical circuits was considered. In differential-algebraic models the variables keep their original physical interpretation, and such implicit models do not require any other simplifications, often necessary to get an purely dynamical system. To solve electrical systems described by differential-algebraic equations, a new hybrid approach, based on the direct shooting method and "stretching" function technique was presented.

The presented method can be used to carry out the circuits simulation in a computationally efficient manner. Quasi-Newton method was applied to obtain a local optimal solution in computationally efficient manner. Simulated annealing algorithm explore promising areas and globalize the obtained solution. To eliminate the local minima, while preserving the global ones, a three-step transformation of the objective function called as the "stretching" technique was applied.

Because the direct shooting method was used, the time interval is to be divided into smaller subintervals. The resulted multistage DAE system can be solved with higher accuracy and parallel using multicore processors. This approach can significantly reduce the computation time [21].

The presented method enables us to solve the DAE systems without known consistent initial conditions and was used to solve the highly nonlinear DAE model of the ring modulator system.

Future research will concern on new decomposition methods of nonlinear optimization problem with differential-algebraic constraints to solve large-scale electrical systems in an effectively manner [6].

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