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Time Domain Simulation of Optical MIMO Channels

Abstract. *In recent years the interest in optical multiple-input multiple-output (MIMO) transmission has increased significantly. Focus of this work is the development of a time domain model of optical MIMO channels using modal diversity. Next to modal and chromatic dispersion, modal crosstalk caused by mode combiner, splitter, splices and also modal coupling within the transmitting fiber can be taken into account. The simulated MIMO impulse responses are validated by practical measurements. The channel measurements confirm the accuracy of the proposed time domain model.*

Streszczenie. *W artykule omawia się optyczny system transmisji typu MIMO (multiple input – multiple output). Zaproponowano model czasowy optycznego kanału MIMO. Wzięto pod uwagę modalną i chromatyczną dyspersję, modalny crosstalk (przesłuch) oraz modalne sprzężenie włókna optycznego.* ***(Czasowa symulacja optycznego kanału typu MIMO)***

Keywords: Multiple-Input Multiple-Output System, Time Domain Simulation, Optical Fiber Transmission, Mode Coupling, Multimode Fiber
Słowa kluczowe: system MIMO (multiple input – multiple oiutput), optyczna transmisja danych

Introduction

Optical multiple-input multiple-output (MIMO) has shown its capability for high-speed data transmission [1]. However, many technological obstacles appear when it comes to practical implementation. This involves mode combining at the transmitter side and mode splitting at the receiver side, in particular. By using fusion or other couplers it is possible to realize a MIMO transmission system over multi-mode fibers (MMFs) as shown in [1, 2]. For a rigorous performance assessment of an optical MIMO system a model simulating the respective impulse responses is essential. However, such a model must also include mode coupling properties of couplers, connectors, splices and the fiber itself. Previously published works have shown the effects of mode coupling within the transmitting fiber and due to splices [3]. So far the impact of couplers and mode excitation has not been studied. Addressing this gap this contribution introduces a time-domain MIMO simulation model, which considers modal crosstalk.

The novelty of our work is a simulation model that takes all the components of the transmission chain such as mode excitation, mode combining, connectors and splices, the transmitting fiber and the mode splitter into account. Additionally the effect of the chromatic dispersion is analyzed. The proposed time-domain model is verified by practical measurements carried out in an optical MIMO testbed. The measured MIMO specific channel impulse responses confirm the proposed theory for setting up a time domain MIMO simulation model.

The remaining part of this paper is structured as follows: first the system model approach is presented. In the subsequent chapter mathematical investigations are made into excitation, combining and splitting of modes. Afterwards the properties of a MMF are briefly reviewed to obtain the corresponding impulse responses. Simulation results of this derived model are then compared to real channel measurements in the testbed. The contribution closes with a brief conclusion.

System Model Approach

In the following a MIMO simulation model is derived, which is able to consider optical components resulting in modal crosstalk like splices, connectors, mode couplers and splitters as they are used in a real MIMO testbed. Fig. 1 shows the testbed setup consisting of a two channel pattern generator driving the electro-optical (E/O) converters. They are followed by mode converters realized by centric and eccentric single-mode fiber (SMF) to MMF splices. The centric splice excites mainly the fundamental mode or low order mode groups (LOM) whereas the eccentric splice activates higher order mode groups (HOM). These are subsequently combined in a fusion coupler and transmitted over a $1.9\mathrm{km}$ long standard gradient index (GI) MMF. The fiber consists of two sections linked by a simple fiber connector. The modes are separated at the receiver side by another fusion coupler. Wideband opto-electrical (O/E) converters are used for detecting the MIMO-specific receive signals, which are finally captured by a high-speed sampling oscilloscope for further off-line signal processing.

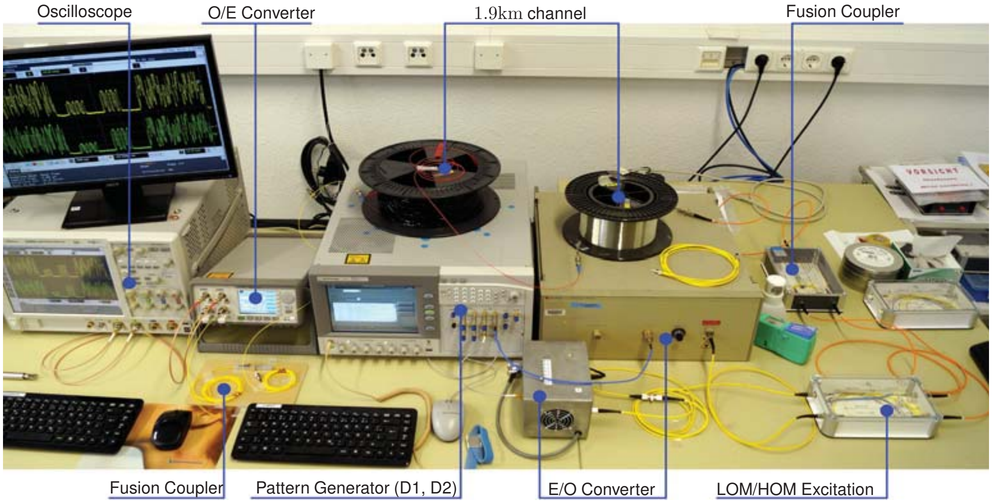


Fig. 1. Testbed for optical MIMO transmission

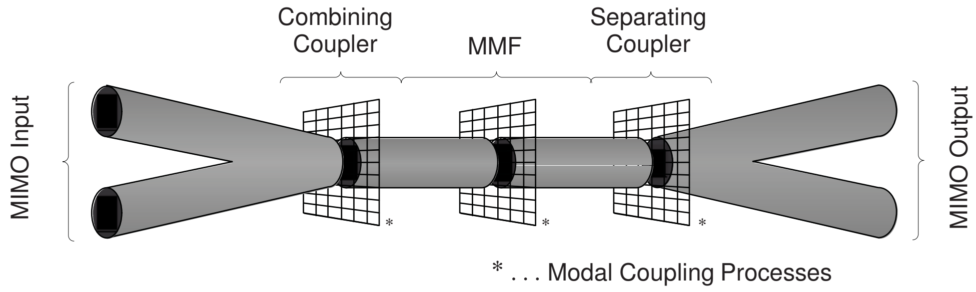


Fig. 2. Scheme of the proposed system model approach

In order to study the impact of the different components on the MIMO transmission channels a detailed simulation model is essential. Fig. 2 shows a simplified scheme of our system model. Next to modal and chromatic dispersion our system model considers also the effect of modal crosstalk within the mode combiner and splitter as well as the effect of splices and modal coupling within the transmitting fiber. Fig. 3 illustrates the corresponding flowchart. The main operation steps are the calculation of all the mode field distributions, the respective power coupling coefficients at fiber-to-fiber-connections with off-sets, the respective delays of the modes and their chromatic dispersion.

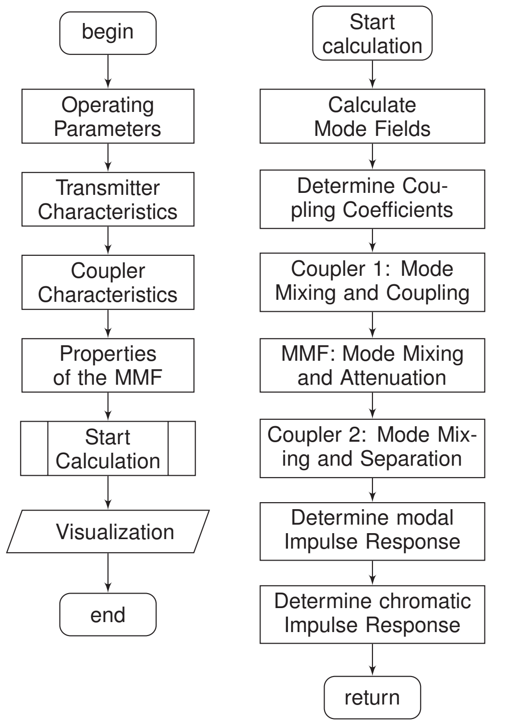


Fig. 3. Flowchart of the simulation

Mode Excitation, Combining and Splitting

At every section i of the optical component transmission chain power transfers between the different LP$_{mp}$-modes occur which can be described by the coefficients $k^{(i)}_{m'p'\to mp}$ and have to be taken into account subsequently. In case pure radial misalignment (e.g., off-set launching, connectors or splices) these coefficients can analytically be determined. At the end of the first MIMO testbed section the excitation of different modes is accomplished by splicing a SMF to MMF with a radial off-set $\vec{\xi}$ as illustrated in Fig. 4. Thereby the vector $\vec{\xi}$ contains the radial and angular off-set. Center launch and off-set launch conditions are applied to form optical channels.

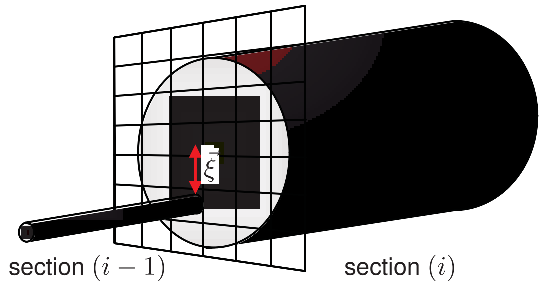


Fig. 4. Principle of mode excitation using a SMF to MMF splice

In general, determining the respective power portions $k^{(i)}_{m'p'\to mp}$ (for $m = 0 \dots m^{(i)}_{\max}$ and $p = 0 \dots p^{(i)}_{\max}$) of each LP$_{mp}$-mode after excitation, the overlapping area ratios of each modal field distribution $\psi_{mp}(r,\varphi)$ have to be calculated,

(1)
$$k^{(i)}_{m'p'\to mp} = \left[\iint_r \int_\varphi \psi^{(i-1)}_{m'p'}(r,\varphi,\vec{\xi})\psi^{(i)}_{mp}(r,\varphi)drd\varphi \right]^2 ,$$

where i indicates the respective MMF section, with its core centered cylindrical coordinates. Hence, the modal field distribution of the previous fiber shows an off-set of $\vec{\xi}$. In general, the modal field distribution is specified for GI-MMFs as given in [4].

(2)
$$\psi_{mp}(r,\varphi) = \left(\sqrt{2}\frac{r}{\omega_0}\right)^m L^{(m)}_{p-1}\left(\frac{2r^2}{\omega_0^2}\right) e^{-\frac{r^2}{\omega_0^2}} \cdot \begin{cases} \cos(m\varphi) \\ \sin(m\varphi) \end{cases}$$

The function $L^{(m)}_{p-1}(\gamma)$ represents the Laguerre polynomial of the order m and the degree $p-1$, which is defined as

$$L^{(m)}_{p-1}(\gamma) = \sum_{\nu=0}^{p-1} \binom{p+m-1}{p-\nu-1} \frac{(-\gamma)^\nu}{\nu!} , \tag{3}$$

and the parameter ω_0 characterizes the mode field radius of the fundamental mode, which is calculated from the normalised frequency V and the MMF core radius a.

$$\omega_0 = a\sqrt{\frac{2}{V}} \tag{4}$$

In order to find the maxima of p and m of the guided modes the Eigenvalue equation [4, p. 69]

$$\frac{V \cdot d}{4} - \frac{m}{2} = p - \frac{1}{2} \tag{5}$$

of a GI-MMF has to be solved, where the normalised phase coefficient d is set to $d = 1$ since only guided modes are considered. The maximum mode indices are found by applying the conditions $m = 0$ and $p = 1$ alternately to (5):

$$p_{\max} = \frac{V+2}{4} \tag{6}$$

$$m_{\max} = \frac{V-2}{2} \; . \tag{7}$$

After excitation of the different modes or mode groups, the channels are combined by a fusion coupler forming nearly independent channels traveling together in a MMF. After each fiber section similar analyses to (1)–(7) have to be applied.

Unfortunately, there is no straightforward analytical solution for calculating the individual mode coupling coefficients of real MMF couplers. The coefficients have to be determined empirically from testbed measurements. The same applies to the mode splitter at the receiver side.

Properties of the MMF

Depending on the quality of a MMF, mode coupling effects can also be observed along a fiber span. Particularely high order modes may couple power to leaky modes and thus become more lossy. The simulation model may also take care of this effect as outlined. However, since an OM4 fiber is used in the testbed the cross coupling coefficients are set to zero. Far more important are the material and the modal dispersion of the MMF.

In order to calculate the material dispersion, the refractive index n_{c} of the fiber core is usually described by an adequate Sellmeier equation [5, p. 103] and subsequently differentiated as given in [4, p. 54].

$$\Delta t_{\mathrm{gc}} \approx -\frac{\lambda_0}{c}\frac{d^2 n_{\mathrm{c}}}{d\lambda_0^2} \cdot \Delta\lambda_0 \cdot L \tag{8}$$

The parameter Δt_{gc} describes the signal full width at half maximum (FWHM) time spread, c the speed of light in vacuum, λ_0 the operating wavelength, $\Delta\lambda_0$ the FWHM spectral width of the light source and L the length of MMF.

The radiation spectrum of a light source is generally approximated by a Gaussian distribution. Thus, the signal FWHM time spread is transferred to the rms time spread

$$\begin{aligned}\sigma_{\mathrm{c}} &= \frac{\Delta t_{\mathrm{gc}}}{2\sqrt{2\ln 2}} \\ &\approx 0.42\Delta t_{\mathrm{gc}}\end{aligned} \tag{9}$$

and the impulse response of the material dispersion can be described as

$$g_{\mathrm{c}}(t) = \frac{1}{\sigma_{\mathrm{c}}\sqrt{2\pi}} e^{\left(\frac{-t^2}{2\sigma_{\mathrm{c}}^2}\right)} \; . \tag{10}$$

This material dispersion is independent of the modal dispersion. Therefore it must be applied to all modes of the MMF channel.

To determine the modal dispersion, all LP$_{mp}$-modes which have a similar propagation speed can be grouped into principal modes numbered by κ.

$$\kappa = m + 2p - 1 \tag{11}$$

The total number of principal modes can be calculated as follows

$$M = \sqrt{\frac{V^2}{4}} \; . \tag{12}$$

Subsequently, the group time delay τ_κ of each principal mode κ can be calculated according to [4, p. 84] and [6]:

$$\tau_\kappa = t_{\mathrm{gAx}} \frac{1 - 2\Delta \cdot \left(\frac{\kappa}{M}\right)^{2g/(2+g)} \cdot \frac{2-P(\lambda_0)}{2+g}}{\sqrt{1 - 2\Delta \cdot \left(\frac{\kappa}{M}\right)^{2g/(2+g)}}} \quad 1 \leqslant \kappa \leqslant M \; , \tag{13}$$

where Δ is the normalised refractive index difference, g describes the profile exponent, t_{gAx} is the time delay in axial direction and $P(\lambda_0)$ the profile dispersion parameter which are described in detail in [4].

Determination of the Impulse Response

The channel impulse response follows from applying the flowchart in Fig. 3 for each fiber section as presented. To follow up the flowchart processing steps and to rule out measurement uncertainties a simple example with a SMF exciting the respective modes of a $1.9\mathrm{km}$ MMF is considered as a proof of concept.

Using different SMF-off-sets $\vec{\xi}$ all the power coupling coefficients with respect to the modes carried by the MMF can be calculated according to (1)–(7). These weighted modes travelling down the MMF span are subject to different delays τ_{mp}. The delays can be calculated by using (11)–(13). Now, the impulse response caused by modal dispersion follows as

$$g_{\mathrm{m}}^{(i)}(t) = \sum_{m=0}^{m_{\max}} \sum_{p=1}^{p_{\max}} \left(\mathrm{P}_{01} \cdot k_{01\to mp}^{(i)} \cdot \delta(t - \tau_{mp})\right) \tag{14}$$

with the power P_{01} of the LP$_{01}$-mode carried by the SMF. This impulse response represents a chain of weighted Dirac pulses with different delays τ_{mp}, where $\tau_{mp} = \tau_\kappa$ with group affiliation followed by (11). Subsequent determination of the consecutive $g_{\mathrm{m}}^{(i)}(t)$ then yields the overall impulse response due to modal dispersion $g_{\mathrm{m}}(t)$.

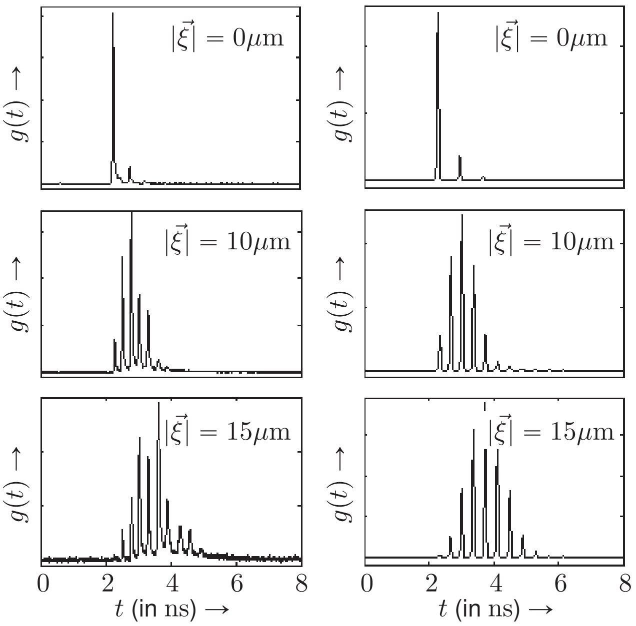
Finally, the chromatic dispersion described by (10) must be taken into account. Since all modes are subject to this dispersion parameter the final impulse response results as

$$g(t) = g_{\mathrm{m}}(t) * g_{\mathrm{c}}(t) \; . \tag{15}$$

In a $(M \times N)$-MIMO system the individual impulse responses $g_{\nu\mu}(t)$ with $\nu = 1 \ldots N$ and $\mu = 1 \ldots M$ are derived by applying the respective launching conditions $\vec{\xi}$ and the coupling coefficients of the corresponding combiner and splitter ports

Simulation Results

Impulse response measurements were carried out in a testbed with a $1.9\mathrm{km}$ OM4-fiber using an operation wavelength of $\lambda_0 = 1327\mathrm{nm}$ to confirm the simulation results. Fig. 5a illustrates the measured impulse responses $g(t)$ for different launching conditions $\vec{\xi}$ whereas Fig. 5b shows the corresponding simulated channels in comparison. From top to bottom the curves depict centered and off-set conditions of $|\vec{\xi}| = 0\mu\mathrm{m}$, $|\vec{\xi}| = 10\mu\mathrm{m}$ and $|\vec{\xi}| = 15\mu\mathrm{m}$, respectively. As can be seen clearly center launch condition excites almost only the LP$_{01}$-mode. With increasing eccentricity the order of modes and their respective delay increase as well. By comparing Fig. 5a and Fig. 5b the proposed simulation model points out a good approximation of the measured channels. However, the minor differences between the measurement and simulation results indicate that small modal crosstalk occurs even in the used OM4 fiber.



(a) measured (b) simulated

Fig. 5. Impulse Responses with centered and offste launching conditions

Conclusion

In this contribution a simulation package has been described which is capable of calculating the impulse responses of an optical MIMO system. Besides an adequate description of the MMF parameters leading to the respective chromatic and modal dispersion, consideration of impacts of components like couplers, connectors and splices are outlined. The mode excitation is analyzed in detail. Furthermore, three simulated impulse responses have been compared to measurements of real testbed channels as a proof of concept. For all the different launching conditions the simulated impulse responses are in good agreement to the corresponding measured optical channels. Hence, the presented time domain model forms the base for further optical MIMO transmission studies on the system level.

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