

Modulation-Mode and Power Assignment in SVD-Assisted Broadband MIMO Systems using Polynomial Matrix Factorization

Abstract. Singular-value decomposition (SVD)-based multiple-input multiple-output (MIMO) systems have attracted a lot of attention in the wireless community. However, applying SVD to frequency-selective MIMO channels results in unequally weighted single-input single-output (SISO) channels requiring complex resource allocation techniques for optimizing the channel performance. Therefore, a different approach utilizing polynomial matrix factorization for removing the MIMO interference is analyzed, outperforming conventional SVD-based MIMO systems in the analyzed channel scenario.

Streszczenie. Analizowano właściwości układu SVD (single value decomposition) w technologii MIMO w bezprzewodowym przesyłaniu informacji. Tego typu układy mają problemy z alokacją kanałów. Dlatego zaproponowano inny system wykorzystujący rozkład macierzy wielomianowej do usuwania interferencji kanałów. (Wykorzystanie rozkładu macierzy wielomianowej w systemie przesyłu informacji MIMO wspomaganego układem SVD)

Keywords: Multiple-Input Multiple-Output, Singular-Value Decomposition, Polynomial Matrix Factorization, Wireless Transmission.

Słowa kluczowe: system MIMO, bezprzewodowe przesyłanie informacji, układ SVD

Introduction

The strategy of placing multiple antennas at the transmitter and receiver sides, well-known as multiple-input multiple-output (MIMO) system, improves the performance of wireless systems by the use of the spatial characteristics of the channel. MIMO systems have become the subject of intensive research over the past 20 years as MIMO is able to support higher data rates and shows a higher reliability than single-input single-output (SISO) systems [1, 2].

Singular-value decomposition (SVD) is well-established in MIMO signal processing where the whole MIMO channel is transferred into a number of weighted SISO channels. The unequal weighting of the SISO channels has led to intensive research to reduce the complexity of the required bit and power allocation techniques [2, 3, 4]. The polynomial matrix singular-value decomposition (PMSVD) is a signal processing technique which decomposes the MIMO channel into a number of independent frequency-selective SISO channels so called layers [1]. The remaining layer-specific interferences as a result of the PMSVD-based signal processing can be easily removed by further signal processing such as zero-forcing equalization as demonstrated in this work.

The novelty of our contribution is that we demonstrate the benefits of amalgamating a suitable choice of MIMO layers activation and number of bits per layer along with the appropriate allocation of the transmit power under the constraint of a given fixed data throughput. Here, bit- and power-loading in both SVD- and PMSVD-based MIMO transmission systems are elaborated. Assuming a fixed data rate, which is required in many applications (e.g., real time video applications), a two stage optimization process is proposed. Firstly, the allocation of bits to the number of SISO channels is optimized and secondly, the allocation of the available total transmit power is studied when minimizing the overall bit-error rate (BER) at a fixed data rate.

Our results, obtained by computer simulation, show that PMSVD could be an alternative signal processing approach compared to conventional SVD-based MIMO approaches in frequency-selective MIMO channels.

State of the Art

A frequency selective MIMO link, composed of n_T transmit and n_R receive antennas is given by

$$(1) \quad \mathbf{u} = \mathbf{H} \cdot \mathbf{c} + \mathbf{n} .$$

In (1), \mathbf{c} is the $(N_T \times 1)$ transmit data signal vector containing the complex input symbols transmitted over n_T transmit

antennas in K consecutive time slots, i.e., $N_T = K n_T$. The vector \mathbf{u} describes the $(N_R \times 1)$ receive signal vector of length $N_R = (K + L_c) n_R$ [3]. The number of non-zero elements of the resulting symbol rate sampled overall channel impulse response between the μ th transmit and ν th receive antenna is given by $(L_c + 1)$. Finally, the $(N_R \times 1)$ vector \mathbf{n} in (1) describes the noise term. The $(N_R \times N_T)$ system matrix \mathbf{H} of the block-oriented system model, introduced in (1), results in

$$(2) \quad \mathbf{H} = \begin{bmatrix} \mathbf{H}_{11} & \cdots & \mathbf{H}_{1n_T} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{n_R 1} & \cdots & \mathbf{H}_{n_R n_T} \end{bmatrix}$$

and consists of $n_R n_T$ SISO channel matrices $\mathbf{H}_{\nu\mu}$ (with $\nu = 1, \dots, n_R$ and $\mu = 1, \dots, n_T$). The system description, called spatio-temporal vector coding, was introduced by Raleigh [5]. Each of these matrices $\mathbf{H}_{\nu\mu}$ with the dimension $((K + L_c) \times K)$ describes the influence of the channel from transmit antenna μ to receive antenna ν including transmit and receive filtering. The removal of the interferences between the different antenna's data streams, which are introduced by the non-zero off-diagonal elements of the channel matrix \mathbf{H} , requires appropriate signal processing strategies. SVD can be considered as a promising solution for transferring the whole MIMO system into a system with non-interfering weighted additive white Gaussian noise (AWGN) channels.

Using SVD the system matrix \mathbf{H} can be written as $\mathbf{H} = \mathbf{S} \cdot \mathbf{V} \cdot \mathbf{D}^H$, where \mathbf{S} and \mathbf{D}^H are unitary matrices and \mathbf{V} is a real-valued diagonal matrix of the positive square roots of the eigenvalues of the matrix $\mathbf{H}^H \mathbf{H}$ sorted in descending order. The conjugate transpose (Hermitian) of \mathbf{D} is denoted by \mathbf{D}^H . For removing the interferences, the MIMO data vector \mathbf{c} is now multiplied by the matrix \mathbf{D} before transmission. In turn, the receiver multiplies the received vector \mathbf{u} by the matrix \mathbf{S}^H . Thereby neither the transmit power nor the noise power is enhanced. The overall transmission relationship is defined as

$$(3) \quad \mathbf{y} = \mathbf{S}^H (\mathbf{H} \cdot \mathbf{D} \cdot \mathbf{c} + \mathbf{n}) = \mathbf{V} \cdot \mathbf{c} + \mathbf{w} .$$

As a consequence, the channel matrix \mathbf{H} is transformed into independent, non-interfering layers having unequal gains [3]. With the proposed system structure, the SVD-based equalization leads to different number of MIMO layers ℓ (with $\ell = 1, 2, \dots, L$) at the time k (with $k = 1, 2, \dots, K$) as

shown in Fig. 1. Here it is worth noting that the number of parallel transmission layers L at the time-slot k is limited by $\min(n_T, n_R)$. The complex-valued data symbol $c_{\ell,k}$ to be transmitted over the layer ℓ at the time k is now weighted by the corresponding positive real-valued singular-value $\sqrt{\xi_{\ell,k}}$ and further disturbed by the additive noise term $w_{\ell,k}$.

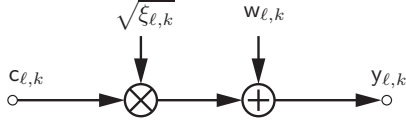


Fig. 1. Resulting layer-specific SVD-based broadband MIMO system model (with $\ell = 1, 2, \dots, L$ and $k = 1, 2, \dots, K$)

Polynomial Matrix Factorization

In contrast to the spatio-temporal vector coding, the polynomial matrix factorization exploits a description of the channel impulse responses in the z -domain. Thus, each frequency-selective channel impulse response $h_{\nu\mu}(k)$ of a $(n_R \times n_T)$ broadband MIMO system is given by

$$(4) \quad \underline{h}_{\nu\mu}(z) = \sum_{k=0}^{L_c} h_{\nu\mu}[k] z^{-k},$$

where the underscore denotes a polynomial and z^{-k} is the unit delay operator. Consecutively, the broadband MIMO channel is formed by grouping these impulse responses into the channel matrix as follows

$$(5) \quad \underline{\mathbf{H}}(z) = \begin{bmatrix} \underline{h}_{11}(z) & \cdots & \underline{h}_{1n_T}(z) \\ \vdots & \ddots & \vdots \\ \underline{h}_{n_R1}(z) & \cdots & \underline{h}_{n_Rn_T}(z) \end{bmatrix},$$

with $\underline{\mathbf{H}}(z) \in \mathbb{C}^{n_R \times n_T}$ being the MIMO channel matrix in polynomial notation. Using this polynomial description in the z -domain the MIMO system is described by

$$(6) \quad \underline{\mathbf{u}}(z) = \underline{\mathbf{H}}(z) \underline{\mathbf{c}}(z) + \underline{\mathbf{n}}(z),$$

where $\underline{\mathbf{c}}(z)$ is the $(n_T \times 1)$ transmit signal vector, $\underline{\mathbf{u}}(z)$ is the $(n_R \times 1)$ receive signal vector and $\underline{\mathbf{n}}(z)$ describes the $(n_R \times 1)$ AWGN vector in polynomial notation.

The polynomial channel matrix $\underline{\mathbf{H}}(z)$ can be orthogonalized calculating the polynomial matrix singular-value decomposition (PMSVD) with the help of the second-order sequential best rotation (SBR2) algorithm as presented in [1]. The decomposition of the polynomial channel matrix results in $\underline{\mathbf{H}}(z) = \underline{\mathbf{S}}(z) \underline{\mathbf{V}}(z) \underline{\mathbf{D}}(z)$, where (\cdot) denotes the para-conjugate operator. The matrices $\underline{\mathbf{S}}(z) \in \mathbb{C}^{n_R \times n_R}$ and $\underline{\mathbf{D}}(z) \in \mathbb{C}^{n_T \times n_T}$ are para-unitary matrices and $\underline{\mathbf{V}}(z) \in \mathbb{C}^{n_R \times n_T}$ is assumed as a diagonal matrix, because the off-diagonal elements are negligibly small when the SBR2 algorithm is set up accordingly [1]. The diagonal matrix for $n_T = n_R$ has the following form

$$(7) \quad \underline{\mathbf{V}}(z) = \begin{bmatrix} \underline{v}_1(z) & 0 & \cdots & 0 \\ 0 & \underline{v}_2(z) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \underline{v}_L(z) \end{bmatrix},$$

where the diagonal polynomial elements are given by $\underline{v}_\ell(z) = \sum_{k=0}^{L_v} v_{\ell,k} z^{-k}$ and consist of $(L_v + 1)$ non-zero elements. In contrast to the singular values $\sqrt{\xi_{\ell,k}}$ using SVD, the polynomial coefficients of $\underline{v}_\ell(z)$ are complex. For removing the interference signal pre-processing at the transmitter

and post-processing at the receiver are applied as shown in (3). Therefore, the transmit data vector $\underline{\mathbf{c}}(z)$ is multiplied by $\underline{\mathbf{D}}(z)$ and the receive vector $\underline{\mathbf{u}}(z)$ is multiplied by $\underline{\tilde{\mathbf{S}}}(z)$ resulting in the orthogonalized system

$$(8) \quad \underline{\mathbf{y}}(z) = \underline{\mathbf{V}}(z) \underline{\mathbf{c}}(z) + \underline{\mathbf{w}}(z).$$

The layer-based discrete-time description with the layer index $\ell = 1, 2, \dots, \min(n_T, n_R)$ is expressed as

$$(9) \quad y_\ell(k) = v_\ell(k) * c_\ell(k) + w_\ell(k),$$

where $*$ denotes discrete convolution. The layer-specific model is depicted in Fig. 2. Here in each layer the input symbols $c_\ell(k)$ are influenced by a finite impulse response

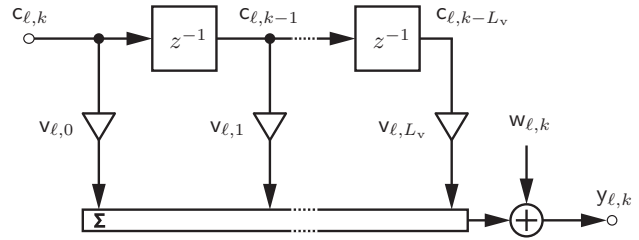


Fig. 2. Layer-specific PMSVD-based broadband MIMO system model assuming $(L_v + 1)$ non-zero coefficients of the layer-specific impulse response

filter $v_\ell(k) = (v_{\ell,0}, v_{\ell,1}, \dots, v_{\ell,L_v})$ and hence intersymbol interference (ISI) occurs on each layer. In order to fully remove the ISI a layer-specific T-spaced Zero Forcing equalizer $f_\ell(k)$ is applied to the received signal $y_\ell(k)$. Thus the equalized receive signal results in

$$(10) \quad z_\ell(k) = y_\ell(k) * f_\ell(k) = c_\ell(k) + w_\ell(k) * f_\ell(k).$$

The corresponding layer-specific ISI free system model is shown in Fig. 3 where the transmitted symbols $c_\ell(k)$ are received unchanged and the noise $w_\ell(k)$ is weighted by the equalizer coefficients $f_\ell(k)$. The PMSVD-based broadband MIMO system model with layer-specific T-spaced equalization is henceforth referred to as T-PMSVD system model.

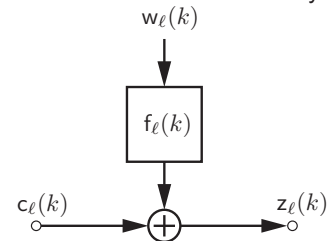


Fig. 3. ISI free layer-specific T-PMSVD-based broadband MIMO system model

Transmission Quality Criterion

In general, the quality criterion for transmission systems can be expressed by using the signal to noise ratio (SNR) at the detector input as follows

$$(11) \quad \rho = \frac{(\text{half vertical eye opening})^2}{\text{noise power}} = \frac{(U_A)^2}{P_R},$$

where U_A and P_R correspond to one quadrature component. Considering a layer-based MIMO system with a given SNR $\rho^{(\ell,k)}$ for each layer ℓ and time k and a M -ary quadrature amplitude modulation (QAM), the bit-error rate (BER) probability is given in [6] by

$$(12) \quad P_{\text{BER}}^{(\ell,k)} = \frac{2}{\log_2 M_\ell} \left(1 - \frac{1}{\sqrt{M_\ell}} \right) \text{erfc} \left(\sqrt{\frac{\rho^{(\ell,k)}}{2}} \right).$$

This BER is averaged at each time slot over all activated layers taking different modulation sizes at each layer into account and results in

$$(13) \quad P_{\text{BER}}^{(k)} = \frac{1}{\sum_{\ell=1}^L \log_2 M_\ell} \sum_{\ell=1}^L \log_2(M_\ell) P_{\text{BER}}^{(\ell,k)} .$$

In order to obtain the total average BER of one data block consisting of K transmitted symbols the time slot dependent BER is averaged as follows

$$(14) \quad P_{\text{BER,total}} = E \left\{ P_{\text{BER}}^{(k)} \right\} \quad \forall k ,$$

where $E\{\cdot\}$ denotes the expectation functional. For QAM modulated signals the average transmit power per layer can be expressed as

$$(15) \quad P_{s,\ell} = \frac{2}{3} U_{s,\ell}^2 (M_\ell - 1) .$$

Intuitively the total available transmit power P_s is equally split between the L activated layers and hence the layer-specific transmit power is given by $P_{s,\ell} = P_s/L$. This guarantees that the condition

$$(16) \quad P_s = \sum_{\ell=1}^L P_{s,\ell}$$

is complied. Rearranging (15) the half-level transmit amplitude for each layer results in

$$(17) \quad U_{s,\ell} = \sqrt{\frac{3 P_s}{2 L (M_\ell - 1)}} .$$

Considering the SVD layer model the noise power is unchanged at the receiver. However, the half vertical eye opening U_A at each time slot k and layer ℓ is influenced by the singular values so that $U_A^{(\ell,k)} = \sqrt{\xi_{\ell,k}} U_{s,\ell}$ holds. Using the T-PMSVD model the equalizer fully removes the ISI and thus for each layer the half vertical eye opening $U_{A,\ell}$ of the receive signal equals the half-level amplitude of the transmitted symbol $U_{s,\ell}$. The drawback of the T-PMSVD is that the noise is weighted differently on each layer by the equalizer coefficients expressed by the factor θ_ℓ so that the noise power on each layer results in

$$(18) \quad P_{R,\ell} = \theta_\ell P_R , \quad \text{where} \quad \theta_\ell = \sum_{\forall k} |f_{\ell,k}|^2 .$$

Taking the influence of the singular values $\sqrt{\xi_{\ell,k}}$ in the SVD based layer model into account and considering the weighting factor of the noise power θ_ℓ induced by the T-spaced equalizer coefficients in the PMSVD based layer model the corresponding SNR values become

$$(19) \quad \rho_{\text{SVD}}^{(\ell,k)} = \frac{\xi_{\ell,k} U_{s,\ell}^2}{P_R} = \frac{3 \xi_{\ell,k}}{L (M_\ell - 1)} \frac{E_s}{N_0}$$

$$(20) \quad \rho_{\text{T-PMSVD}}^{(\ell)} = \frac{U_{s,\ell}^2}{\theta_\ell P_R} = \frac{3}{\theta_\ell L (M_\ell - 1)} \frac{E_s}{N_0} ,$$

with E_s being the energy of the transmit signal and the parameter N_0 describing the power spectral density of the noise.

Power Allocation

The overall bit-error rate of a decomposed MIMO system is largely determined by the layer with the highest BER. In order to balance the bit-error rates on all layers the mean of choice is to equalize the SNR values $\rho^{(\ell,k)}$ over all layers. This is clearly not the optimal solution for minimizing the overall BER but it is easy to implement and not far away from the optimum as shown in [2]. Therefore, the half-level transmit amplitude $U_{s,\ell}$ is adjusted on each layer by multiplying it with $\sqrt{p_{\ell,k}}$ in order to apply the power allocation (PA) scheme. Consequently, the half vertical eye opening of the received symbols for the SVD-based model becomes

$$(21) \quad U_{A,\text{PA}}^{(\ell,k)} = \sqrt{p_{\ell,k}} \sqrt{\xi_{\ell,k}} U_{s,\ell} ,$$

whereas in the T-PMSVD model the factor $\sqrt{\xi_{\ell,k}}$ is dropped. With this adjustment the SNR values are resulting in

$$(22) \quad \rho_{\text{PA}}^{(\ell,k)} = p_{\ell,k} \rho^{(\ell,k)} .$$

The respective system models for T-PMSVD and SVD equalization including PA are depicted in Fig. 4 and Fig. 5. In order to achieve the above mentioned equal SNR PA and considering the limited total transmit power, the PA factors $p_{\ell,k}$ can be calculated for SVD and T-PMSVD based MIMO systems as follows [3]

$$(23) \quad p_{\ell,k}^{(\text{SVD})} = \frac{(M_\ell - 1)}{\xi_{\ell,k}} \frac{L}{\sum_{\lambda=1}^L \frac{(M_\lambda - 1)}{\xi_{\lambda,k}}}$$

$$(24) \quad p_{\ell}^{(\text{T-PMSVD})} = \theta_\ell (M_\ell - 1) \frac{L}{\sum_{\lambda=1}^L \theta_\lambda (M_\lambda - 1)} .$$

Using the equal-SNR criterion on all activated layers nearly the same BER can be obtained.

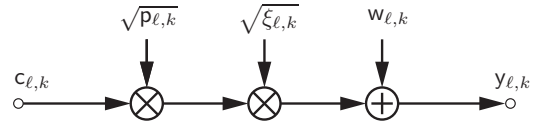


Fig. 4. Resulting layer-specific SVD-based model with PA

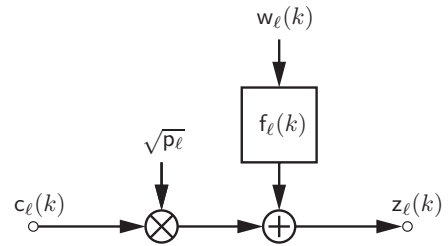


Fig. 5. Resulting layer-specific T-PMSVD-based model with PA

Results

In this contribution fixed transmission modes are used regardless of the channel quality. Assuming predefined transmission modes, a fixed data rate can be guaranteed. The obtained BER curves are depicted in Fig. 6 and Fig. 7 for the different QAM constellation sizes and MIMO configurations of Tab. 1, when transmitting at a bandwidth efficiency of 8 bit/s/Hz. The BER of the uncoded MIMO system is dominated by the specific layer having the smallest SNR. As a remedy, a MIMO transmit power allocation (PA) scheme is required for minimizing the overall BER under the constraint of a limited total MIMO transmit power. Here, a suboptimal PA solution is proposed which guarantees an equal SNR on

Table 1. Transmission modes

throughput	layer 1	layer 2	layer 3	layer 4
8 bit/s/Hz	256	0	0	0
8 bit/s/Hz	64	4	0	0
8 bit/s/Hz	16	16	0	0
8 bit/s/Hz	16	4	4	0
8 bit/s/Hz	4	4	4	4

all activated layers as highlighted in [2]. Assuming a uniform distribution of the transmit power over the number of activated MIMO layers, it turns out that not all MIMO layers have to be activated in order to achieve the best BERs. More explicitly, our goal is to find that specific combination of the QAM mode and the number of MIMO layers, which gives the best possible BER performance at a given fixed bit/s/Hz bandwidth efficiency. A direct comparison depicted in Fig. 8 shows that PMSVD-based ZF equalization outperforms the SVD model. By replacing the ZF equalization in the PMSVD model with a layer-specific maximum likelihood sequence estimation, further improvements in the BER performance can be expected. Therefore, PMSVD could be an alternative signal processing approach compared to conventional SVD-based MIMO approaches in frequency-selective MIMO channels.

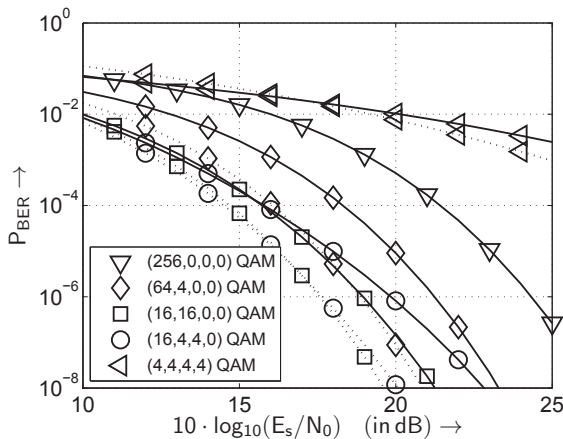


Fig. 6. Bit-error rate (BER) with PA (dotted lines) and without PA (solid lines) applying SVD-based equalization when transmitting over a Rayleigh distributed (4×4) MIMO two path channel ($L_c = 1$) with 8 bit/s/Hz using the transmission modes introduced in Table 1

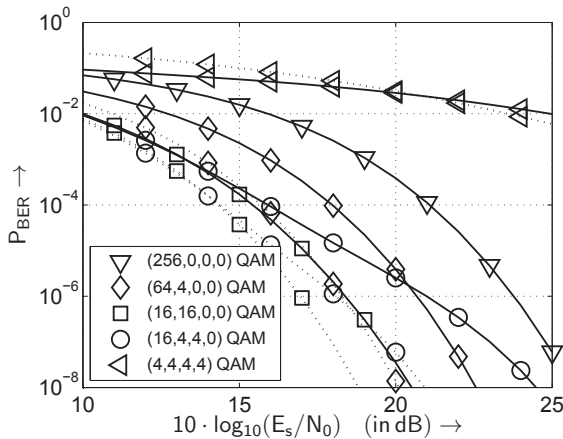


Fig. 7. BER with PA (dotted lines) and without PA (solid lines) applying T-PMSVD equalization when transmitting over a Rayleigh distributed (4×4) MIMO two path channel ($L_c = 1$) with 8 bit/s/Hz

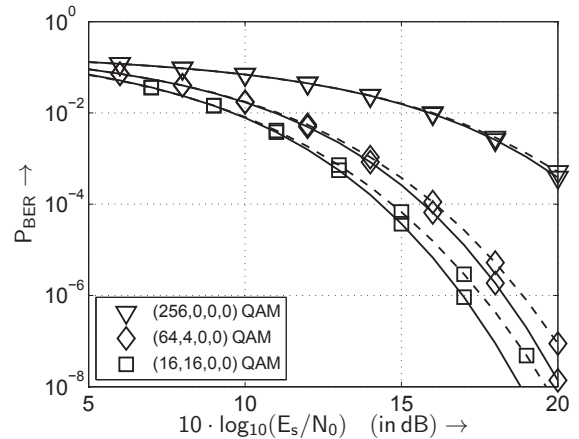


Fig. 8. BER comparison between the SVD-based (dashed lines) and T-PMSVD-based equalization results (solid lines) when transmitting over a Rayleigh distributed (4×4) MIMO two path channel ($L_c = 1$) with 8 bit/s/Hz and applying equal SNR power allocation

Conclusion

In this contribution broadband MIMO systems have been analyzed using polynomial matrix factorization. In order to remove the MIMO channel interference a particular singular-value decomposition algorithm for polynomial matrices (PMSVD) including layer-specific T-spaced equalization for eliminating the remaining intersymbol interference has been studied. This T-PMSVD technique has been compared in terms of the bit-error rate performance with the well-known spatio-temporal vector coding description applying SVD equalization. Using T-PMSVD equalization the BER performance is superior compared with conventional SVD. In addition, T-PMSVD offers some implementation advantages as a block-oriented system structure can be avoided. For both equalization types bit loading schemes have been combined with equal SNR power allocation so as to optimize the BER performance. Furthermore, the bit and power loading analogies between both equalization types have been shown. The analyzed Rayleigh channel clarifies that the activation of all transmission layers is not always beneficial.

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