Pareto - ABC Analysis of Temperature Field in High Voltage Three-Phase Cable Systems

Abstract. The paper presents the "Pareto Principle" used for temperature analysis of a high voltage cable core of 110kV working in a three-phase system. The study makes use of a multi-criteria Pareto model and ABC charts to determine the qualitative and quantitative impact of some specific parameters on the temperature of the cable core.

Streszczenie. W pracy przedstawiono "zasadę PARETO" zastosowaną do analizy temperatury żyły kabla wysokiego napięcia 110kV pracującego w układzie trójfazowym. W analizie wykorzystano wielokryterialny model PARETO oraz wykresy ABC w celu ilościowego i jakościowego zidentyfikowania wpływu określonych parametrów na temperaturę żyły kabla. (Pareto-ABC analiza pola temperatury w układach trójfazowych kabli wysokiego napięcia).

Keywords: Pareto principle, ABC charts, temperature field, core temperature, 110kV cables, FEM, Słowa kluczowe: Zasada PARETO, wykresy ABC, pole temperatury, temperatura rdzenia, kable 110kV, MES.

Introduction

The Pareto principle together with the resultant ABC charts is used in economic sciences, quality management systems and to some lesser degree in technical sciences [1, 2, 3, 4, 5]. In paper [5] the principle was used for analysis of the temperature field in a single high-voltage cable located at different depths in the ground. The paper studied the influence of several parameters such as the temperature of the air above the ground surface, conductivity of the ground, conductivity of the concrete block, thermal conductivity of the cable core, impact of current load I on the temperature of the cable core.

However, designers of high voltage cable systems are primarily concerned with the distribution of the temperature field in three-phase cables laid in the ground. In the case of a single core cable, the temperature of the core \(T_{cp}\) depends on the seven parameters given below [4, 5]:

\[
T_{cp} = f(\lambda_{s}, T_{p}, \rho_{c}, \varepsilon, \lambda_{d}, \lambda_{Cu}, \lambda_{pl})
\]

where: \(T_{cp}[^\circ C]\) – core temperature, \(\lambda_{s} [W/(m·K)]\) – thermal conductivity of the ground, \(T_{p}[^\circ C]\) – temperature of the air above the ground surface, \(I[A]\) - cable core current, \(\rho_{c} [\Omega·m]\) – cable core resistivity, \(\varepsilon [W/(m^2·K)]\) – convection heat transfer coefficient above the ground surface, \(\lambda_{d} [W/(m·K)]\) – dielectric’s thermal conductivity, \(\lambda_{Cu} [W/(m·K)]\) – thermal conductivity of the cable core.

Additionally, in the case of the three phase system, the analysis includes thermal conductivity of the concrete block denoted by \(\lambda_{d} [W/(m·K)]\) and \(\lambda_{Cu} [W/(m·K)]\) standing for the thermal conductivity of air duct where the cable is laid (see Fig. 1). Thus, \(T_{r}\) i.e. the temperature of the cables in a three phase system, will depend on a larger number of parameters:

\[
T_{r} = f(\lambda_{s}, T_{p}, I, \rho_{c}, \varepsilon, \lambda_{d}, \lambda_{Cu}, \lambda_{pl}, \lambda_{b})
\]

Making use of the "Pareto Principle", it is possible to define A, B and C sets which exert a decisive, medium, and marginal impact on the temperature of the 3 phase cable system placed in the ground at different depths. The principle also allows us to carry out an analysis of the elements of the sets [5].

Numerical model of the system

Numerical analysis of the temperature field was developed for the three-phase high-voltage (110kV) power cables arranged in the ground as shown in Fig. 1 [7].

![Fig.1 Three-phase high-voltage cable system of 110kV laid in a concrete block](image)

To analyze temperature field \(T_{x,y}\) in the steady state for the system illustrated in Fig. 1, we use thermal conductivity equation given below [6]:

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = -\frac{g}{\lambda}
\]

where: \(g = j^2\rho[A/m^3]\) is the rate of heat production in cable cores, \(j[A/m^2]\) is current density, \(\rho[\Omega·m]\) - resistivity, \(\lambda [W/(m·K)]\) is the thermal conductivity of individual elements of the system.

Equation (3) is solved using finite elements method [8]. For this purpose, a numerical model of the system shown in Fig. 2 was developed. The model also includes the following boundary conditions assumed for the calculation:
- non-homogeneous boundary condition of the first kind (Dirichlet’s) on the bottom edge of the analyzed area (constant ground temperature \(T_0 = 8[^\circ C]\))
- non-homogeneous boundary condition of the third kind on the ground surface corresponding to heat transfer in accordance to Newton’s law.
\[
\frac{\partial T}{\partial n} \bigg|_s = -\frac{\varepsilon}{\lambda} (T_s - T_P)
\]

where: \(\varepsilon\) [W/(m²·K)] is the convective heat transfer coefficient describing the speed of wind over the ground surface \([9]\), \(T_s\) is the ground surface temperature and \(T_P\) - the air temperature above the ground.

- on the vertical edges of the analyzed area (Fig. 2), the condition of heat flux continuity must be used.

**Effect of selected parameters on temperature \(T_r\) of the cable core**

In order to illustrate the effect of some selected parameters on the cable core temperature in the three-phase system, three exemplary charts: \(T_r = f(h)\), \(T_r = f(T_P)\), and \(T_r = f(\lambda_z)\) are presented below. Other parameters have been assumed as fixed by adopting the calculation values given in Table 1. Considering the fact that the differences between the upper core \(T_r\) and the lower cores are less than \(1^\circ\mathrm{C}\), the charts present only the upper core values.

The \(T_r = f(h)\) chart is shown in Fig. 4.

![Fig. 4 Dependence of cable core temperature \(T_r\) on the system’s distance \(h\) from the ground surface.](image)

Taking into account the climatic conditions, it is also worth presenting the temperature chart \(T_r\) as dependent on air temperature \(T_P\) above the ground surface. The chart \(T_r = f(T_P)\) is shown in Fig. 5. We observe a linear relationship between the temperature \(T_r\) cable wire from the air temperature changes \(T_P\) above the ground.

![Fig. 5 Chart of three phase system cable core temperature \(T_r\) as dependent on air temperature \(T_P\).](image)

According to the literature the thermal conductivity of the ground varies from 0.2 to 1.4 [W/(m·K)], \([7]\). Fig. 6 shows core temperature chart \(T_r\) depending on the thermal conductivity of the ground \(\lambda_z\). The lowest values of thermal conductivity correspond to dry ground (sand) , whereas the highest ones correspond to wet soil (peat). As can be observed \(\lambda_z\) exerts a critical influence on the core temperature within the thermal conductivity ranging from 0.2 [W/(m·K)] to 0.6 [W/(m·K)].

![Fig. 6 Chart of three phase system cable core temperature \(T_r\) as dependent on thermal conductivity of the ground.](image)
Pareto ABC analysis of temperature field in three-phase high-voltage cable system.

Basing on the research methodology developed for a single core cable laid in the ground [5] the Pareto-ABC analysis of the cable system shown in Figs. 1 and 2 was carried out. Fig. 1 shows the configuration of a three-phase cable system placed in a concrete block. This type of layout model is actually used in power industry. The use of concrete blocks increases the number of factors influencing the distribution of the temperature field. They include the thermal conductivity of air duct $\lambda_p$, in which a single cable is placed, and $\lambda_b$, i.e. the thermal conductivity the concrete coating with its dimensions. The depth of the concrete block was varied from 1 to 20m. The scope of the changes of the base parameters [5] used in the calculation is shown in Table 2.

Table 2 Scope of base parameter changes for the three-phase system placed at depths ranging from 1 to 20 meters

<table>
<thead>
<tr>
<th>Parameter number</th>
<th>Parameter symbol</th>
<th>Unit</th>
<th>Minimum value</th>
<th>Maximum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$T_p$</td>
<td>°C</td>
<td>-30</td>
<td>+30</td>
</tr>
<tr>
<td>2</td>
<td>$\lambda_p$</td>
<td>W/(m·K)</td>
<td>0.2</td>
<td>1.4</td>
</tr>
<tr>
<td>3</td>
<td>$I$</td>
<td>A</td>
<td>914</td>
<td>1218</td>
</tr>
<tr>
<td>4</td>
<td>$\lambda_{Cu}$</td>
<td>W/(m·K)</td>
<td>360</td>
<td>400</td>
</tr>
<tr>
<td>5</td>
<td>$\lambda_{b}$</td>
<td>W/(m·K)</td>
<td>0.7</td>
<td>1.4</td>
</tr>
<tr>
<td>6</td>
<td>$\lambda_{d}$</td>
<td>W/(m·K)</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>$\varepsilon$</td>
<td>W/(m²·K)</td>
<td>16.6</td>
<td>150</td>
</tr>
<tr>
<td>8</td>
<td>$\rho_{Cu}$</td>
<td>Ω·m</td>
<td>1.57E-8</td>
<td>1.88E-8</td>
</tr>
</tbody>
</table>

Fig. 7 ABC chart for the three-phase cable system located at the depth of 1m

Fig. 8 ABC chart for the three-phase cable system located at the depth of 2m

Fig. 9 ABC chart for the three-phase cable system located at the depth of 3m

Fig. 10 ABC chart for the three-phase cable system located at the depth of 4m

Fig. 11 ABC chart for the three-phase cable system located at the depth of 5m

Fig. 12 ABC chart for the three-phase cable system located at the depth of 10m
using the results of the calculations carried out for different cable laying depths ranging from 1m to 20m, it was possible to create corresponding ABC charts shown in Figs. 7-13 below. The cumulative values S and the values of relative temperature $T_w$ were calculated in accordance with the procedure developed for a single cable laid in the ground [5]. The values refer to the temperature of the upper cable $T_r$. (Fig. 1).

Table 3 provides the relative changes in temperature $T_w$ of cable cores for the assumed base parameters listed in Table 2 and different values of $h$.

Analysis of Figs. 7-13 makes it possible to define A, B and C subsets. At a depth of 5m system set A includes two elements: temperature above the ground $T_p$ and ground’s thermal conductivity $\lambda$ (A = $\{T_p, \lambda\}$). Set B includes long term cable load, copper resistivity $\rho_{Cu}$, thermal conductivity of the concrete block $\lambda_b$ and $\lambda_a$ - thermal conductivity of the air duct $B = \{I, \rho_{Cu}, \lambda_b, \lambda_a\}$. On the other hand, the elements of subset C that have a minimal effect on the temperature of the cable core include: thermal conductivity of the dielectric $\lambda_d$, convective heat transfer coefficient on the ground surface $\epsilon$, and thermal conductivity of the cable core $\lambda_{Cu}$, $C = \{(\lambda_d, \rho_{Cu}), \lambda_{Cu}\}$.

With an increase of depth $h$ above 10m, the situation is changed. Set A is composed of ground’s thermal conductivity of the earth $\lambda_e$, long term cable load $I$ and $\lambda_a$ - thermal conductivity of the air duct $A = \{\lambda_e, I, \lambda_a\}$. Set B includes air temperature $T_e$, copper resistivity $\rho_{Cu}$ and thermal conductivity of the concrete block $\lambda_b$ (B = $\{T_e, \rho_{Cu}, \lambda_b\}$) whereas set C is composed of dielectric thermal conductivity $\lambda_d$, convective heat transfer coefficient $\epsilon$ and the core thermal conductivity $\lambda_{Cu}$, $C = \{(\lambda_d, \rho_{Cu}), \lambda_{Cu}\}$.

We should note the changes in the structure of the elements affecting sets A and B. Thus, at the depth $h = 1m$ the greatest influence on the cable core temperature is exerted by changes of air temperature $T_e$, it is followed by ground’s thermal conductivity of the earth $\lambda_e$. With the increase of the depth $h$ above 2m, we can observe a change. Here the greatest impact is exerted by the thermal conductivity of the ground with changes in air temperature in the second position. It is possible to determine the depth $h$, at which a transition takes place from set $\{T_p, \lambda\}$ into ordered set $\{h, \lambda_a\}$). It is illustrated by the charts shown in Fig. 14 presenting the dependence of relative temperature changes $T_{w,p} = f(h)$ and $T_{w,\lambda_e} = f(h)$ based on data contained in Table 3. The intersection point of these two functions determines the depth $h$ at which the exchange of the elements in set A takes place. The depth equals 2.2m.

$T_{w,p} = 0.9248h^{-0.643}$

The function dependent on the thermal conductivity of the ground $T_{w,\lambda_e}$ is expressed by the following formula:

$T_{w,\lambda_e} = \frac{(T_{max} - T_{min})}{(T_{max} - T_{min})}$

Fig. 14 Dependence of relative changes of cable core temperature $T_w$ in the function of the cable system’s distance from the ground surface for two parameters $T_p$ and $\lambda_a$. The intersection point of the two curves defines the depth $h = 2.2m$ at which the exchange of set elements occurs.

The function approximating the dependence of the relative changes of cable core temperature on the depth $h$ of the laid cable system for the base changes of air temperature $T_{w,x}$ is described by the following formula:

$$T_{w,x} = \frac{(T_{max} - T_{min})}{(T_{max} - T_{min})}$$
\[ T_{w,A} = 0.0453 \ln(h) + 0.5226 \]

With further changes of the three-phase system's location depth the impact of air temperature \( T_p \) becomes smaller and smaller and at a depth of \( h = 6.48 \) m it is equivalent to the changes caused by cable current load \( I \). The function approximating the dependence of the relative changes of cable core temperature on the depth \( h \) of the laid cable system and its current load \( I \) is as follows:

\[ T_{w,I} = 0.0301 \ln(h) + 0.2392 \]

The dependence \( T_{w,p} = f(h) \) and \( T_{w,I} = f(h) \) is shown in Fig. 15.

By analyzing the curves presented in Figure 16, it should be noted that the greatest impact on cable core temperature \( T_w \) of the three-phase system is exerted by the temperature of the air above the ground, thermal conductivity of the ground \( \lambda_z \) and current load \( I \). The curve \( T_{w,p} = f(h) \) intersects all the other curves at five points specifying the position of parameter \( T_p \) in the \( A \), \( B \) and \( C \) sets. Air temperature plays a marginal role at a depth of 20 m.

**Comments and conclusions**

In general, the research paper "The Pareto-ABC analysis of energy systems" presents a multi-parameter analysis of the temperature of high voltage cable cores operating in a single and three-phase system. In Part One of this paper we discussed the methodology concerning the analysis of the influence of base parameters such as air temperature above the ground surface \( T_p \), thermal conductivity of the ground \( \lambda_z \) and current load \( I \). The study made use of a multi-criteria Pareto model [1, 2] and the ABC charts to determine the qualitative and quantitative impact of the parameters on the temperature of the cable core. The research methodology was also applied in the analysis of real three-phase systems, which is presented in this paper.

Based on the "Pareto principle" the previously developed analysis method of the temperature field in a single phase HV cable laid in the ground [1, 2, 3, 5] was been used for the analysis of a three-phase system. The analysis required an introduction of some additional parameters affecting the temperature of cable cores, namely thermal conductivity of air duct \( \lambda_p \) holding a single cable and thermal conductivity of concrete coating \( \lambda_b \). It was found that the greatest impact on the temperature of cable cores laid at the depths up to 5 m is exerted by air temperature above the ground’s surface \( T_p \) and ground thermal conductivity \( \lambda_z \).

Analyzing the three-phase system of high-voltage cables placed in the ground, it was found that basic changes in the impact of individual parameters take place at the following depths:

- 2.2 m for set \( A \) where: \( A = \{T_p, \lambda_z\} \rightarrow A = \{\lambda_z, T_p\} \)
- 6.48 m and also for set \( A = \{\lambda_z, T_p, I\} \rightarrow A = \{\lambda_z, I, T_p\} \)

**Fig. 17 Temperatures of the three phase cable core and single cable core at various location depths under the ground surface.**

**Fig. 18. Temperatures of the three phase cable core and single cable core and their temperature difference accounting for the changes of air temperature above the ground surface.**

**Fig. 19. Temperatures of the three phase cable core and single cable core and their temperature difference depending on thermal conductivity of the ground.**

It is interesting to compare the temperature of the cable core in the three phase system with the temperature of a single core cable [5]. Figs 17, 18 and 19 shows the curves of \( T_{r3} = f(h), T_{r1} = f(T_p) \) and \( T_{r3} = f(\lambda_z) \) for a single cable and the three phase system. Taking into account the research
results for $T_r = f(h)$, $T_r = f(T_p)$, it was found that there is an average increase of more than a dozen degrees in temperature for the three-phase system in comparison with the single core cable. The maximum difference being at 15°C. In the case of $T_r = f(\lambda)$ there is a lesser impact observed. It was of the order of several degrees at low values of the ground’s thermal conductivity caused by worse conditions of heat exchange between the analyzed systems and the environment (Fig. 19).

The paper, making use of both the "Pareto principle" and ABC charts, has determined the parameters exerting a fundamental impact on the temperature of cable cores in the high-voltage three-phase systems. The reliability of cable systems is affected in a primary way by the thermal field and a good knowledge of the parameters having a decisive influence on the temperature of the cable core. The cable core temperature constitutes a basic criterion in the design of high-voltage cables.

**REFERENCES**


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