

The System Functions MAOPCs for Analysis and Optimization Of Linear Periodically Time-Variable Circuits Based on the Frequency Symbolic Method

Abstract. In this paper we considered the following provisions: the architecture of system functions MAOPCs and examples of its application to solve tasks of multivariate analysis of LPTV circuits based on the frequency symbolic method. The method is based on approximation of transfer functions of LPTV circuits in the form of trigonometric polynomials of the Fourier. The system functions MAOPCs is implemented in the environment of MATLAB

Streszczenie. W pracy zaprezentowano układ funkcji do symbolicznej analizy i optymalizacji obwodów parametrycznych, zmiennych w czasie. Metoda jest oparta na aproksymacji transmitancji obwodu parametrycznego za pomocą trygonometrycznego wielomianu Fouriera. Zaproponowany zbiór funkcji jest zaimplementowany w środowisku programu MATLAB. (Metoda symboliczna analizy i optymalizacji obwodów parametrycznych, zmiennych w czasie).

Keywords: linear periodically time-variable circuits, frequency symbolic method, multivariate analysis, optimization.

Słowa kluczowe: liniowe obwody parametryczne, wieloparametrowa analiza częstotliwościowa, optymalizacja.

Introduction

It is shown that [1,2,3] the frequency symbolic method (FS method) is an effective tool for the analysis of steady states of LPTV circuits in the frequency domain. The method is based on solving the equation of L.A. Zadeh [1] and the approximation of transfer function $W(s,t)$ (s - complex variable, t - time) of LPTV circuit by trigonometric polynomial of the Fourier if some or all parameters of the circuit are defined by symbols [1]. This paper presents the architecture and application examples of system functions MAOPCs (Multivariate Analysis and Optimization of the Parametric Circuits) to solve tasks of multivariate analysis of LPTV circuits. The application of the system functions to solve the optimization tasks of linear parametric circuits, provided their stability control, are presented also in the papers [2,3].

The feature of the system MAOPCs lies in the fact that in this system it is possible to estimate the asymptotic stability for correct determination of the parametric transfer function, sensitivity function, multivariate analysis of LPTV circuits and particular analysis of tolerances and optimization. Evaluation of the asymptotic stability of the investigated LPTV circuit is within specified limits change of the parameters x_1, \dots, x_n and are determined by allowable areas of their changes, providing stability of the circuit. Exclusively in these areas is the analysis of the tolerance and optimization [2,3].

Evaluation of stability of LPTV circuit is held by the values of the real parts of the roots of the denominator $\Delta(s, x_1, \dots, x_n)$ of the normal transfer function $G(s, \xi, x_1, \dots, x_n)$ of the inertial part of LPTV circuit [4], which is determined by the approximated trigonometric polynomial of Fourier with symbolic values x_1, \dots, x_n (ξ - the time of filing to the input Delta function).

Architecture of the system functions MAOPCs

The system consists of 17 functions that are implemented in the MATLAB environment [5]. Each function has arguments and global variables and performs over them defined transformations. Functions and global variables form the input data program for research LPTV circuit and should be defined (set) at the time of the function call. The results obtained in the process of performing one

function (in some cases several functions) can be global variables (input data) for other functions. The sequence of the functions is arbitrary. Thus, the developed system functions with other standard functions of the MATLAB package is the tool to develop the necessary algorithms and programs of computer experiments for multivariate analysis and optimization of LPTV circuit that is investigated. The system MAOPCs is open to replenish it with new functions.

To investigate the of LPTV circuits in MATLAB environment using system MAOPC, first need:

- 1) create a m-file [5] with text of the program that describes the research of circle and global variables, and save it in a folder;
- 2) set functions of the system MAOPCs and MATLAB in a sequence that comply with the algorithm of research, in the created m-file;
- 3) perform m-file in the MATLAB environment using the option «Run" m-file "».

The results of the research are saved in mat-file [5], which is in the same folder (in the computer memory) as the m-file. The results of research and intermediate calculations that were stored by default during the research, we can see in the window «Workspace» of MATLAB environment [5]. It is clear that the set of folders (in the computer memory) creates a library of algorithms and computational experiments of LPTV circuits that were investigated.

Purpose of the system functions MAOPCs are following:

- 1) *TrFunc* — the formation of parametric transfer function in symbolic form;
- 2) *Stability* — evaluation of the asymptotic stability of a circuit;
- 3) *FunctionOfZoneStability* — approximation of the boundary of the stability region by polynomial of n-th order;
- 4) *FSM* τ and *MFSM* — a determination of parametric transfer functions with the conventional and modified frequency symbolic model of linear parametric circuit, accordingly;
- 5) *OutVar* — the function formation of the output variables;
- 6) *SensFO* and *SensSO* — the calculation of sensitivity functions of the first and second order, accordingly;
- 7) *RelativeDeviation_FirstOrder*, *RelativeDeviation_SecondTerm* and *RelativeDeviationOffFunction* — the calculation of the relative deviation of function;

8) *FormOfFunOfGoal*, *FormOfFunCharacteristic* and *FormOfObjectiveFun* — the formation of the objective function;

9) *Table* — the formation of the table of function values;

10) *Graph_2D* and *Graph_3D* — visualization of the function in the plane and in space, accordingly.

To use the system functions on your personal computer, you must install MATLAB 7.6.0, in particular its components Symbolic Toolbox, Extended Symbolic Math, Optimization Toolbox, Genetic Algorithm and Direct Search Toolbox [5].

Computer computational experiments

Task for experiment 1. To conduct the tolerance analysis of single circuit parametric amplifier of fig. 1 under control its stability. To determine the relative deviation of module parametric transfer function $Z_1(s,t) = U_1(s,t)/I(s)$

by changing the parameters elements within: $m = 0.04:0.06$, $Y_1 = 0.2375:0.2625 S$, $Y_2 = 0.38:0.42 mS$, $c_0 = 9.5:10.5 pF$,

$L = 0.2433:0.2633 \mu H$.

Experiment 1. According to the procedure of tolerances analysis the researcher performs the following steps [2]:

Step 1: **Stability** function forms the denominator $\Delta(m, c_0, Y_1, Y_2, L, s)$ of the function $G(s, \xi)$ of current $I(s)$ to voltage $U_c(s, t)$ parametric capacitance amplifier. The denominator $\Delta(m, c_0, Y_1, Y_2, L, s)$ in one harmonic component $k = 1$ in the approximation of function $G(s, \xi)$ has the form:

$$\begin{aligned} \Delta(m, c_0, Y_1, Y_2, L, s) = & (60e-25 * L^3 * c_0^3 * m^2 - 12e-24 * \\ & L^3 * c_0^3) * s^6 + (60e-25 * L^3 * c_0^2 * m^2 * Y_2 - 36e-24 * L^3 * \\ & c_0^2 * Y_2) * s^5 + (-36e-24 * L^3 * Y_2^2 * c_0 + 60e-25 * c_0^2 * L^2 * m^2 - \\ & 39e-6 * L^3 * c_0^3 - 36e-24 * c_0^2 * L^2 + 19e-6 * L^3 * c_0^3 * \\ & m^2) * s^4 + (-12e-24 * L^3 * Y_2^3 - 75e-6 * L^3 * c_0^2 * Y_2 + 96e- \\ & 7 * L^3 * c_0^2 * m^2 * Y_2 - 72e-24 * L^2 * Y_2 * c_0) * s^3 + (-36e-24 * c_0 * L - \\ & 30e12 * L^3 * c_0^3 - 57e-6 * L^3 * Y_2^2 * c_0 - 96e-7 * c_0^2 * L^2 * m^2 - \\ & 36e-24 * L^2 * Y_2^2 + 15e12 * L^3 * c_0^3 * m^2) * s^2 + (-30e \\ & 12 * L^3 * c_0^2 * Y_2 - 19e-6 * L^3 * Y_2^3 - 36e-24 * L * Y_2) * s - 19e- \\ & 6 * L^2 * Y_2^2 + 39e-6 * c_0 * L - 30e12 * c_0^2 * L^2 - 12e-24. \end{aligned}$$

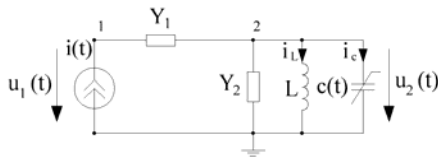


Fig.1. Single-circuit parametric amplifier

$$i(t) = I_m \cdot \cos(\omega_s \cdot t + \varphi), \quad c(t) = c_0 \cdot (1 + m \cdot \cos(\Omega \cdot t));$$

$$c_0 = 10 pF, \Omega = 4 \cdot \pi \cdot 10^8 \text{ rad} / s, Y_1 = 0.25 S, Y_2 = 0.0004 S,$$

$$L = 0.2533 \mu H, s = j\omega_c, \omega_c = 2 \cdot \pi \cdot 10^8 \text{ rad} / s; m = 0.05$$

Step 2. By the function *Stability* and the denominator $\Delta(m, c_0, Y_1, Y_2, L, s)$, which in $k = 3$ [6] provides satisfactory accuracy, conducted the estimation of the amplifier stability within changing each of the parameters. The results of the stability estimation boil down by means of MATLAB in the so-called stability table [2]. The data of the stability table indicate such inadmissibility of deviations and the need to reduce them.

Step 3. The stability table of stability is defined relative deviations of the parameters of the circuit elements in which it is stable (completed by researcher). For the performed experiment it follows that the relative changes $\delta m, \delta c_0, \delta Y_1, \delta Y_2, \delta L$ of the parameters elements of the amplifier, in which it is stable, are: $\delta m = 6\%$, $\delta c_0 = 2\%$,

$\delta L_1 = 2\%$, $\delta Y_1 = 5\%$, $\delta Y_2 = 5\%$. Thus, all further calculations will be obtained from the stability limits table.

Step 4: By functions *TrFunc* and *FSM* is obtained the expression for the transfer function of the amplifier $Z_1(s, t) = U_1(s, t)/I(s)$:

$$(1) \quad Z_1(s, t) = \frac{((Y_1 + Y_2 + 1/Ls_i) - Y_1 \cdot A_c(s, t))}{Y_1 \cdot (Y_2 + 1/Ls_i)}$$

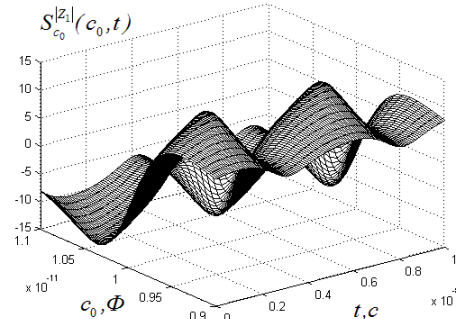


Fig.2. The dependence of the module sensitivity $S_{c_0}^{|Z_1|}(c_0, t)$ of the transfer function from c_0 and the time t at $\omega = 2 \cdot \pi \cdot 10^8 \text{ rad/s}$

Step 5: By functions *SensFO* and *SensSO* and symbolic expression (1), which is the input data for these functions and in $k = 3$ [6] provides satisfactory accuracy, defines the sensitivity functions of the first and second orders of the module of parametric transfer function $Z_1(s, t)$ by parameters m, c_0, Y_1, Y_2, L . For example, the sensitivity function:

$$S_{c_0}^{|Z_1(s, t, c_0)|} = (\partial |Z(s, t, c)| / \partial c) \cdot (c / |Z(s, t, c)|)$$

of the module of parametric transfer function $Z_1(s, t)$ before the parameter was changed c_0 graphically displayed by using the function *Graph_3D* and shown in fig. 2.

Step 6: By function *RelativeDeviationOfFunction* and the sensitivity functions defined in step 5, calculated the relative deviation $\delta |Z_1(t)|$ of the module of parametric transfer function of the amplifier when the relative changes $\delta m = 6\%$, $\delta c_0 = 2\%$, $\delta L_1 = 2\%$, $\delta Y_1 = 5\%$, $\delta Y_2 = 5\%$ of parameters of elements that ensuring the stability of this amplifier.

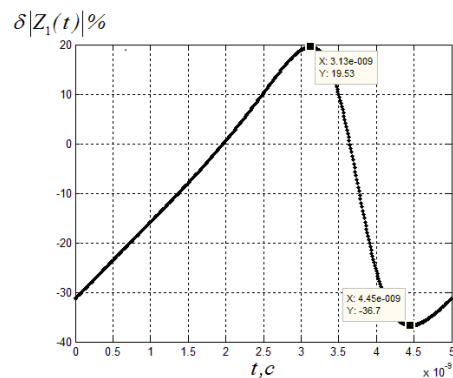


Fig.3. The time dependence $\delta |Z_1(t)|$ of the function module $Z_1(t)$ when the relative change of parameters of elements

The results of experiment 1. The result of using the system MAOPCs to tolerance analysis of amplifier from

fig.1 is the dependence of the relative deviation $\delta|Z_1(t)|$ of the module of the transfer function $Z_1(t)$ from time t , which is formed in step 6 and graphically displayed by using the function **Graph_3D** and shown in fig. 3.

From fig. 3 it follows:

a) the relative deviation $\delta|Z_1(t)|$ of the module of the transfer function $Z_1(t)$ of the amplifier is a time-dependent function, because that the transfer function itself $Z_1(t)$ is time-dependent;

b) the relative deviation of the transfer function $Z_1(t)$ varies from -36.7% to 19.53%.

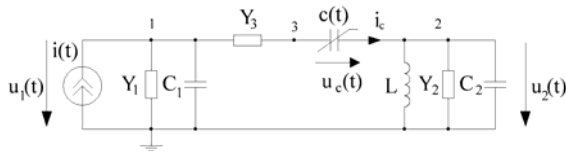


Fig.4. Parametric modulator

$$i(t) = I_m \cdot \cos(\omega_s \cdot t + \varphi), \quad c(t) = c_0 \cdot (1 + m \cdot \cos(\Omega \cdot t)); m = 0.1;$$

$$c_0 = 0.9 \text{ pF}; \Omega = 2 \cdot \pi \cdot 4.5 \cdot 10^8 \text{ rad/s}; I_m = 0.01 \text{ mA};$$

$$s = j\omega_s; \omega_s = 2 \cdot \pi \cdot 10^7 \text{ rad/s}; \varphi = 0; Y_1 = 0.01 \text{ S}; Y_2 = 0.001 \text{ S};$$

$$Y_3 = 0.0125 \text{ S}; C_1 = 1.812 \text{ pF}; C_2 = 20 \text{ pF}; L = 65.3 \text{ nH}.$$

Task for experiment 2. To conduct the tolerance analysis of parametric modulator of fig. 4 under control of its stability. To determine the relative deviation of module of the parametric transfer function $Z_2(s,t) = U_2(s,t)/I(s)$ by changing the parameters of elements within: $m = 0.46 : 0.54$
 $L = 62.04 : 68.57 \text{ nH}, c_0 = 0.87 : 0.93 \text{ pF}, Y_1 = 0.97 : 1.03 \text{ mS},$
 $Y_2 = 0.0097 : 0.0103 \text{ S}, Y_1 = 0.97 : 1.03 \text{ S}, Y_3 = 0.0118 : 0.0132 \text{ S},$
 $C_2 = 1.721 : 1.903 \text{ pF}, C_1 = 19.6 : 20.4 \text{ pF}.$

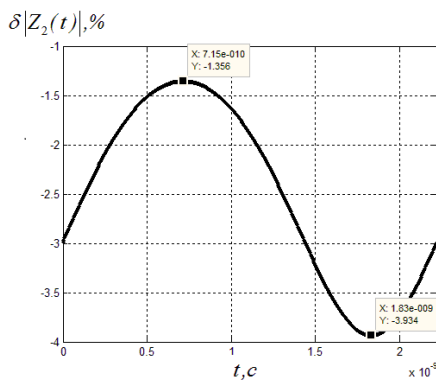


Fig.5. The time dependence $\delta|Z_2(t)|$ of the function module $Z_2(t)$ at a relative change of parameters of elements

The results of experiment 2. The result of using the system MAOPCs to tolerance analysis of modulator from fig. 4 is formed by the procedure of tolerance analysis (similar to the procedure of Experiment 1) the dependence of the relative deviation $\delta|Z_2(t)|$ of the module parametric transfer function $Z_2(t)$ from time t . Function $\delta|Z_2(t)|$ which is formed within the limits of amplifier stability that were previously determined by **Stability** function, and graphically displayed using by function **Graph_2D** and shown in fig. 5.

From fig. 5 follows that the relative deviation of the transfer function $Z_2(t)$ varies from -3.9% to 1.36%.

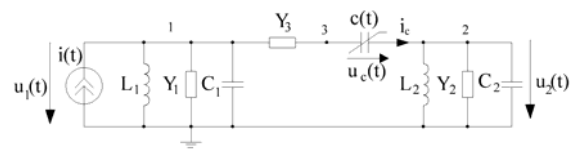


Fig.6. Double-circuit parametric amplifier

$$i(t) = I_m \cdot \cos(\omega_s \cdot t + \varphi), \quad c(t) = c_0 \cdot (1 + m \cdot \cos(\Omega \cdot t)),$$

$$c_0 = 1 \text{ pF}; m = 0.1; \Omega = 2 \cdot \pi \cdot 298.573 \cdot 10^6 \text{ rad/s};$$

$$I_m = 0.1 \text{ mA}; s = j\omega_s; \omega_s = 2 \cdot \pi \cdot 10^8 \text{ rad/s}; \varphi = \pi / 4;$$

$$Y_1 = Y_2 = 0.0001 \text{ S}; Y_3 = 0.55 \text{ S}; C_1 = C_2 = 68 \text{ pF};$$

$$L_1 = 36.70795 \text{ nH}; L_2 = 9.312609 \text{ nH}.$$

Task for experiment 3. To conduct the tolerance analysis of double-circuit parametric amplifier of fig. 6 under control of its stability. To determine the relative deviation of module parametric transfer function $Z_1(s,t) = U_1(s,t)/I(s)$ by changing the parameters of elements within: $m = 0.09 : 0.11,$

$$L_1 = 36.70422 : 36.71165 \text{ nH}, Y_1 = 0.091 : 0.108 \text{ mS},$$

$$c_0 = 0.9 : 1.1 \text{ pF}, L_2 = 9.311671 : 9.313545 \text{ nH}, Y_2 = 0.091 : 0.108 \text{ mS},$$

$$Y_1 = 0.091 : 0.108 \text{ mS}, C_2 = 67.980 : 68.12 \text{ pF},$$

$$C_1 = 67.993 : 68.012 \text{ pF}, Y_3 = 0.471 : 0.528 \text{ S}.$$

The results of experiment 3. The result of using the system MAOPCs to tolerance analysis of amplifier from fig. 6 is formed by the procedure of tolerance analysis (similar to the procedure of Experiment 1) the dependence of the relative deviation $\delta|Z_1(t)|$ of the module parametric transfer function $Z_1(t)$ from time t . Function $\delta|Z_1(t)|$ which is formed within the limits of stability of the modulator, which is previously determined by the **Stability** function, and graphically displayed using function **Graph_2D** and shown in fig. 7

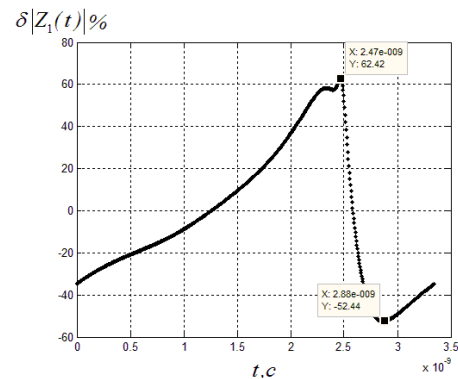


Fig.7. The time dependence $\delta|Z_1(t)|$ of the function module $Z_1(t)$ at a relative change of parameters of elements

From fig. 7 shows that the relative deviation of the transfer function $Z_1(t)$ varies in from -52.44% to 62.42%.

Conclusions

The system functions MAOPCs allows to:

- solve a wide range of design tasks and investigate LPTV circuits, setting in the program input data algorithms for their research;
- use a powerful symbolic machine and other standard functions of the package MATLAB in full, without delving into the depth of mathematical apparatus of the implemented methods;

From a number of experiments it follows that symbolic sensitivity functions are enough bulky and their calculation requires a lot of computer time. However, the presence of symbolic sensitivity function allow you quickly calculate the relative deviations of transfer functions when you change the tolerances of the parameters elements.

The obtained results show that the realized system functions MAOPCs that is based on the FS- method is an effective tool for computer modeling of the established modes of parametric devices which are represented by LPTV circuits.

REFERENCES

- [1] Shapovalov Yu. The peculiarities of analysis of linear parametric circuit performed by frequencysymbolic method / Yu. Shapovalov, B. Mandziy, S.Mankovsky // *Przegląd Elektrotechniczny* (2010), Vol.86, № 1, 158–160
- [2] Shapovalov Yu. Tolerances analysis and optimization of linear periodically time-variable circuits based on the frequency symbolic method/ Yu Shapovalov, B.A. Mandziy, D.R. Bachyk // *Modern problems of radio engineerings, telecommunications*

- and computer science (TCSET'2014) : Int. Conf., 25 Feb.-1 Mar. 2014 : Proc.Lviv-Slavsko, Ukraine, (2014), 120-122.
- [3] Shapovalov Yu.I. Optimization of parametric balanced modulator on frequency symbolic method / Yu.I. Shapovalov, B.A. Mandziy, D.R. Bachyk // *Computational Problems of Electrical Engineering*, (2012), Vol.2, No.2, 93–101
- [4] A. Solodov, F. Pietrov, *Linear Automatic Systems with Varying Parameters*. – Moscow, Ruais: Nauka (1971), 620 p. (Russian)
- [5] Dyakonov. V.P. *MATLAB 7. * / R2006 / R2007: Tutorial / V.P. Dyakonov*. - M.: DMK Press, (2008), p.768
- [6] Bachyk D.R. The means of computer circuit engineering modelling of parametric radioengineering nodes based on the frequency symbolic method. Thesis for a candidate of technical sciences degree in specialty, 05.12.17, Dariya Bachyk – Lviv, (2014), p. 220

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