Use of Orthogonal Components to Determine the Active Power of AC Arc Furnace Based on Measurements of Voltages and Currents

Abstract. The present article shows the results of an analysis of active power consumed by AC arc furnace at various stages of the melting process, taking harmonics into consideration. The analysis was based on measurement data in the form of computer recorded voltage and current waveforms in the power circuit of the arc furnace. Next the frequency analysis and the evaluation of the content harmonics of recorded waveforms were performed. Computation made using the fast Fourier transform FFT built-in in MATLAB. The active power of the arc furnace was determined on the basis of the classical theory of harmonic power distributions and instantaneous power after Clarke transformation to the orthogonal coordinate system α-β. Harmonic distribution phase currents and voltages were transformed using the FFT procedure, as well as distorted voltage/current waveforms registered as a sequence of samples. The obtained results demonstrate a high level of compatibility between the instantaneous power and active power for the methods under consideration.

Streszczenie. W artykule przedstawiono wyniki analizy mocy czynnej pobieranej przez piec łukowy AC w różnych etapach procesu wypotu z uwzględnieniem wyższych harmonicznych. Do analizy wykorzystano dane pomiarowe w postaci zarejestrowanych komputerowo przebiegów napięć i prądów w torze załania pieca łukowego. Następnie przeprowadzono analizę częstotliwościową i ocenę zawartości wyższych harmonicznych zarejestrowanych przebiegów. Obliczenia wykonano za pomocą szybkiej transformacji Fouriera FFT w programie MATLAB. Moc czynną pieca łukowego wyznaczono w oparciu o klasyczną teorię mocy dla rozkładów harmonicznych oraz na podstawie mocy chwilowej po transformacji Clarke’a do układu ortogonalnych współrzędnych α-β. Transformacji poddano uzyskane w procedurze FFT rozkłady harmoniczne prądów i napięć fazowych, a także zarejestrowane w postaci ciągu próbek odkształcone przebiegi napięciowo-prądowe. Uzyskane wyniki wykazują dużą zgodność mocy chwilowej i mocy czynnej dla rozpatrywanych metod. (Zastosowanie składowych ortogonalnych do wyznaczania mocy czynnej pieca łukowego AC na podstawie pomiarów napięć i prądów).

Keywords: harmonics, orthogonal components, arc furnace, power flow

Słowa kluczowe: harmoniczne, składowe ortogonalne piec łukowy, przepływ mocy

Introduction

Operators of power systems, due to the sensitivity of some of the equipment and in order to reduce losses, seek to ensure that proper power quality is supplied to the end customers. Power quality is mainly influenced by voltage fluctuations and distortion of voltage and current waveforms deviating from the sinusoidal shape [1], basically caused by customers connected to a power grid. These distortions are caused by harmonics in the current drawn by powerful, nonlinear loads such as DC and AC arc furnaces, and power electronic devices used for electrical drives [2,3,4]. Electric AC arc furnaces are technological devices which consume a lot of electric power. Their voltage/current characteristics are highly non-linear and non-stationary due to the properties of the electric arc. AC arc furnaces generate a continuous spectrum of current harmonics, causing distortions voltage fluctuations. The harmonics of the 2nd, 3rd, 5th and 7th order relative to the mains frequency have the greatest impact [5-7]. Instability of the arc in the initial phase of steel melting cause’s stochastic supply current changes [8], which is confirmed by the voltage and current waveforms in the power waveforms in the power circuit of the AC arc furnace referred to in publications [5,6]. From the viewpoint of furnace operation and efficiency of the melting process, an important matter is to understand the mechanism of active power flow in the arc furnace installation. For the analysis of this issue, measurement data in the form of computer recorded voltage and current waveforms in the power circuit of the arc furnace were used, which afterwards underwent numeric processing using the Fast Fourier Transform (FFT) procedures. The active power of the arc furnace was determined on the basis of the classical theory of harmonic power distributions and instantaneous power after transformation to the orthogonal coordinate system α-β. Harmonic distribution phase currents and voltages were transformed using the FFT procedure, as well as distorted voltage/current waveforms registered as a sequence of samples.

Measurements of currents and voltages in the power supply circuit of ac arc furnace

A power system diagram of AC arc furnace with the medium voltage circuit highlighted and the locations of voltage and current measurements indicated are shown in Figure 1.

Electric power is fed into the arc furnace installation from an electric power system via TR1 network transformer, which supplies 30 kV medium voltage switchgear. At the point of common coupling (PCC), the short-circuit power relative to the power supply transformer is \( S_{KZ}/S_{Tt}=135 \). In the medium voltage switch station passive HF harmonics filters are installed, which also ensure compensation of inductive reactive power consumed by the electric arc furnace (EAF). The TR2 furnace transformer is powered via LR reactors. This transformer is equipped with tap changer with make it possible to regulate the voltage supplied to the electrodes of the furnace under load.

A PC with data acquisition board and DASYLab software was used for recording the phase voltage and current waveforms in the EAF power supply system. One of the benefits of recording the waveforms of instantaneous voltages and currents using a computer is that the measurement data can be digitally processed. It allows us to perform calculations in order to learn the effects of active power flow, taking into account the harmonics, and determine the power quality parameters. The principal procedures used in the calculations are the Fast Fourier Transform (FFT) [9] and transformation of non-sinusoidal waveforms from a three-phase neutral system \( l=2-3 \) to orthogonal coordinate’s \( \alpha-\beta-0 \) [10-12].

In the measurement system MS (Fig. 1), the necessary galvanic separation was achieved by the incorporation of LEM current and voltage transducers into the secondary circuit of VT and CT transformers installed in medium voltage circuit. Using the processed, standard signals from the LEM transducer circuits does not introduce any
significant measurement errors with respect to the module and phase changes of the measured waveform in a 2 kHz band, i.e. to the 40th harmonic [2,13].

The sampling frequency of the recorded signals was \( f_s = 20 \) kHz, which equals \( N = 400 \) samples per a period of \( T = 20 \) ms and is a sufficient value for a precise reproduction of the shape of the voltage and current, taking into account the harmonic spectrum to the 40th order, as required by the standards [14].

The measurements were taken during a full steelmaking cycle. An analysis of the active power flow was carried out for three operating states of the arc furnace, as indicated in Figure 2, respectively: point 1 marks the stage of melting the first scrap basket, point 2 marks the refining phase, shortly before shutting the furnace down to control the process parameters, and point 3 marks the refining phase just before the melting process is done.

Active power harmonic analysis based on voltage-current waveforms

Figure 2 presents the waveform of an average effective value of furnace transformer current in real time, but unsuitable for frequency analysis. To determine the harmonic components of the studied signals, it was necessary to extract at least one period (\( T = 20 \) ms) or a suitable window of e.g. eight periods. For this purpose, two representative furnace operation states were selected, for which the recorded waveforms of voltages and currents during a single full period of \( T = 20 \) ms are shown in Figures 3 and 4.

The content of the harmonic components for the measured voltages and phase currents of the load (Fig. 1) was determined on the basis of the recorded measuring signal samples \( \{u(1), \ldots, u(N)\}; \{i(1), \ldots, i(N)\} \) using Fast Fourier Transform (FFT) in MATLAB. Knowing the distributions in the Fourier series, the voltages measured for the particular phases can be expressed using relations (1÷3):

\[
(1) \quad u_{L1}(t) \Rightarrow \sum_{v=1}^{m} U_{vL1} \sin(\nu vt + \xi_{vL1})
\]

\[
(2) \quad u_{L2}(t) \Rightarrow \sum_{v=1}^{m} U_{vL2} \sin(\nu vt - \frac{2\pi}{3} + \xi_{vL2})
\]

\[
(3) \quad u_{L3}(t) \Rightarrow \sum_{v=1}^{m} U_{vL3} \sin(\nu vt + \frac{2\pi}{3} + \xi_{vL3})
\]
similarly, the load current equations can be expressed in the form of relations (4–6):

\[ i_{L_1}(t) = \frac{\text{FFT}(i_{L_1}(t))}{m_{\omega}} \sum_{v=1}^{m} I_{L_1}(v) \sin(v \alpha t + \phi_{v,L}) \]

\[ i_{L_2}(t) = \frac{\text{FFT}(i_{L_2}(t))}{m_{\omega}} \sum_{v=1}^{m} I_{L_2}(v) \sin\left(v \alpha t + \frac{2\pi}{3} + \phi_{v,L}\right) \]

\[ i_{L_3}(t) = \frac{\text{FFT}(i_{L_3}(t))}{m_{\omega}} \sum_{v=1}^{m} I_{L_3}(v) \sin\left(v \alpha t + \frac{2\pi}{3} + \phi_{v,L}\right) \]

where the symbols used in the equations (1+6) mean:

- \( \nu \) – harmonic order \( \nu = 1, \ldots, m \),
- \( \omega = 2\pi f_0 \); \( f_0 = 50 \text{ Hz} \),
- \( \phi_{v,L}, \psi_{v,L} \) – angles of phase shifts for voltages and currents, \( k \in \{1, 3\} \),
- \( j \in \{1, \ldots, N\} \) – measuring point number,
- \( \text{FFT} \) – Fourier transform procedure.

The modules and angles of phase shifts of harmonic voltages and currents in the medium voltage circuit for the particular phases: L1, L2, and L3, during the steel melting process can be determined for each of the periods \( T \) using formula (8):

\[ P_{v,k} = \frac{1}{T} \int_{0}^{T} u_{L,k}(t) \cdot i_{L,k}(t) dt = \sum_{v=1}^{V} U_{v,L,RMS} I_{v,L,RMS} \cos \phi_{v,L} = \sum_{v=1}^{V} P_{v,L,k} \]

where: \( \phi_{v,L} = \psi_{v,L} - \phi_{v,L} \) for \( k \in \{1, 3\} \),

\[ U_{v,L,RMS} = \sqrt{\frac{2}{T} \int_{0}^{T} u_{v,L}^2(t) dt} \; I_{v,L,RMS} = \sqrt{\frac{2}{T} \int_{0}^{T} i_{v,L}^2(t) dt} \] for \( k \in \{1, 3\} \).

The total active power of the furnace \( P_{M} \) determined directly using the measurements, is the sum of active powers calculated according to relation (8) for the particular phases:

\[ P_{M} = P_{M_1} + P_{M_2} + P_{M_3} = \sum_{v=1}^{\omega} \sum_{k=1}^{3} P_{v,k,L} = \sum_{v=1}^{\omega} P_{v,M} \]

The presence of voltage and current harmonics has a significant impact on the manner of active power flow in the medium voltage circuit of an arc furnace. In order to determine the impact of the particular harmonics in the process of transferring electric energy to the charge, a distribution of harmonic active power \( P_{v,k,L} \) was determined in accordance with relation (8) in each of the phases for the studied states of arc furnace operation. The obtained calculation results are shown in Figure 5 and Tables 1 and 2. They indicate that the active power of the arc is determined by the fundamental component, which equals approximately 15 MW/phase for the refining stage. The percentage and directional share of the power harmonics depends on the stage of the melting process. The impact of the harmonics is best seen at the stage of melting, where the resultant third power harmonic has the opposite direction (negative value) and reaches a value of approx. 0.5% of the fundamental power. The resultant fifth power harmonic for the melting stage has a compatible direction and its value is more than 10 times lower than the resultant third power harmonic. Different relations take place at the refining stage, i.e. the final stage of the melting process. At that time, the third and fifth power harmonics have opposing directions with relatively low power, whereas one of the phase harmonic powers has compatible direction.

The detailed values and distribution of harmonic active power flow directions for the various phases and operating states of an arc furnace as a nonlinear load with variable parameters are shown in Tables 1 and 2. From the calculations it is apparent that the highest value is the power of the third harmonic at the melting stage. While the flow directions at the melting stage are repeatable, at the refining stage, due to a lesser degree of distortion of currents and lower THD I, the harmonic direction flows are random and the markings in Table 2 should not be taken as meaningful.

Table 1. Determination of the values and directions of harmonic active phase power flows of ac arc furnace at the melting stage

<table>
<thead>
<tr>
<th>( \nu )</th>
<th>( P_{v,11} ) kW</th>
<th>( P_{v,12} ) kW</th>
<th>( P_{v,13} ) kW</th>
<th>( P_{v,1M} ) kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+14612,8</td>
<td>+14074,1</td>
<td>+9274,4</td>
<td>+37961,3</td>
</tr>
<tr>
<td>2</td>
<td>+4,4</td>
<td>+1,0</td>
<td>+4,1</td>
<td>+9,5</td>
</tr>
<tr>
<td>3</td>
<td>-30,5</td>
<td>-30,6</td>
<td>-112,4</td>
<td>-173,5</td>
</tr>
<tr>
<td>4</td>
<td>-6,6</td>
<td>-0,5</td>
<td>-4,3</td>
<td>-11,4</td>
</tr>
<tr>
<td>5</td>
<td>+2,0</td>
<td>+2,6</td>
<td>+9,5</td>
<td>+14,1</td>
</tr>
<tr>
<td>6</td>
<td>+1,8</td>
<td>+0,4</td>
<td>+2,5</td>
<td>+4,7</td>
</tr>
<tr>
<td>7</td>
<td>-0,7</td>
<td>-0,8</td>
<td>+1,9</td>
<td>+0,4</td>
</tr>
<tr>
<td>8</td>
<td>-2,6</td>
<td>-0,4</td>
<td>-5,0</td>
<td>-8,0</td>
</tr>
<tr>
<td>9</td>
<td>+0,8</td>
<td>-0,5</td>
<td>+1,2</td>
<td>+1,5</td>
</tr>
<tr>
<td>11</td>
<td>-2,0</td>
<td>+0,0</td>
<td>-2,8</td>
<td>-4,8</td>
</tr>
</tbody>
</table>

Table 2. Determination of the values and directions of harmonic active phase power flows of ac arc furnace at the refining stage

<table>
<thead>
<tr>
<th>( \nu )</th>
<th>( P_{v,11} ) kW</th>
<th>( P_{v,12} ) kW</th>
<th>( P_{v,13} ) kW</th>
<th>( P_{v,1M} ) kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+15022,7</td>
<td>+15966,3</td>
<td>+14748,7</td>
<td>+45737,7</td>
</tr>
<tr>
<td>2</td>
<td>-0,1</td>
<td>-0,2</td>
<td>-0,6</td>
<td>-0,9</td>
</tr>
<tr>
<td>3</td>
<td>-0,5</td>
<td>-0,7</td>
<td>+0,3</td>
<td>-0,9</td>
</tr>
<tr>
<td>4</td>
<td>-0,0</td>
<td>+0,0</td>
<td>+0,2</td>
<td>+0,2</td>
</tr>
<tr>
<td>5</td>
<td>-3,6</td>
<td>+0,7</td>
<td>-3,9</td>
<td>-6,8</td>
</tr>
<tr>
<td>6</td>
<td>-0,0</td>
<td>+0,0</td>
<td>+0,0</td>
<td>+0,0</td>
</tr>
<tr>
<td>7</td>
<td>-0,5</td>
<td>-1,4</td>
<td>-0,4</td>
<td>-2,3</td>
</tr>
<tr>
<td>8</td>
<td>+0,0</td>
<td>+0,0</td>
<td>+0,0</td>
<td>+0,0</td>
</tr>
<tr>
<td>9</td>
<td>-0,0</td>
<td>-0,1</td>
<td>-0,0</td>
<td>-0,2</td>
</tr>
<tr>
<td>11</td>
<td>-0,0</td>
<td>-0,1</td>
<td>+0,0</td>
<td>-0,1</td>
</tr>
</tbody>
</table>

Some of the harmonic active phase power values in Table 2 are very low, in the order of Watts or dozens of Watts, therefore they are marked as 0.0, assigning a relevant flow direction mark.
Comparison of the three-phase active power of ac arc furnace with active power of the orthogonal components

The three-phase active power, with the use of definitional integral relation according to equations (8) and (9) contains a component characterized by a variability of pulse equal to twice the mains frequency. For non-sinusoidal waveforms of voltages and currents, the shape of instantaneous values of active power is also distorted, with a clear fundamental harmonic equal to \( f_P = 2f = 100 \text{ Hz} \), imposed on the constant component representing the value of the resultant active power consumed by the nonlinear load. This waveform is marked in purple and shown in Figures 6 and 7. In analogy to the dynamics of three-phase electrical machines [15-16], transformation from a three-phase neutral system 1-2-3 to orthogonal coordinates [10-12] is also used in the analysis of the phenomena occurring in three-phase circuits with non-sinusoidal currents and voltages. The Fortescue transformation [11] is used in the general case it is, while Clarke transformation [11-12] can be used in the case of three-phase circuits (without neutral conductor). If axes 1 and \( \alpha \) overlap, the transformation matrix takes the form [12]:

\[
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix}
1 & -1& 1 \\
\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & 0 \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix} \begin{bmatrix}
I_x \\
I_y \\
I_z
\end{bmatrix}
\]

where \( F \) refers to the voltage (\( u \)) or the current (\( i \)), and the parameter \( \frac{\sqrt{3}}{2} \) ensures invariance of the power transformation.
Since in the three-phase three-wire system the phase currents and the phase voltages relative to the neutral point satisfy the relationship (11):
\[ i_{L1}(t) + i_{L2}(t) + i_{L3}(t) = 0 \]
\[ u_{L1}(t) + u_{L2}(t) + u_{L3}(t) = 0 \]
(11)
Clarke transformation of phase voltages and currents can be simplified to the following form (12):
\[
\begin{bmatrix}
  u_\alpha \\
  u_\beta
\end{bmatrix} = \frac{2}{\sqrt{3}} \begin{bmatrix}
  \frac{1}{2} & 0 \\
  \frac{1}{2} & \frac{-1}{\sqrt{2}} \\
  \frac{-1}{2} & \frac{-1}{\sqrt{2}}
\end{bmatrix} \begin{bmatrix}
  u_{L1} \\
  u_{L2} \\
  u_{L3}
\end{bmatrix} = \frac{2}{3} C_{\alpha\beta} \begin{bmatrix}
  u_{L1} \\
  u_{L2}
\end{bmatrix}
\]
(12)
\[
\begin{bmatrix}
  i_\alpha \\
  i_\beta
\end{bmatrix} = \frac{2}{\sqrt{3}} \begin{bmatrix}
  \frac{1}{2} & 0 \\
  \frac{-1}{2} & \frac{1}{\sqrt{2}} \\
  \frac{-1}{2} & \frac{-1}{\sqrt{2}}
\end{bmatrix} \begin{bmatrix}
  i_{L1} \\
  i_{L2} \n\end{bmatrix} = \frac{2}{3} C_{\alpha\beta} \begin{bmatrix}
  i_{L1} \\
  i_{L2}
\end{bmatrix}
\]
Clarke transformation to the orthogonal coordinate system \( \alpha\beta \) in the form (12) was applied to the particular harmonics of the \( \nu \)-th order taking into account the harmonic transformation matrix \( C_{\nu \alpha \beta} \) (13):
\[
\begin{bmatrix}
  u_{\nu \alpha} \\
  u_{\nu \beta}
\end{bmatrix} = C_{\nu \alpha \beta} \begin{bmatrix}
  u_{\nu L1} \\
  u_{\nu L2}
\end{bmatrix} ; \quad \begin{bmatrix}
  i_{\nu \alpha} \\
  i_{\nu \beta}
\end{bmatrix} = C_{\nu \alpha \beta} \begin{bmatrix}
  i_{\nu L1} \\
  i_{\nu L2}
\end{bmatrix}
\]
(13)
where:
\[
C_{\nu \alpha \beta} = \begin{bmatrix}
  \frac{\sqrt{3}}{2} & 0 \\
  \frac{-1}{2} & \frac{1}{\sqrt{2}}
\end{bmatrix}
\]
(14)
Then, with the use of the \( p-q \) instantaneous power theory [10-12], the value of instantaneous power for each harmonic voltages and currents were determined according to the equations (15) for the first harmonic and (16) for the \( \nu \geq 2 \) harmonics:
\[
P_j = u_{j \alpha} \cdot i_{j \alpha} + u_{j \beta} \cdot i_{j \beta}
\]
(15)
\[
P_\nu = u_{\nu \alpha} \cdot i_{\nu \alpha} + u_{\nu \beta} \cdot i_{\nu \beta}
\]
(16)
The total value of instantaneous power in orthogonal axes \( \alpha\beta \) is:
\[
P_{\Sigma \nu \alpha \beta} = \sum_{\nu = 1}^{m=40} P_\nu
\]
(17)
The changes of total instantaneous power in the orthogonal \( \alpha\beta \) axis system described by the relation (17) is presented in black on the graphs in Figures 6 and 7 for both states of operation of the AC arc furnace.

The third way to determine the instantaneous power with the use of orthogonal components of voltages and currents brought to a \( \alpha\beta \) coordinate system involved transformation of a discrete set of \( N \) points of measurement data (voltages and currents) using matrix notation (12).

The instantaneous power for each \( j \)-th measurement point is defined as:
\[
p_{\alpha\beta}(j) = u_{\alpha}(j) \cdot i_{\alpha}(j) + u_{\beta}(j) \cdot i_{\beta}(j)
\]
(18)
A graph of the instantaneous power in the discrete form for \( N \) measuring points is presented in Figures 6 and 7 in the form of points.

By averaging the sum of instantaneous powers for all \( N \) points according to the relation (19), the active power from the period consumed by the nonlinear three-phase load is calculated:
\[
P_{\alpha\beta} = \frac{1}{N} \sum_{j=1}^{N} p_{\alpha\beta}(j)
\]
(19)
where the \( j \) index used in the equations (18) and (19) is specified as \( j \in \{1, ..., N\}, N = f_m \cdot f \).

The similarity of calculations for the proposed methods is satisfactory both for the stage of refining and melting, when the greatest distortions of phase voltages and currents occur.
Table 3. The values of active power

<table>
<thead>
<tr>
<th>Stage</th>
<th>Active power for $\nu=1$</th>
<th>$\sum_{\nu=1}^{40} P_{\nu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melting $\tau=16$ minute</td>
<td>37.78 MW 37.79 MW 37.78 MW 37.80 MW</td>
<td></td>
</tr>
<tr>
<td>Refining $\tau=76$ minute</td>
<td>45.73 MW 45.73 MW 45.73 MW 45.73 MW</td>
<td></td>
</tr>
<tr>
<td>Refining $\tau=82$ minute</td>
<td>46.68 MW 46.68 MW 46.68 MW 46.68 MW</td>
<td></td>
</tr>
</tbody>
</table>

Conclusion
The results of the present measurement analysis of AC arc furnace operation at the level of average voltage allow us to identify the phenomena of active power flow for different stages of the melting process, taking particular power harmonics into account. Owing to the conducted measurements using dedicated computer software, it was possible to select the harmonic components of active harmonic power. The highest average phase power value was observed for the third harmonic, whose flow had the opposite direction to the fundamental component (Figure 5a). This phenomenon is recorded in a metering and billing system equipped with four-quadrant Landis+Gyr meters for harmonic power. The highest average phase power value was recorded in a metering and billing system equipped with four-quadrant Landis+Gyr meters for the AC arc furnace. In order to verify the results, calculations were performed using Clarke transformation in an orthogonal $\alpha-\beta$ axis system for the harmonic distributions of voltage and current signals and transformation of discrete measurement data on a set of four hundred elements corresponding to the period of the fundamental component. It was observed that there is high similarity between the results obtained using these different methods concerning the instantaneous and average active power consumed by the AC arc furnace.

REFERENCES