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A Novel Methodology for Signal Recovery in Linear Time Invariant Systems for Medicine Applications

***Abstract**. This paper presents a deconvolution based method that allows the improvement of the time response of a Swan-Ganz catheter. The goal of the deconvolution method is to obtain the input signal of an LTI system from the knowledge of its output and its impulse response. The noise causes degradation of the recovered signal, and this phenomenon is also discussed. The deconvolved signal is enriched in function of the cutoff frequency of the filter. It is shown that as the cutoff frequency increases, the deconvolved signal becomes more nosier. A Butterworth digital filter of third order, multiplied by the reverse impulse response was used in the frequency domain, and the response in frequency for the 1/H(s) function, limited for the various cutoff frequency of the filter are shown. It can be clearly observed that for higher cutoff frequency magnitude, the signal becomes completely distorted. Experiments with animals were used to measure the impulse response and the obtained results were satisfactory, with average error of 8.9%. The results suggest that the method can be useful in applications of linear systems.*

Streszczenie.** Opisano metodę poprawy odpowiedzi czasowej cewnika typu Swan_ganz bazująca na rozplocie. Umożlia to ocenę sygnału wejściowego na podstawie odpowiedzi układu I znajomosci odpowiedzi impulsowej. . W pracy analizowano wpływ filtru cyfrowego na parametry sygnału. Metode z dobrym rezultatem przetestowano na zwierzętach.* ***Nowa metoda odzyskiwania sygnału w układach cyfrowych stosowanych w medycynie bazująca na rozplocie

Keywords: thermodilution. deconvolution. Signal recovery. medicine.
Słowa kluczowe: odzyskiwanie sygnału, rozplot, cewnik

Introduction

In many situations, in intensive care units (ICUs), it is important to know the values of some cardiovascular parameters, including cardiac output and ejection fraction of the ventricles. The heart and the vascular bed form a system for distribution of gas and nutrients and extraction of residues generated by the human metabolism. By bringing the oxygen and nutrients to the cells, and removing the carbon dioxide and the metabolites, the circulatory system establishes a communication between the external world and the living organism. The circulatory system also allows the distribution of hormones and helps several defense, aiding in the distribution of leukocytes and substances related to blood coagulation [11].

Since the early 1970s, the pulmonary artery catheter, also known as Swan-Ganz catheter, has been used to estimate the cardiac output. These catheters are based on the principle of thermodilution. In this principle, a physiological solution in a temperature lower than the blood temperature is rapidly injected into the right atrium or the right ventricle of the heart, mixing with the blood in these chambers, changing the blood temperature. Then, the blood temperature is measured in the pulmonary artery as a function of time, yielding the thermodilution curve.

It can be shown [3] that cardiac output (amount of blood pumped by the right ventricle per minute), is approximately equal to a constant divided by the area under the thermodilution curve. This method can measure, with good accuracy, the cardiac output of the right ventricle.

At the end of the 1960s [9] researchers realized that it would also be possible to determine, by means of thermodilution, ejection fraction and end-diastolic volume of the right ventricle. However, to make this possible, it was necessary that the catheter was equipped with a much faster temperature sensor that the sensor used in standard catheters. Initially this assembly was feasible only in research laboratories. In 1980, catheters in which a faster temperature probe was manually assembled in the catheter were released. With these faster sensors, it became possible to measure, besides the cardiac output, ejection fraction and maximum diastolic volume. However, the price of these special catheters was much higher than the standard catheters because of the difficulty in assembling the fast sensor.

In [3], it was shown that is possible to measure the right ventricular ejection fraction even with the slow sensor standard catheter. For a full discussion on the improvement of the catheter response performance using techniques of digital signal processing we suggest reading the following references [1, 4, 6]. It was proposed that the reduction of the effects of thermal inertia of the slow sensor could be accomplished through a mathematical deconvolution operation, that would yield a signal that would allow the measurement of ejection fraction.

The proposed method [3] had limitations, taking around 30 minutes to run. The method was later improved in [5], and began to run in a few minutes. However, the method in question had a disability: its convergence depended on a certain degree of the initial estimates of the catheter parameters. The method was based on spectral analysis.

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In this context, this paper proposes a method based on the time domain [7], which has better convergence characteristics that the methods presented in [3] and [5], requiring a processing time of a few seconds and converging to virtually all cases. Furthermore, as an additional contribution, the catheters used in this work are cheaper than those catheters with fast sensors, with a difference of a few hundred dollars. This work presents the method, its implementation, and experimental to assess its precision.

The Deconvolution Operation

Convolution is a mathematical way of combining two signals to form a third signal. It is the most important technique in Digital Signal Processing. Using the impulse decomposition strategy, systems are described by a sign called impulse response. Convolution is important because it relates to the three signals of interest [10].

The deconvolution is an inverse mathematical operation of convolution. In this operation, the input signal is obtained through a mathematical solution involving the Fourier transform of the input and output. Dividing them point by point in the frequency domain the output by the input, and obtaining the inverse Fourier transform of this result, the entry is obtained, thereby characterizing the deconvolution operation. This operation is shown below.

The output of a linear time-invariant system can be represented as a convolution of an input signal with the impulse response of the system, as shown in equation (1) in the time domain.

$$T_{measured}(t) = h(t) * T_{real}(t) \tag{1}$$

where $T_{measured}(t)$ is the temperature signal in the time domain, $h(t)$ is the response of the sensor impulse, and $T_{real}(t)$ is the temperature been measured. The asterisk means that the convolution operation in this case. In the frequency domain, the Eq. (1) becomes a point to point multiplication, as can be seen in Eq. (2).

$$T_{measured}(j\omega) = H(j\omega).T_{real}(j\omega) \tag{2}$$

In Eq. (2), $T_{measured}$(jω) is the FFT - Fast Fourier Transform - of the temperature signal, H(jω) is the FFT of the sensor impulse response, T_{real}(jω) the FFT of the temperature been measured.

The deconvolution [1, 2] can then be obtained by taking the FFT of the impulse response of the system, which in this case is a sensor of a catheter, and the FFT of the measured signal which is the signal obtained of the thermodilution sensor. Equations (1) and (2) indicate that the temperature measured by the sensor actually corresponds to the convolution between the actual temperature and the impulse response of the sensor. The division point to point FFT signal measured by the FFT of the impulse response of the system is the deconvolution proposed in this paper. The deconvolution can be applied to various types of physical systems from the system to be LTI [5,7].

The situation can be better understood in Fig. 1 wherein is shown that the obtained response is emanating from a convolution in the frequency, multiplied point by point, for an inverse response of a digital filter of the FIR type (Finite Impulse Response) or IIR (Infinite Impulse Response). In Fig. 1the real temperature signal in the time domain, T(jω), is modified by the impulse response of the sensor H(jω) to give the measured signal T(jω). If the impulse response of the sensor, H(jω), is precisely known, then you can determine what was the original signal, T (jω), passing the signal through a filter that corresponds to the frequency domain, 1/H(jω). The obtained result, $T_{filtered}$(ω), is an estimate of the actual temperature signal.

In fact, in [3] it is shown that to obtain sensor impulse response may be performed by differentiating a step response. The output of a linear time-invariant system is the convolution of an input signal with the impulse response of the system, according to Eq.1.

For discrete systems that are linear and time-invariant, the expression of the convolution can be given by the following equation.

$$y[n]_n = \sum_{m=0}^{N-1} h[n-m]\,x[m] \tag{3}$$

where $x[n]$ is the discrete signal system input, $h[n]$ is the discrete response in time for the impulse of the system, and $y[n]$ is the discrete output signal in time. Equation (3) can be rewritten in a more synthetic way as:

$$y_n = \sum_{m=0}^{N-1} h_{n-m} x_m \tag{4}$$
,

where x_n is the input discrete signal of a system, h_n is the discrete response in time for the impulse of this system, and y_n is the discrete output signal in time. The problem of the Eq. (4) can be described by a matrix ratio, which is indicated more synthetically by Equation (5).

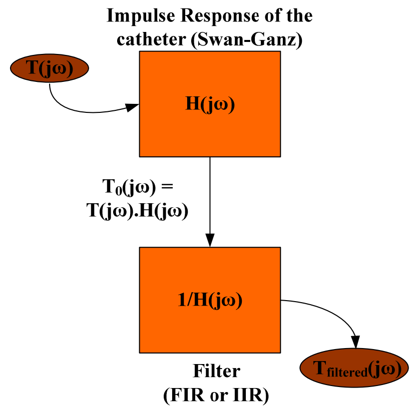


Fig. 1: Block diagram illustrating the problem of deconvolution. H(jω) is the FFT of the impulse response of the sensor, T(jω) is the FFT of the real temperature, and T_0(jω) is the FFT in module for the measured temperature. The measured signal is filtered by a filter with a frequency response equivalent to 1 / H(jω), and the result, $T_{filtrado}$ (jω) is an estimate for the actual signal.

$$[y] = [h][x] \tag{5}$$

where [y] is the system output vector, [h] is the Toeplitz matrix representing the discrete convolution and [x] is the discrete input vector. In this case, M = N, and expanding Equation (5) comes to the circulant matrix shown in Equation (6), which is a special case of a Toeplitz matrix.

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ \bullet \\ \bullet \\ y_{N-4} \\ y_{N-3} \\ y_{N-2} \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} h_0 & 0 & 0 & 0 & \bullet\bullet\bullet\bullet & 0 & 0 & 0 & 0 \\ h_1 & h_0 & 0 & 0 & \bullet\bullet\bullet\bullet & 0 & 0 & 0 & 0 \\ h_2 & h_1 & h_0 & 0 & \bullet\bullet\bullet\bullet & 0 & 0 & 0 & 0 \\ h_3 & h_2 & h_1 & h_0 & \bullet\bullet\bullet\bullet & 0 & 0 & 0 & 0 \\ & & & & \bullet\ \bullet\ \bullet & & & & \\ & & & & \bullet\ \bullet\ \bullet & & & & \\ h_{M-7} & h_{M-8} & h_{M-9} & h_{M-10} & \bullet\ \bullet\ \bullet & h_0 & 0 & 0 & 0 \\ h_{M-3} & h_{M-4} & h_{M-5} & h_{M-6} & \bullet\ \bullet\ \bullet & h_1 & h_0 & 0 & 0 \\ h_{M-2} & h_{M-3} & h_{M-4} & h_{M-5} & \bullet\ \bullet\ \bullet & h_2 & h_1 & h_0 & 0 \\ h_{M-1} & h_{M-2} & h_{M-3} & h_{M-4} & \bullet\ \bullet\ \bullet & 0 & h_2 & h_1 & h_0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \bullet \\ \bullet \\ x_{N-4} \\ x_{N-3} \\ x_{N-2} \\ x_{N-1} \end{bmatrix} \tag{6}$$

In the matrix above, [y0...y_{N-1}] is the output vector of the system, [h] MxN is a Toeplitz matrix, and [x0 ... x_{N-1}] is the discrete input vector. The problem of deconvolution can be regarded as the inverse solution of the linear system proposed by Eq. (6). Thus, to perform the deconvolution can adopt the methodology for obtaining the vector xn such that:

$$[x] = [h]^{-1}[y] \tag{7}$$

Note that [y] is the system output vector, $[h]^{-1}$ is the inverse Toeplitz matrix and [x] is the discrete input vector. The matrix of the DFT (Discrete Fourier Transform) is shown in Eq. (8),

$$[W]=\frac{1}{N}\begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & W^{-1} & W^{-2} & \dots & W^{-(N-1)} \\ 1 & W^{-2} & W^{-4} & \dots & W^{-2(N-1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & W^{-(N-1)} & W^{-2(N-1)} & \dots & W^{-(N-1)^2} \end{bmatrix} \quad (8)$$

where $W=e^{-j\frac{2\pi}{N}}$. Multiplying both sides of Eq. (5) by Fourier matrix, we obtain Eq. (9).

$$\begin{aligned}&[W][y]=[W][h][x]\\ &[W][y]=[W][h][W]^{-1}[W][x]\Leftrightarrow[Y]=[H][X]\end{aligned} \quad (9)$$

The W^{-1} matrix is the complex conjugate matrix of W. It is not difficult to show that $[W].[h].[W]^{-1}$ is a diagonal matrix. Therefore, in the frequency domain, the Eq. (10) must be solved:

$$X=[H]^r.[Y] \quad (10)$$

where [H]r is a diagonal matrix whose elements are the reciprocals of the matrix [H]. It is important to note that an advantage of using the DFT, is run away from a matrix inversion that it is often slow in the time domain, to a more efficient operation because it can use the FFT of the obtained signals, from both the impulse response, as the system output. In this way, the proposed algorithm is optimized, there is seen that avoids a matrix inversion in the time domain.

Validation of the Proposed Methodology

The proposed system in this paper can be modelled by a temperature sensor based on a thermocouple, and the impulse response thereof, can be characterized as a sum of one or more exponentials. So, suppose the function h[n] below:

$$h[n]=2e^{-3n} \quad (11)$$

The graph corresponding to h[n] is shown in Fig. 2. The time section in question is 10s sampling and the period is 0.1s (sampling frequency 10 Hz).

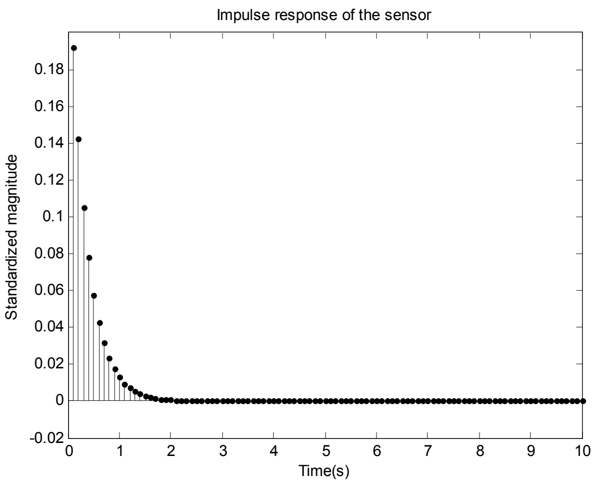
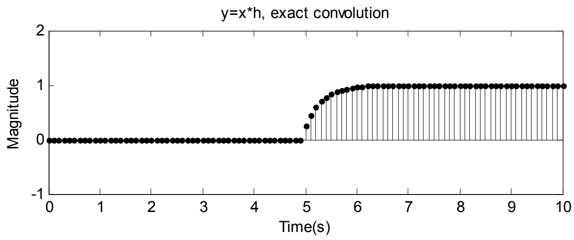


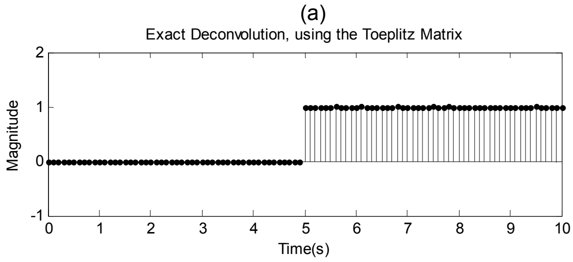
Fig. 2. Response to the exponential impulse, which corresponds to a hypothetical h[n], representing the impulse response for the sensor. The exposed function is normalized by dividing the amplitude of each point by the sum of the amplitudes of all samples, so the system does not enter the signal energy. The time interval in question is 10 s and the sampling period is 0.1 sec (sampling frequency 10 Hz).

An offset unit step (u (n-50)), is shown in Fig. 3(b). The convolution of the discrete signals of Figs. 2 and 3(b) is shown in Fig. 3(a). Applying the mathematical exposed above, and using the Toeplitz matrix is obtained the curve that is shown in Fig. 3 (a), faithfully reproducing a step unit.

A Gaussian noise is added to the output y[n], as illustrated at the top of Fig. 4(a). The noise has zero mean, standard deviation zero, with values ranging from 0 to 0.01 [7]. It is clearly seen that a small noise may distort the deconvolved signal displayed in Fig. 4(b).

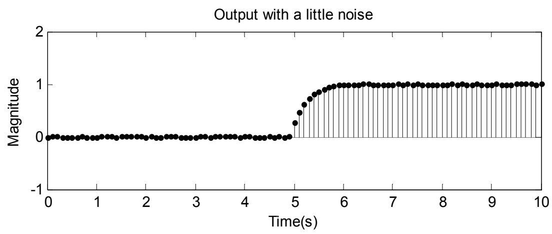


(a)

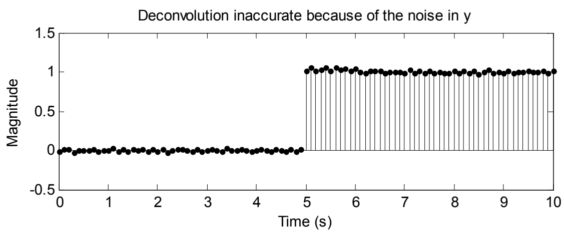


(b)

Fig. 3. A unit step. A signal step was used because the same is more common, and is a classical input for analysis of linear time invariant systems. In this case the input signal is shifted a unit step (u (n-50)). The convolution of the discrete signals of Figs. 2 and 3 are shown in the upper curve of Figure 4. Applying mathematical exposed in Deconvolution The Operation section using the Toeplitz matrix is obtained the curve shown in the lower part of Fig. 4, which is not accurate, and clearly not faithfully reproduces the input of the curve of the expected system that would be a step without any noise.



(a)



(b)

Fig. 4. Response to the exponential impulse. In the first figure got the deconvolution inaccurate, and in figure below has an entry with noise. System output with a small noise in Fig.4 (a) and inexact deconvolution shown in Fig. 4 (b) due to noise. The system output, illustrated in the upper curve, now has a small superimposed noise. Note the effect of this little noise deconvolution illustrated in the bottom graph of this figure in getting the input, the unit step.

A signal obtained through three exponential sensor deconvolution method is shown in Fig. 6 [2,3,4,7].

(12) $$h(t) = K\left(e^{-at} + B.e^{-bt} + C.e^{-ct}\right)$$

where h(t) the impulse response, a, B, b, C, c are parameters obtained in the characterization of the sensor, and K is a constant that is set such that the area under h(t) is equal to 1, that is:

(13) $$K = \frac{1}{\frac{1}{a} + \frac{B}{b} + \frac{C}{c}}$$

It can be observed the system input with a small noise in Fig. 4 (a), and the inexact deconvolution in Fig. 4 (b) due to noise. The output system, illustrated in the upper curve in Fig. 4, has a small overlapped noise. Note the effect of this little noise deconvolution illustrated in the bottom graph of this figure. It is clearly seen that a small noise can greatly distort the deconvolved signal. The result for a noise slightly higher in y [n] is shown in Figure 4 bellow, where it can be seen that the deconvolved time for input signal is distorted.

Results

Figure 5 shows an automated system for obtaining the impulse response in laboratory and Figure 6 shows a result with the ideal curve, provided by the laws of thermodynamics and a real answer. The curve have the same area, and this area is used to measure the ejection fraction and cardiac output of a given circulation.

The computer simulations show what are the limits of the proposed algorithms: the algorithm is not satisfactory operation for heart rates above 200 and ejection fractions above 0.8. There are two reasons for the failure algorithm for high heart rates and higher ejection fractions.

The first reason is that for high ejection fractions fall is too abrupt, and at the first stroke, the curve drops to 10% of its value, and second, to 1%. Thus, the algorithm has no reference points sufficient for deconvolution, which leads to failure. It is important to note, however, that even ideal systems with very fast sensors allow the medical needs of larger ejection fractions 0.7. The reason is as follows: the first plateau just over 30%, the second to 9%, 3% the third and fourth 1%. The first pair, which is the most accurate, their accuracy is impaired by the fact that immediately after the start of decay, there are still remaining mixture injection. The second and third pairs are inaccurate due to its small amplitude. Thus, this limitation is not only for the system with deconvolution, but also with the system with fast sensor.

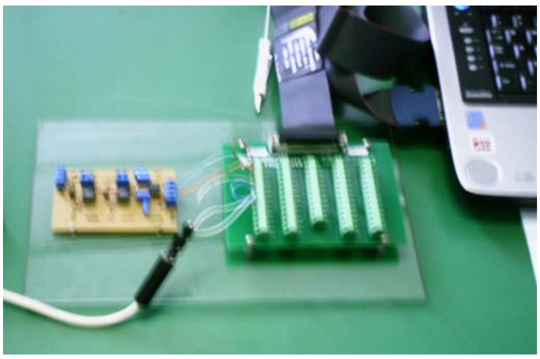


Fig. 5.Automated data acquisition that obtains the impulse response in an experiment with animals.

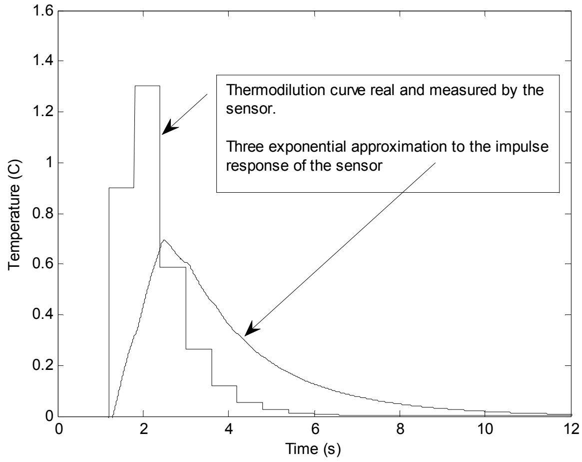


Fig. 6. In the above figure is shown an idealized version of the actual temperature and the temperature measured by the Swan-Ganz catheter.

The second reason for the error is that to heart rates above 150, the plateaus are very close, and the algorithm does not have strong support on the plateau, which damages the functioning. This is a fundamental limitation of the algorithm proposed here, it is perhaps only resolved in future developments. However, if a measure with lower ejection fractions has been made before the measure with very high heart rates, an estimated sensor response can be properly used for high frequencies.

The filter response 1/H(jω) strongly enhances the high frequencies. Clearly, when there is additive noise, this is also highly amplified, distorting the results of the signals that could be deconvolved .

Figure 7 shows the deconvolved signal to four cutoff frequencies of the lowpass filter. It can be clearly observed that on the 5 Hz cutoff frequency, the signal is better reconstructed, and the distortion significantly increases for higher cutoff frequencies.

The answer with excessive noise is due to the enrichment of the high frequency noise by the factor 1/H(jω). To delimit this problem, the response of the inverse filter must be restricted. In this case, it was used a Butterworth digital filter of third order, multiplied by the inverse impulse response in the frequency domain.

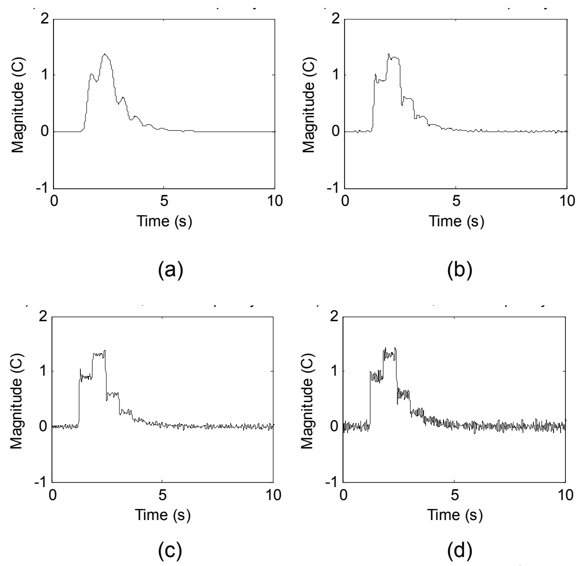


Fig. 7. Deconvolved signal using a Butterworth lowpass filter with the following frequency cutoffs: (a) 2Hz; (b) 5Hz; (c) 10Hz; and (d) 15 Hz.

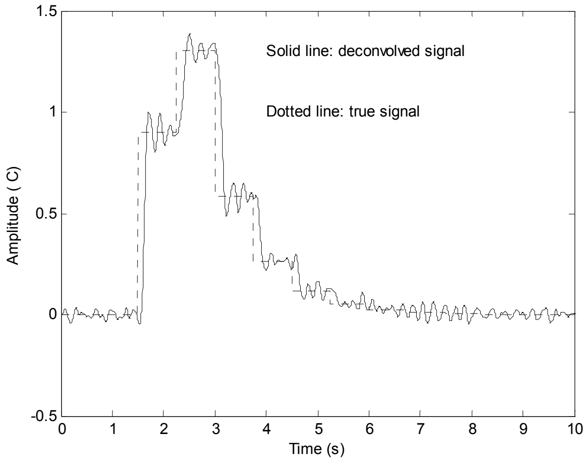


Fig. 8. Deconvolved signal with delay due the digital filter for the 5Hz cutoff frequency. The filter imposes a delay on the system response, and with the deconvolution the same problem appears. This is solved by moving the signal for an approximate amount of samples.

In Fig. 7 it is not possible to see, but there is a delay imposed by the filter response. This can be observed in Fig. 8 by a 5 Hz cutoff frequency.

The choice of the filter used in this paper may not have been the ideal one, and many parameters could be defined to impose a more accurate response. The fact is that the optimization depends on the definition of a function error. An extensive discussion of these enhancements can be found in [3] and [8].

Conclusions

The new methodology has a different principle, working in the time domain. The computer simulations have shown an excellent performance for heart rate below 180 beats per minute and ejection fractions less than 0.8. Experimental results with real signals, obtained from mechanical model for the cardiovascular system, have led to fairly good results, with an average error of 8.9%.

The maximum convergence time for the algorithm proposed in this work was 9s for the worst case. It is possible that, with further development and more comprehensive testing, the method might have clinical applications.

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REFERENCES

[1] Bickle, L. W., (1971). “A time domain deconvolution technique for the correction of transient measurements,” Report SC-RR-71 06658, Sandia Laboratories, Albuquerque, New Mexico, Exploratory Measurements Division 1442, November.

[2] da Rocha, A. F.; dos Santos, I.; Nascimento, F. A. O. & de Melo, M. D. B. (2005). Dieter Haemmerich and Jonathan W Valvano, “Effects of the time response of the temperature sensor on thermodilution measurements”, Physiol. Meas. 26.

[3] da Rocha, A. F. (1997). “The Dynamic Behavior of Thermistor Probes”, Dissertation for degree of Doctor of Philosophy, University of Texas, Austin, May.

[4] Dantzig, J. A. (1985). “Improved transient response of thermocouple sensors,” Rev. Sci. Instrum., v. 56, no. 5, pp. 723-725.

[5] dos Santos, Í. (2000). Proposta de um método para medição da fração de ejeção do ventrículo direito, Dissertação (Mestrado em Engenharia Biomédica), Departamento de Engenharia Elétrica, Universidade de Brasília, Brasília.

[6] Hori, J.; Saitoh, Y. & Kiryu, T. (1994). “Improvement of the time-domain response of a thermodilution sensor by the natural observation system,” IEICE Trans. Fundamentals, v. E77-A, n. 5, may.

[7] de Melo, M. D. B. (2007). Algoritmo para Recuperação de Sinais de Temperatura de Cateteres de Artéria Pulmonar. Tese de Doutorado, Departamento de Engenharia Elétrica (ENE), Universidade de Brasília.

[8] Riad, S., M., “The deconvolution problem: an overview,” Proceedings of the IEEE, v. 74, pp. 82-85, January 1986

[9] Salgado, C. R., Galletti, P. M., “In vitro evaluation of the thermodilution technique for the measurement of ventricular stroke volume and End-Diastolic Volume,” *Cardiologia*, vol. 49, n.2, pp. 65-78, 1966.

[10] Smith, S. W., The Scientist and Engineer's Guide to Digital Signal Processing, Second Edition. www.dspguide.com

[11] Mouncastle, Vernon B., “Medical Physiology”, Copyright 1974 by The C. V. Mosby Company.