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doi:10.15199/48.2016.11.17

Decomposition of mechanical vibration signals – the Hilbert-Huang Transform in the time domain and the Fourier Transform in the frequency domain

***Abstract**. The investigation results concerning comparisons of effects of the application of the Hilbert-Huang Transform (HHT), Fourier Transform (FT) and Short Time Fourier Transform (STFT) for the vibration signals decomposition, are presented in this paper. The record of the vibration accelerations signals of the electric torque tool was applied in investigations. It was shown that the FT transform, often applied in practice for vibration signals analysis can provide uncertain or erroneous information. Thus, it requires verifications by means of other methods, e.g. STFT or HHT.*

Streszczenie.** W artykule przedstawiono wyniki badań dotyczących porównania skutków zastosowania do dekompozycji sygnałów drgań transformacji Hilberta Huanga HHT, Fouriera FT oraz krótko-czasowej transformacji Fouriera STFT (Short Time Fourier Transform). Do badań wykorzystano zapis przykładowych sygnałów przyspieszeń drgań zakrętarki elektromechanicznej. **(Dekompozycja sygnałów drgań mechanicznych – transformacja Hilberta-Huanga HHT w dziedzinie czasu a transformacja Fouriera w dziedzinie częstotliwości).

Keywords: signals decomposition, vibration signals, Hilbert transform, HHT Huang-Hilbert transform, Fourier transform.
Słowa kluczowe: dekompozycja sygnałów, sygnały drgań, transformacja Hilberta, transformacja Huanga HHT, transformacja Fouriera.

Introduction

The research scope related to mechanical vibrations analysis is very wide. Vibration signals decomposition is one of its essential branches. It concerns quantitative assessments, performed by mathematical methods, of phenomena occurring in mechanical objects.

One of the often applied in practice mathematical method is the frequency analysis with using either the Fourier transform (FT) or short-time Fourier transform (STFT) [1]. They allow, among others, determinations of the amplitudes decomposition of sinusoidal components of vibration signals for frequency values occurring in the analysed signal. Resonance properties of mechanical systems can be identified in this way, too. The Fourier analysis enables also obtaining information concerning phase dependencies between individual components which can be utilised in the systems diagnostics. However, the most essential information obtainable due to the Fourier transform is the determination of frequencies occurring in the vibration signal.

In contrast to the often applied Fourier signal analysis, where it is assumed that a signal is a sum of a certain number of sinusoidal waveforms, the Hilbert transform is important in the analysis. In this transform it is assumed that a signal has a form of an individual but modulated sinusoidal waveform [2–5]. The nature of vibration signals is often identical with this definition. Therefore, in several cases, the Hilbert transform and decomposition methods based on it, can be fulfilled much better than methods based on the Fourier transform. In addition, in case of analysis of phenomenon's having both a non-linear and non-stationary character [3], the Fourier transforms FT and STFT of signals can generate spectral characteristics, difficult for interpreting. Huang, in his paper [2] presented the new adaptation technique of representing signals characterising such phenomena as the sum of simpler components in the time domain. This technique called the Empirical Mode Decomposition (EMD) is able to separate – from each other – components overlapping in time and in frequency domains, which can not be separated by other standard filtration techniques. The EMD algorithm, by means of the intrinsic mode function (IMF), allows the signal decomposition into components from the highest to the lowest frequency. Broadening the EMD algorithm by the Hilbert transform is called the Hilbert-Huang Transform (HHT). The Hilbert transform allows determining instantaneous frequencies and amplitudes of the IMF components of the signals decomposed by the EMD method [6–13].

The Hilbert-Huang Transform (HHT)

The principle of operation of the EMD algorithm is based on the adaptative decomposition of the primary signal $x(t)$ into component functions IMF, which fulfil the given below conditions [2, 3, 7].

Condition 1: the number of extremes and the number of signal transitions via zero must be equal or can differ by one only.

Condition 2: the average value of the envelope interpolating local maxima and the envelope interpolating local minima equals zero.

The process of looking for each i-th IMF $h_i(t)$ component consists of subtraction of the local average $m_k(t)$ from the signal:

$$m_k(t) = \frac{e_{uk}(t)+e_{lk}(t)}{2} \quad for \quad k = 1,2,\dots,n \tag{1}$$

where: k – subsequent iteration during determining the i-th IMF component, $e_{uk}(t)$ and $e_{lk}(t)$ – upper and lower signal envelope, being spline functions of the third degree interpolating local maxima and minima of the signal, respectively, up to obtaining the IMF functions fulfilling both conditions given above.

This process is aimed at the elimination of small deformations in the vicinity of local minima and maxima and at leading to the quasi-symmetric form in relation to zero. When conditions 1 and 2 are fulfilled, the process of looking for the i-th IMF component $h_i(t)$ will be stopped. The signal $h_i(t)$ will be assumed as the i-th IMF component and the algorithm will start looking for the next IMF.

When performing the decomposition of signal $x(t)$ we obtain, in subsequent steps of the EMD algorithm the individual IMF components: $h_1(t)$, $h_2(t)$,...,$h_N(t)$ and the residual signal $r_N(t)$. Thus, finally, equation is satisfied [3,7]:

$$x(t) = \sum_{i=1}^{N} h_i(t) + r_N(t) \tag{2}$$

Each IMF component $h_i(t)$ is described by slow-changing functions of amplitude $A_i(t)$ and pulsation $\omega_i(t)$. This allows to understand a vibration process, in every time instant as a quasi-harmonic oscillation, which is modulated in the amplitude and frequency way by time variable amplitude $A_i(t)$ and pulsation $\omega_i(t)$ functions [3,6]:

$$h_i(t) = A_i(t)\cos\left(\int_0^t \omega_i(t)dt\right) \quad for \;\; i \in [1 \div N] \tag{3}$$

When $\hat{h}(t)$ is the Hilbert transform of signal $h(t)$, then using the properties of the analytical signal:

$$H(t) = h(t) + j\hat{h}(t) \tag{4}$$

it is possible to determine the instantaneous amplitude and pulsation of each i-th IMF component, on the bases of:

$$A_i(t) = \pm\sqrt{h_i^2(t) + \hat{h}_i^2(t)} \tag{5}$$

and

$$\omega_i(t) = \frac{d\psi_i(t)}{dt} = \frac{d}{dt}\left(\arctan\frac{\hat{h}_i(t)}{h_i(t)}\right) \tag{6}$$

A frequency resolution of the HHT method plays an essential role in differentiation of components. This resolution can be understood as the minimum difference between two harmonics, which are discriminable in the signal. The EMD algorithm is able to discriminate two harmonics only when their frequencies are significantly different. The boundary dividing two components, being in the vicinity, is the hyperbola described by the equation [2]:

$$\frac{A_2}{A_1} \le \left(\frac{\omega_2}{\omega_1}\right)^{-2} \quad for \;\; \frac{\omega_2}{\omega_1} > 1{,}5 \tag{7}$$

where: A_1, A_2 – amplitudes of successive harmonics,
ω_1, ω_2 – pulsations of successive harmonics.

Purpose and subject of investigations

The research purpose presented in the hereby paper was to investigate certain aspects of the application of the Hilbert-Huang Transforms (HHT) and Fourier FT and STFT transforms for decompositions of signals of vibration accelerations of the electric torque tool. The dynamometric torque tool, used for screwing and unscrewing bolts, is driven by an electric motor, and its drive is transferred by the intermediate and planetary gears. Measurements were carried out at various work conditions of this electric torque tool. The decomposition results of vibration signals by the Hilbert-Huang method in the time domain and by the Fourier method in the frequency domain were compared.

Experiments and their results

Measurements were performed by means of the piezoelectric accelerometer cooperating with the charge amplifier of a voltage output. The following parameters of the measuring system were assumed:

- amplifier frequency band: 0.1 Hz–10 kHz,
- accelerometer frequency band: 0.1 Hz–4.8 kHz,
- sampling frequency: f_p = 20 kHz,
- number of recorded samples: n = 65536,
- individual recording time: $T = N \cdot T_p$ = 3.2768 s.

The piezoelectric accelerometer was mounted on the positioning clamp attached on the electric torque tool housing. The nominal rotational speed of the torque tool equals 24 000 rot/min and there are two possible transmission ratios: in the 1-st gear transmission ratio: 4096, in the 2-nd gear transmission ratio: 2048.

The acceleration signal of vibrations of the electric torque tool operating in the 2-nd gear in the screwing mode, is presented as an example in Figure 1. The frequency spectrum of the signal is shown in Figure 2. The time-frequency diagram – of the electric torque tool vibration accelerations – obtained due to applying the short-time Fourier transform (STFT) is presented in Figure 3. Simulation calculations of the FT and STFT were carried out in the Matlab environment [14]. The frequency spectrum of the signal was shown within the range up to 6 kHz, and of the STFT up to 7 kHz, in consideration of the fact that above these frequencies none essential amplitudes of harmonic components were found.

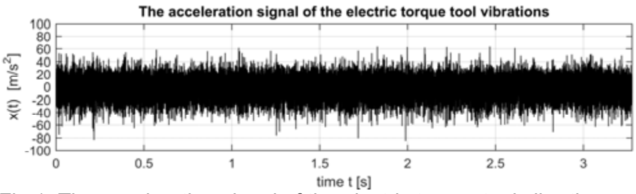

Fig.1. The acceleration signal of the electric torque tool vibrations

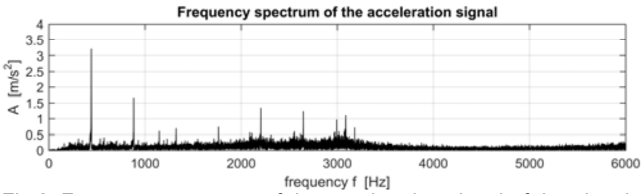

Fig.2. Frequency spectrum of the acceleration signal of the electric torque tool vibrations

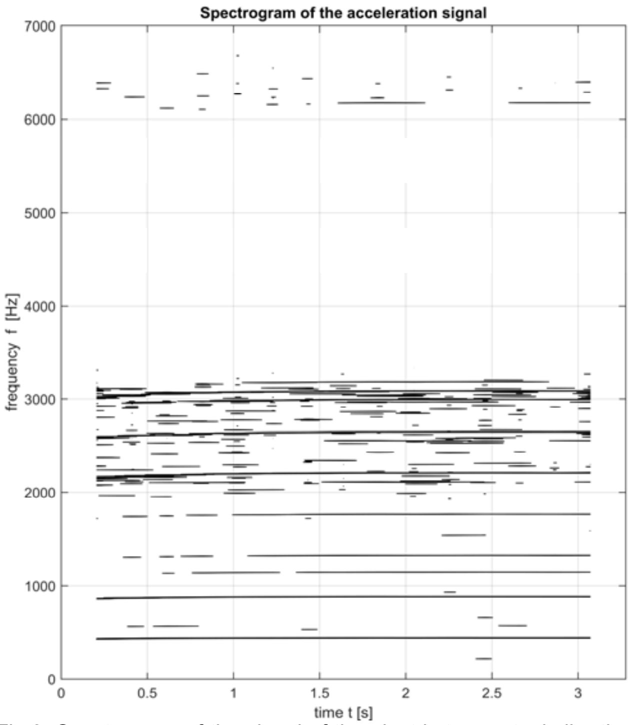

Fig.3. Spectrogram of the signal of the electric torque tool vibration accelerations

Results of the signal decomposition $x(t)$ into the IMF components – by means of the EMD algorithm – are presented in Figure 4, while instantaneous frequencies and amplitudes of the IMF signal components in Figure 5 and 6. Six dominating IMF components are presented within the time interval between 560 and 580 millisecond of the recording time. Narrowing of the presented time interval allows precise observations of the time waveforms character.

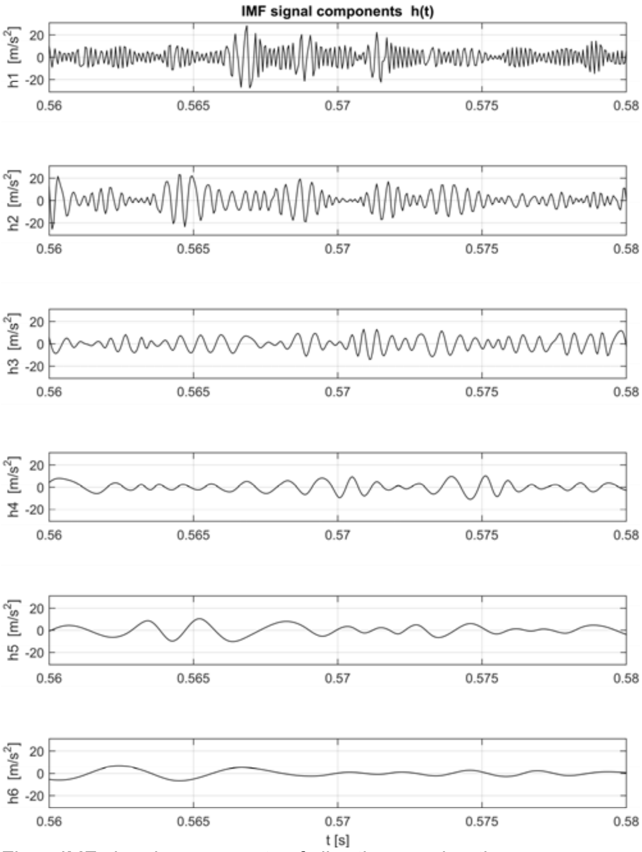

Fig.4. IMF signal components of vibration accelerations

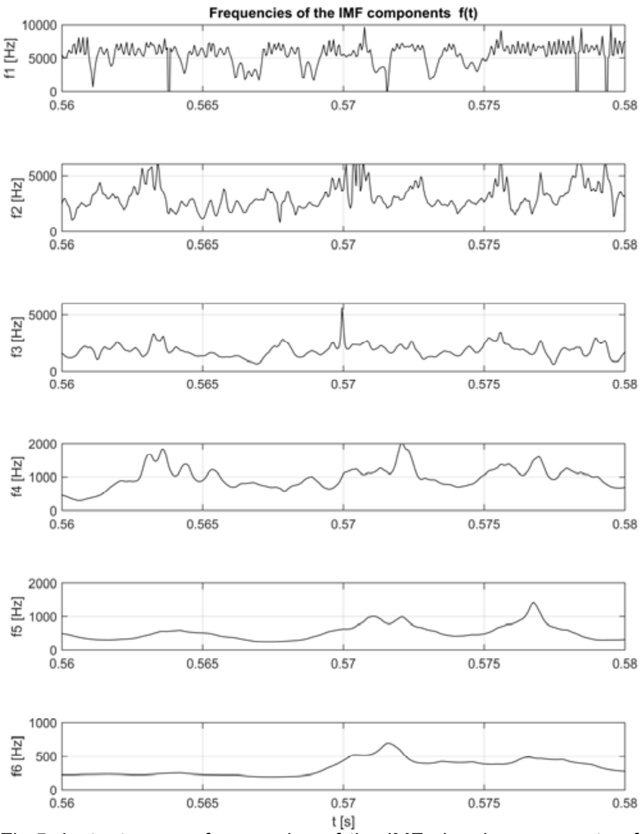

Fig.5. Instantaneous frequencies of the IMF signal components of vibration accelerations

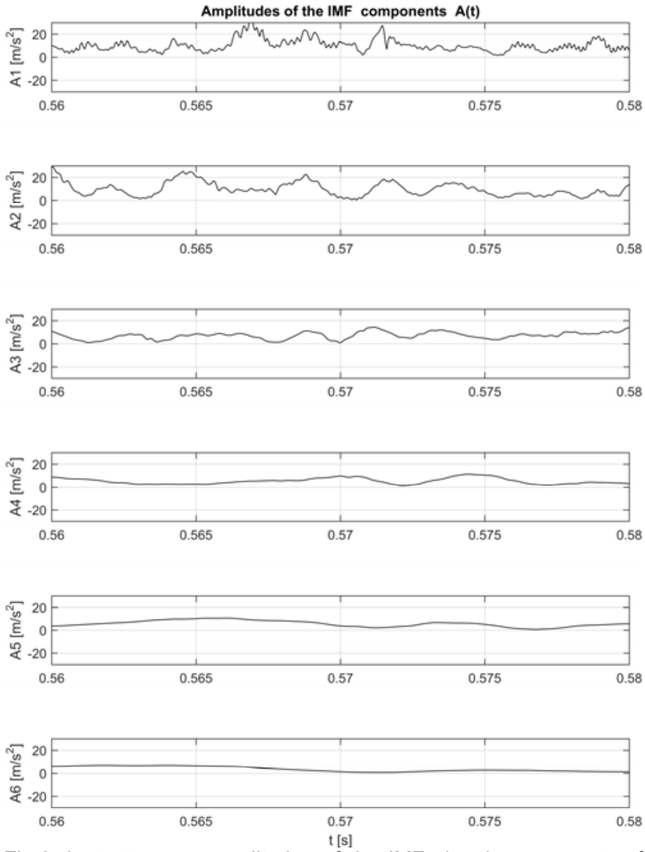

Fig.6. Instantaneous amplitudes of the IMF signal components of vibration accelerations

Analysis of the obtained results

As the result of the performed simulation experiments, with using the HHT, component functions $h_i(t)$ (2) of the signal of the electric torque tool vibration accelerations $x(t)$ were distinguished (Fig. 4). The IMF component functions are of the sinusoidal functions character with the modulated amplitude and frequency, in accordance to (3). The sequence of components distinguishing runs from the highest to the lowest frequencies. The HHT method separated 17 components, out of which an essential acceleration amplitudes has 6 components of average frequencies: 5782 Hz, 3249 Hz, 2082 Hz, 1231 Hz, 638 Hz, and 359 Hz. The first six IMF components are shown in Figure 4. For each component $h_i(t)$ time waveforms of frequencies (Fig. 5) and of amplitudes (Fig. 6) were determined on the basis of (4)–(6).

When comparing frequency values, distinguished by means of the HHT method, with frequencies obtained in the FT (Fig. 2) and STFT (Fig. 3) analyses the following should be stated:

- the FT and STFT methods did not find the occurrence of vibrations of frequency 5782 Hz and 638 Hz in the acceleration signal (which were shown by the HHT method),
- remaining frequency values occurring in the Fourier spectrum signal are only similar to the ones distinguished in the HHT method and differ also in amplitude values of vibration accelerations (Fig. 6),
- the occurrence of frequencies – from 6 to 6.5 kHz – was found in the vibration signal, by means of the STFT method (but not found by the FT method),
- however, the periodical occurrence of frequencies – from 6 to 6.5 kHz – was found in the component $h_1(t)$ of vibration signal (Fig. 4) of the frequency waveform $f_1(t)$ (Fig. 5), determined by the HHT method,

- the spectrogram of the vibration accelerations signal (Fig. 3) exhibits either a periodical occurrence or periodical fading of various frequencies in the vibration signal, which indicates a non-stationary character of the electric torque tool vibrations.

The source of the shown above differences between the results obtained by means of the FT, STFT and HHT methods, can constitute a non-stationary and non-linear character of phenomenon's occurring during the electric torque tool operations. The Fourier transform, FT, allows for the correct analysis of signals describing phenomenon's of a stationary and linear character, while the short-time Fourier transform, STFT, allows for the correct analysis of non-stationary phenomena when they are linear. The application of the Hilbert-Huang transformation, which decomposes signals in the time domain, is possible in relation to signals describing phenomena which are both non-linear and non-stationary.

The Fourier transform, FT, as often – in practice – used tool for analysis of signals describing phenomenon's occurring in mechanics, must be applied with care and should be verified by means of other methods, e.g. STFT and HHT. Its not proper application can lead to erroneous analyses and false conclusions concerning the analysed physical effects, including vibrations of mechanical objects.

Conclusions

On the bases of the presented investigation results several conclusions can be drawn.

- The presented experimental results confirm the possibility of the correct decomposition of vibration signals by means of the Hilbert-Huang transformation, HHT, in the time domain.
- Applying the HHT allows to determine components of the analysed vibration signal in a form of narrow-band sinusoidal signals of modulated amplitude and frequency.
- The HHT method enables the proper determination of components of vibration signals describing phenomenon's of a non-stationary and non-linear character.
- The vibration signal components, obtained due to their decomposition, allow to determine instantaneous frequency and amplitude values of vibration signal components as well as phase dependencies between them.
- Determined values of parameters of the vibration signals components are essential from the diagnostics point of view of the analysed vibrating object.
- Certain limitations in the HHT method application constitutes the problem of ensuring the adequate frequency resolution, expressed by dependence (7).
- The application of the Fourier transform, FT, for the decomposition of mechanical vibrations signals can lead to erroneous conclusions if these signals describe phenomena of a non-stationary or non-linear character.
- The short-time Fourier transform, STFT, applied for the decomposition of vibration signals can lead to wrong interpretations of non-linear phenomena.

The investigation results concerning comparisons of effects of the application of the Hilbert-Huang Transform (HHT), Fourier Transform (FT) and Short Time Fourier Transform (STFT) for the vibration signals decomposition, are presented in this paper. It would be also interesting to compare of HHT with other transformations, for example Wavelet transform [15].

This research was financed by resources for statutory activities of the Department of Metrology and Electronics, AGH.

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