Estimation size of aerosol droplets produced by explosion

Abstract. This paper presents methodology of droplet diameter measurement with a recorder designed to catch droplets from an aerosol jet produced by explosion. A substantial part of the paper is devoted to the problem of determination of the scaling coefficient allowing to recalculate the diameters of droplet traces left on the recorder’s glass plate into the diameters of droplets while in the air. In this context the methodology of determination of diameters of droplets in laboratory using diffraction images of the laser light beam scattered on the aerosol droplets is discussed.

Streszczenie. W artykule przedstawiono metodologię pomiaru kropl obraszku wodnego wytworzonego metodą wybuchową w zaprojektowanym rejestratorze. Znacząca część artykułu skupia się na określeniu współczynnika skali między kroplami schwytanymi przez rejestrator, a tymi będącymi w powietrzu tuż po wytworzeniu aerozolu. Określenie współczynnika skali wielkości kropl wykonano w laboratorium za pomocą lasera i analizy obrazów dyfrakcyjnych. (Wyznaczanie wielkości kropel aerozolu wodnego tworzonego metodą wybuchową).

Keywords: aerosol production, droplet size measurement, light scattering.

Słowa kluczowe: aerozol wodny, pomiar wielkości kropel, rozpraszanie światła.

Introduction

The problem of measurement to the broad class of metrological problems of size analysis of dispersed phase particles applicable in many branches of science and technology [1, 2, 3].

The explosive method of production of aerosol consists in detonating explosive charge placed inside a water capsule, which results in both fragmentation and acceleration of the aerosol medium [4, 5]. Measuring sizes of droplets produced by such method is difficult due to high pressure and high speed of droplets [4, 6, 7] and measurement of explosion-produced droplet sizes may be executed with an apparatus called droplet trap-box or droplet recorder [6, 7].

The water aerosol is prepared in special explosive system which gives an appropriate droplets diameters, energy of droplets and energy of aerosol incoming in an extinguishing area, which gives very good results in fire damping [8, 9]. Especially the droplets diameter is a very important parameter in such process [5]. Shock fragmentation and driving of a water layer is an effective way of producing quite immense cloud of water droplets in the air. The explosive charge can deliver quite significant amount of energy that is necessary to perform the work of partition of a volume of homogeneous water into droplets and to provide the droplets with an initial velocity. The spray dissolves the free-feeding oxygen and the explosion removes a part of it from the surrounding. Explosive formation and spreading of a spray water cloud is a complex phenomenon which completely and unified theoretical description may be extremely difficult and may ultimately require extensive numerical modelling [5]. At first we presented relatively simple models that represent different aspects of the phenomenon that provide a framework for designing and interpretation of high precision measurement. We devise special attention to the process of drag-related slowing down and slowing perpendicular to the axis of explosion of a water droplet resulting in limited diameter of the water-spray cloud. All tests have been performed for the cylindrical configuration (Fig. 1), which our theoretical an experimental investigation had shown to be the best [4–9]. The droplet recorder requires determination of scaling coefficient β, which allows to estimate the diameter of the aerosol droplet from the size of the trace left by it at the recorder’s glass plate. This has to be done in a laboratory where diameters of the sprayer-generated droplets are measured both after adhering to a glass plate which is done by a rather direct method and in the air using diffraction images generated on a screen by the scattered laser beam. The method of extracting droplet diameter from such an image is pretty complicated and requires application of the Fraunhofer diffraction theory [1, 2] supported with elements descriptive statistics.

Application of the droplet recorder requires determination of the so called scaling coefficient β, which allows to estimate the diameter of the aerosol droplet from the size of the trace left by it at the recorder glass plate. In the measurements to be discussed a constant value of the coefficient was assumed

\[ \beta = \text{const.} \] (1)

In reality its value depends on many parameters, including: the size of aerosol droplets, surface tension coefficient of the droplet, velocity of the droplet at the collision with the recorder’s microscope glass, properties of the glass plate surface, overpressure at the moment of registration, the inclination angle of the glass plate at the moment of collision with a droplet, atmospheric conditions, first of all the temperature and humidity of the air surrounding the recorder. The assumption on the constant value of the scaling coefficient (1) instead of taking it as a function of the above factors is an obvious simplification following from the impossibility to determine the form of the function.

To calibrate droplet recorder one has to solve the difficult problem of measurement size of free droplets. In the research reported here the optical method of measurement based on the Fraunhofer diffraction of laser light on aerosol droplets was used. Applying it requires not only a proper design of experimental apparatus, which is relatively simple,
but first of all adoption of suitable theoretical tools securing efficient analysis of measuring results. This subject is in the focus of the present paper that continues a series of papers devoted to the explosive production of water aerosol [4-9].

**Research methodology**

In this work the following symbols are used for the aerosol droplet size distribution:

- \( f \) – size distribution function of the aerosol droplets under investigation,
- \( f^* \) – size distribution function of the aerosol droplets determined with a method based on the Fraunhofer theory of diffraction,
- \( f^\hat{ } \) – size distribution function of the aerosol droplets on the plate.

For such defined size distribution functions of the aerosol droplets the scaling coefficient (1) may be symbolically expressed as

\[
\beta = \frac{f^*}{f}
\]

Size distributions can be compared using their characteristic parameters. In this paper it is assumed that expectation values will be used in this role.

In this paper we assume the following form of the size distribution of the aerosol droplets [10]:

\[
n(d) = N_0 \left( \frac{d}{2} \right)^{\mu+1} \frac{\mu+1}{\Gamma(\mu+1)} e^{-\frac{\mu+1}{2a_0}}
\]

This distribution appears in many problems connected with the production of water-aerosol. It is characterized by the independent parameters \( a_0 \), \( N_0 \), \( \mu \) and the droplet diameter \( d \) is the independent variable. After normalization the distribution assumes the form (by using the Euler's gamma function \( \Gamma \))

\[
n_\mu(d) = \left( \frac{a_0}{\mu} \right)^{\mu+1} \frac{\Gamma(\mu+1)}{\Gamma(\mu+1)} \left( \frac{d}{2} \right)^{\mu} e^{-\frac{\mu+1}{2a_0}}
\]

and its expectation value and standard deviation, respectively, read [13]

\[
E = 2 \frac{a_0}{\mu} (\mu+1)
\]

\[
\sigma = 2 \frac{a_0}{\mu} \sqrt{\mu+1}
\]

Fig. 2 shows a schematic view of the experiment in which sprayer-generated aerosol was investigated via scattering He-Ne laser beam with the wave-length of 632.8 nm.

**Physical foundations of the droplet size measurement**

Within the Fraunhofer diffraction theory [1, 2, 12, 13] the intensity of the light scattered by a material particle is given by the formula:

\[
\Gamma_c(\theta, r) = \frac{1}{\kappa} \left| r \left( 1 + \cos \theta \right) J_1 \left( r \sin \theta \right) \right|^2 \cdot I_0
\]

where: \( I_0 \) is the intensity of illumination of the particle by the incident light beam, \( \theta \) – scattering angle, \( r \) – particle size parameter (Mie parameter), \( J_1 \) – first order Bessel function of the first kind, \( \kappa \) – wavenumber of the incident light.

Function (7) has interference-diffraction minima at the distances \( X_i \) from the optical axis of the measuring system. Minima registered in the course of experiment allow to calculate droplet diameter with the formula

\[
d = \frac{p_i \cdot \lambda}{X_i^2 + L^2} \frac{\pi}{X_i}
\]

where: \( i \) – the minimum number from the range \( i = 1, 2, \ldots N \); \( p_1 \) – \( i \)-th root of the first order Bessel function of the first kind, \( \lambda \) – wavelength of the scattered light, \( L \) – the distance of the scattering particle from the screen.

Equation (8) allow one to connect radii of diffraction minima with the size of the light scattering aerosol droplet.

**Simulator of the measurement of the size of the aerosol droplet**

For the sake of the present research a simulator of the size of aerosol droplets had been created. It was based on (9) obtained within the Fraunhofer diffraction theory [1, 2, 12, 13]. It allows one to generate read-offs of the diffraction minima for particular values of the following parameters like: diameter of the aerosol droplets, wavelength of the used laser light, the distance of the scattering aerosol from the screen and minima determination errors.

In the course of simulation the minima read-off error was assumed as the tripled standard deviation obtained for the experimental data, which, depending on the number of the minimum ranged from 2 to 3 mm. The assumption of the tripled standard deviation allowed one to simulate even such an extreme situation as positive superposition of various errors: scale error, distortion of the optical system of the camera used for registration images, overextended exposition time leading to distortion of the minima, paralactic error following from a particular location of the camera etc. The minima read-off error was simulated with a uniform distribution, which leads to a larger error than in the case of the normal distribution.

The simulation consisted in theoretical computing values of the diffraction minima read-offs for the assumed size of the aerosol droplet. In the next step the droplet size was determined using (8). The arithmetic mean of various obtained diameters was used as the estimator of the proper diameter and the measurement error was based on the standard deviation. An array of the values obtained in the process of estimation for the droplet with the diameter of 100 μm leading to the reconstructed value of (106 +/-12) μm. The simulator allowed to check the role of various parameters affecting the result of measurement. With this knowledge taken into account a procedure of determining the diameter of aerosol droplets based on
pictures of the diffraction images was worked-out. In the course of the simulations boundary conditions had been superimposed on each parameter in the way assuring realistic representation of actual experiments.

The procedure of determining the number of minima hidden inside the laser-light beam diameter

The Fraunhofer theory allows one to connect the radii of the diffraction minima with diameter of the light scattering particles (8). It’s very difficult to achieve such a concentration of the dispersed medium in experiment to quench the incident laser beam completely and obtain the diffraction image of the scattered light exclusively (cf. Fig. 2). In practice most frequent is the situation when the diffraction image results from superposition of the incident and scattered light. As the result the laser beam can cover the lowest order minima especially in the case of large droplets for which they are very close to the beam axis. This is analogous to the diffraction of light on a circular aperture. For this reason we will use here the term of Airy disk to describe the area of the screen inside the first order dark ring. The size of the Airy disk is considerable and the relative intensity in its area is so large that the radiation from that area should be suppressed.

In the process of analysis of diffraction images generated by droplets of aerosol determining the size of the Airy disk and the diameter of the laser beam are of crucial importance. If they are known one can determine the number of diffraction minima (dark rings) hidden inside the laser beam diameter, using the estimated of the rings diameters obtained from (8). In the case of an unknown aerosol it is impossible, however, to estimate accurately the diameter of the Airy disk and consequently the number of diffraction minima hidden within the laser beam is unknown. This problem can be solved with the simulator of light scattering described in section 4 of this article.

To determine the number of diffraction minima hidden within the laser beam diameter a simulation was carried on for seven values of droplet diameter: 10 µm, 30 µm, 50 µm, 100 µm, 150 µm, 200 µm, 300 µm. For each of those values diameters of the diffraction minima had been computed. In the next step covering of the subsequent dark rings from the 1st to the 7th with the laser beam diameter have been simulated. For each of the cases the diameter of the droplets and the standard deviations have been computed. It was deduced on the basis of the simulation that the most accurate reconstruction of the actual particle diameter corresponds to the case with the smallest standard deviation. Taking into account the results from simulator we can assume (Fraunhofer theory) that droplets which diameter should be smaller than 150 µm.

Determination of the droplet diameter using the least-square method

In section 3 we have presented a method of determined aerosol droplets diameters as the mean of the values obtained with (8) for various diffraction minima. The simulations have shown that the method gives good results only in the case of small diameter droplets. For example droplets with diameters 30 µm have been reconstructed with multiple simulations as particles of the diameter (30 +/-1) µm. On the contrary, large droplets, e.g. 300 µm, with the same procedure have been viewed as particle of the diameter (380 +/-171) µm. Here we discuss an alternative method of determining droplet diameters on the ground of the diffraction images.

One can estimate diameters of droplets by fitting them to the measured diameters of diffraction minima in such a way to minimize the distances between their estimated and measured diameters. In practice the least squares method may be used. For random read-out errors this method should be essentially more accurate than the one described in section 4. The idea had been tested for droplets of various diameters and the results are shown in table 1. The first column presents the “true” diameters \( d \), for which the simulation was carried on. In the second column one has values of the diameters \( d_1 \) (reconstructed using the least squares method) and the last column shows maximal distance between \( d \) and \( d_1 \) observed in the course of many simulations.

Table 1. The results of simulations: \( d \) – diameter of the droplet; \( d_1 \) – mean value of the diameter obtained with the simulation; \( \max(\vert d-d_1 \vert) \) – the largest absolute difference between \( d \) and \( d_1 \) (this variable may be used for estimating measurement error).

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<th>( d ) [µm]</th>
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Determination of the droplet size distribution on the glass plate

To determine the scaling factor \( \beta \), one has to know the droplet diameter distribution of the sprayed aerosol. To this end microscopic measurements of the diameter of aerosol droplets’ traces on the glass plate using the methodology described earlier have been carried on.

Since in reality shapes of the traces vary it was necessary to work-out a procedure allowing to determine the effective diameter of the droplet. For the sake of determining the diameter of a trace with maximum accuracy the pictures have been converted using the program ImageJ for microscopic analysis to the binary image. For such an image areas of traces have been computed. Next the areas were converted into the equivalent trace diameters (\( d_1 \)). Such an approach contains simplifications but gives quite reasonable results. The results of such measurements were used for fitting the distribution function (3) using the least square method. In this way nonlinear regression of the form (3) and the basic parameters of the descriptive statistics were obtained: number of cases 1354; mean 121.8 µm; maximum 355 µm; minimum 50.46 µm; standard deviation 53.49.

Summary and conclusions

Determination of the scaling coefficient \( \beta \) was based on the comparison of the distribution of the droplets diameters in the air and their traces on the plate of the recorder. The expectation value of droplet diameter in the air \( \mu_d = 171 \) µm, standard deviation \( \sigma_1 = 91 \) µm and determination coefficient \( r_{d1} = 0.745 \). The expectation value of droplet trace diameter on the plate \( \mu_{d1} = 124 \) µm, standard deviation \( \sigma_1 = 62 \) µm and determination coefficient \( r_{d2} = 0.987 \).

For the comparison the expectation values of the both distributions were (somewhat arbitrarily) taken:

\[
\beta = \frac{\mu_{d2}}{\mu_{d1}} = 1.38 \approx 1.4
\]

For comparison one may also estimate the value of the same coefficient with standard deviations. For \( \sigma_1 \) and \( \sigma_2 \) one obtains \( \beta = 1.47 \), which is very close to the quoted above value of the scaling coefficient (in the whole range of the diameters). On this basis one may assume that the value of the sought parameter is:

\[
\beta = (1.4 +/-0.1)
\]
and the measurement uncertainty considers only the parameters of the distribution function. The small difference of the value of the $\beta$ coefficient between the estimate based on the expectation value and the standard deviation gives an evidence that both distribution scale linearly with a considerable accuracy. The performed experiments do not allow to determine the complete measurement uncertainty $\Delta \beta$. This error depends on many factors of which most important seems to be uncertainty of the distribution of droplet diameter in the air essentially depending on the number of analyzed pictures. Apart from that one has to take into account the approximate character of number of analyzed pictures. Although its value is burdened with an error which exact value is difficult for accurate estimation.

The final result of the paper consists in determination of the scaling coefficient defined as the quotient of diameter expectation values for the free and adhering droplets of the same aerosol produced in laboratory with a sprayer. The obtained solutions allowed to estimate the value of $\beta$ scaling coefficient. Determination of this parameter allows one to determine the complete measurement uncertainty of the value of the scaling coefficient and working-out the protocol occupying the lion's share of this paper. The results and the protocol are presented in the subsequent paper.

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