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Phase shift measurement method using conditional averaging of the delayed signal module

***Abstract**. A measurement method of sine voltage phase shift disturbed with noise using conditional averaging of the delayed signal module was proposed. The advantages of the proposed principle were indicated. The results of basic experiments confirming the obtained theoretical results were also provided.*

***Streszczenie.** Zaproponowano metodę pomiaru przesunięcia fazowego napięcia sinusoidalnego zakłóconego szumem, wykorzystującą warunkowe uśrednianie modułu sygnału opóźnionego. Podano i analizowano modele teoretyczne pomiaru. Pokazano korzystne cechy zaproponowanej zasady. Zamieszczono wyniki podstawowych eksperymentów potwierdzające uzyskane wyniki teoretyczne. (**Metoda pomiaru przesunięcia fazowego z zastosowaniem warunkowego uśredniania modułu sygnału opóźnionego**).*

Keywords: phase shift, additive disturbances, conditional averaging of signals.
Słowa kluczowe: przesunięcie fazowe, zakłócenia addytywne, warunkowe uśrednianie sygnałów.

Introduction

The methods of measuring the phase shift expected value using phase processing into a time interval use information about the phase on the basis of only two signal values in their time implementations. Such a way of measuring does not eliminate the impact on the result of the measurement of random disturbances or distortions of the analysed sine wave signals.

For periodic signals as well as for repetitive transitional ones distorted by noise, an improvement in signal-to-noise ratio and the extraction of the useful signal from the noise along with an evaluation of its parameters can be obtained by using integral methods of statistical signal processing, and in particular the correlation techniques and signal averaging in the field of time and frequency, as well as approximation algorithms [1–6]. One of the directions in research and application in this area of metrology is the use of conditional signal averaging [7–9].

The principle of phase shift measurement

In the signal processing model with period T presented in Fig.1 it is assumed that the original sine signal $x(t)$ is free from distortions, whereas the secondary signal $y(t)$ with phase shift φ_{xy} in relation to the signal $x(t)$ is subject to additive disturbance by independent noise $n(t)$ with distribution $N(0, \sigma_n)$ and autocorrelation function $R_n(\tau) = \sigma_n\rho_n(\tau)$. The signals available in the phase shift analysis are the signals $x(t)$ and $z(t) = y(t) + n(t)$.

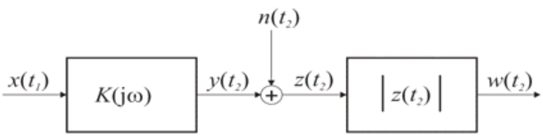


Fig.1. Model of measurement signal processing

The values of signals in the moments t_1 and t_2 ($\tau = t_2 - t_1$) were taken into account. In the measurement model the signal $z(t)$ is subjected to a non-inertial nonlinear transformation $w_2 = |z_2(t)|$. Next, in the range $0 \div \pi$ one determines a characteristic of the conditional expected value of the module (*CAEV*) for the signal $z(t)$ with the condition initiating the averaging $x(t) = 0$ and $\frac{dx}{dt} > 0$ [9].

Behaviours of the signals in a measurement model with time delay τ_0 and arguments τ_i and τ_{i+1} of the local minimums are presented in Figure 2.

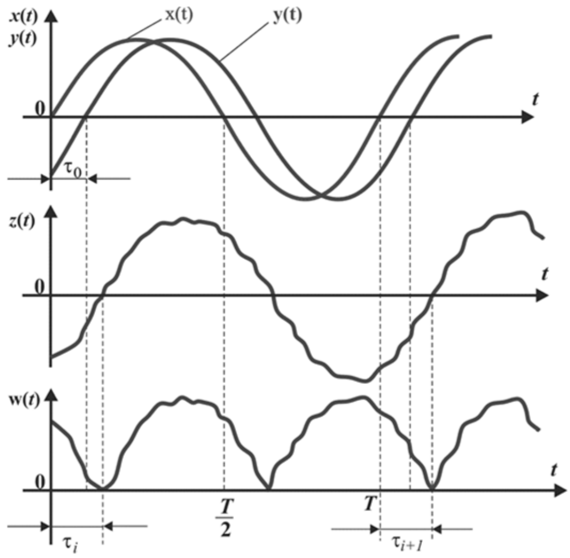


Fig.2. Behaviours of the signals in a measurement model (Fig. 1)

Identifying the time argument τ_m of the minimum of the *CAEV* characteristic obtained as a result of arithmetic averaging of M conditional implementations makes it possible to experimentally determine the phase shift φ_{xy}. The aim of the present paper is to evaluate the influence of correlating $R_n(\tau)$ the disturbance $n(t)$ on the experimentally determined *CAEV* characteristic in the neighbourhood of its minimum.

Applying the conditional expected value of the delayed signal module

In publication [9] the properties of a *CAEV* characteristic were analysed for a disturbance in the form of broadband noise without taking into account the correlation of arithmetically averaged values of the disturbing signal. The sine signals under analysis: $x(t)$ and $y(t)$ with the phase shift φ_{xy} are described by the basic models:

$$x(t) = X_m \sin \omega t \,;\; y(t) = Y_m \sin\left(\omega t + \varphi_{xy}\right);\;\; \varphi_{xy} = \frac{2\pi}{T}\tau_0 \,. \tag{1}$$

The relation for *CAEV* is as follows [5]:

$$\overline{w}_2 = \overline{w}(\tau) = 2y_0 \cdot \Phi(\eta_0) + \sqrt{\frac{2}{\pi}}\sigma_n \cdot e^{-\frac{y_0^2}{2\sigma_n^2}} , \tag{2}$$

where:

$y_0 = -Y_m \sin(\omega \cdot \tau + \varphi_{xy})$; $\eta_0 = -\dfrac{y_0}{\sigma_n}$; $\Phi(\eta_0)$ – Laplace functional.

In the minimum place, *CAEV* has the value of:

(3) $$\overline{w_2}_{\,\min} = \sqrt{\frac{2}{\pi}}\sigma_n .$$

The variance of variable w_2 can be determined from the relation:

(4) $$Var[w_2] = \int_0^{\infty} \left(w_2 - \overline{w_2}\right)^2 p(w_2)\,dw_2 = \sigma_n^2 \left(1 - \frac{2}{\pi} e^{\left[-\frac{y_0^2}{2\sigma_n^2}\right]^2}\right) - y_0^2\left(4\Phi^2(\eta_0) - 1\right) - 4\sqrt{\frac{2}{\pi}}\sigma_n y_0 \Phi(\eta_0) e^{-\frac{y_0^2}{2\sigma_n^2}}$$

The variance of the estimate of a *CAEV* characteristic, which is the conditional average value of the module (*CAAV*), based on M implementations in the minimum point for $\tau = \tau_m$ equals:

(5) $$Var\left[\overline{w_2}(\tau_m)\right] = \frac{Var[w_2(\tau_m)]}{M}\left[1 + 2\sum_{k=1}^{M-1}\left(1 - \frac{k}{M}\right)\rho_n(kT)\right].$$

For broadband noise with no correlation of samples based on (4) and (5):

(6) $$Var_{nk} = Var\left[\overline{w_2}(\tau_m)\right] = \frac{\sigma_n^2}{M}\left(1 - \frac{2}{\pi}\right).$$

With the disturbance $n(t)$ in the form of low-pass noise and samples with period *T* significantly correlated for the evaluation of the *CAEV* variance, the autocorrelation function $R_{|n|}(\tau)$ ought to be determined previously, equalling:

(7) $$R_{|n|}(\tau) = E(|n_1||n_2|) - E^2(|n|) = \frac{2\sigma_n^2}{\pi}\rho_n(\tau) arc\sin\rho(\tau) + \sqrt{1 - \rho_n(\tau)} - 1 \approx \frac{\sigma_n^2}{\pi}\rho_n^2(\tau).$$

Relation (7) can be the basis for the evaluation of the variance (5), which for example for a model of exponential correlation of disturbance $n(t)$ equals:

(8) $$Var\left[\overline{w_2}(\tau_m)\right] = \frac{\sigma_n^2}{M}\left(1 - \frac{2}{\pi}\right)\frac{1 + \rho_n^2(T)}{1 - \rho_n^2(T)} = Var_{nk}\frac{1 + \rho_n^2(T)}{1 - \rho_n^2(T)} = Var_{nk} \cdot \eta_k.$$

and further allows to calculate and evaluate the estimates $\hat{\tau}_m$ and $\hat{\varphi}_{xy}$. Based on simplified characteristics (2) in the neighbourhood of point $\overline{\tau}_m$ determining the minimum of the function $\overline{w_2}(\tau)$ to:

(9) $$\overline{w_2}(\tau) = \overline{w_2}(\overline{\tau}_m)\left[\left(\frac{Y_m}{\sigma_n}\omega\Delta\tau\right)^2 + 1\right]$$

one can determine the relation for the model of deviation $\Delta\tau$ from $\overline{\tau}_m$ in the form of $N(0, \sigma_{\Delta T})$ [5]. When taking into account the variance (8), the evaluation of experimental deviation of the average value $\hat{\tau}_m$ with M averages equals:

(10) $$\hat{\sigma}_{\overline{\tau}_m} = 0{,}12\frac{\sigma_n T}{Y_m\sqrt{M}}\left(\frac{\sigma_n^2\left(1 - \frac{2}{\pi}\right)\left(1 + \rho_n^2(T)\right)}{\left(\sqrt{\frac{2}{\pi}}\sigma_n\right)^2\left(1 - \rho_n^2(T)\right)}\right)^{\frac{1}{4}}$$

Determining the value of the estimate $\hat{\tau}_m$ for the minimum of the characteristic can be done by applying one of the appropriate methods of seeking the extreme of a function obtained experimentally [10]. The argument of the minimum of the *CAAV* characteristic $(\overline{\tau}_m)$ allows to determine the estimate of phase shift in degrees:

(11) $$\hat{\varphi}_{xy} = -\overline{\tau}_m \frac{360^\circ}{T}$$

with standard uncertainty:

(12) $$\hat{\sigma}_{\hat{\varphi}_{xy}} = \hat{\sigma}_{\overline{\tau}_m}\frac{360^\circ}{T}.$$

Experimental research results

During the experimental works and signal processing a multi-function generator, inertial system, adder and dual-channel digital oscilloscope were used as well as ORIGIN software. In studying the role of the disturbance $n(t)$ physically generated low-pass white noises in two bands B_1 = 25 kHz (noise generator NRG201) and B_2 = 55 Hz (noise generator DPGS2) were used (Fig.3).

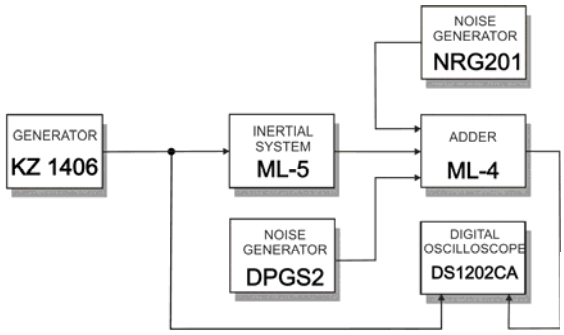


Fig.3. The block diagram of measuring stand

Normalised autocorrelation functions of disturbing signals $\rho_1(\tau)$ and $\rho_2(\tau)$ are provided in Figures 4 and 5.

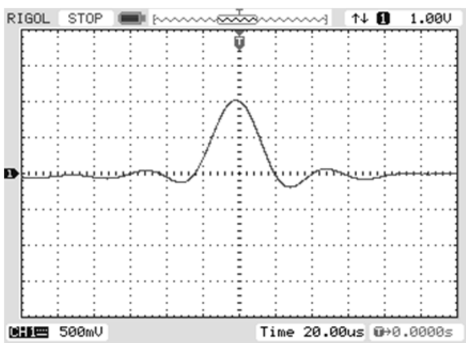


Fig.4. Normalised autocorrelation function $\rho_1(\tau)$ of white noise in the band B_1 = 25 kHz

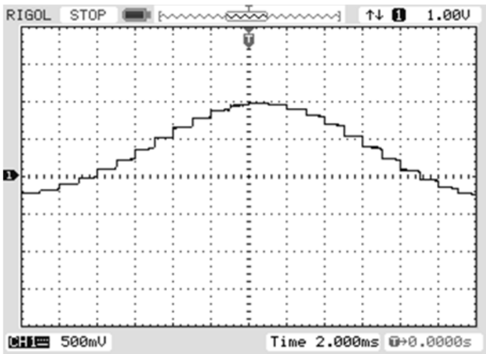
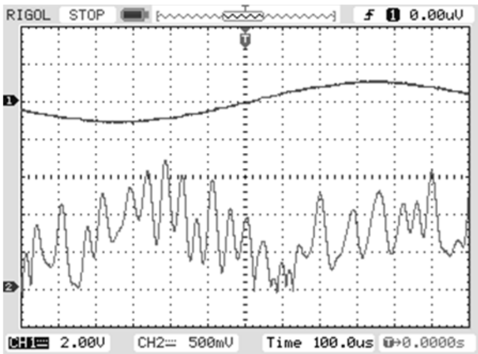


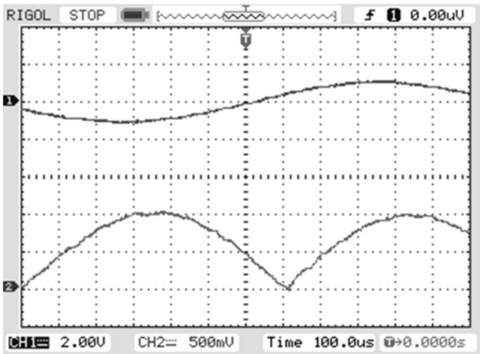
Fig.5. Normalised autocorrelation function $\rho_2(\tau)$ of white noise in the band B_2 = 55Hz

Examples of input signals disturbed by white noise in the 25 kHz band are presented in Figure 6.

a)



b)



c)

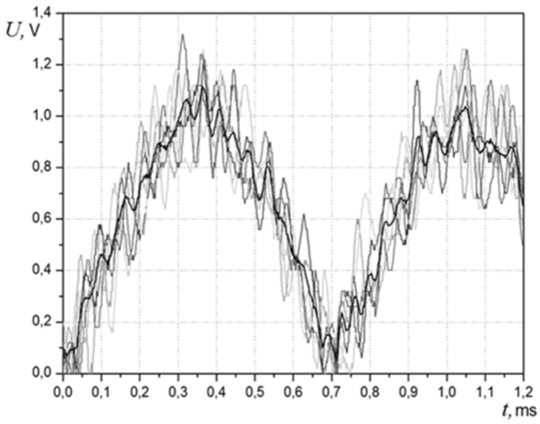
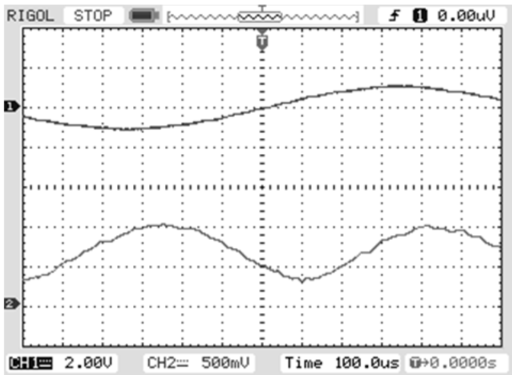


Fig.6. Behaviours of signals: input signal $x(t)$ - 1 and the delayed signal – 2; a) delayed signal disturbed by noise $N_1(0,\ 0.14)$; B_1 = 25 kHz; b) delayed signal disturbed by a noise on the level of -30 dB; c) examples of implementation of delayed signals disturbed by a noise on the level of -10 dB and the average value of the implementation (marked in black)

Examples of *CAAV* characteristics are presented in Figures 7a÷c. Behaviour 1 is the signal $x(t)$ with the period T = 1.4 ms.

The output signal $y(t)$ with the period T = 1.4 ms is disturbed with noise on various levels and then averaged conditionally. For a disturbance with the value of standard deviation σ_1 = 0.45 V the minimum of the *CAAV* characteristic equals 0.36 V, which is in accordance with expression (3).

a)



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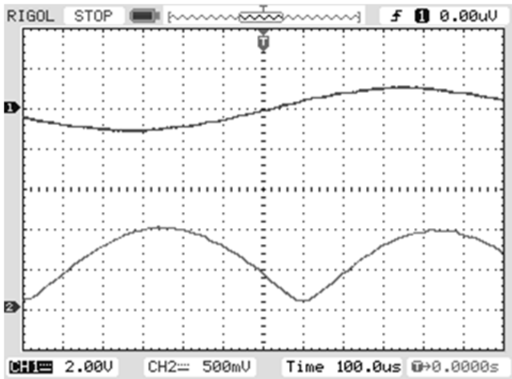

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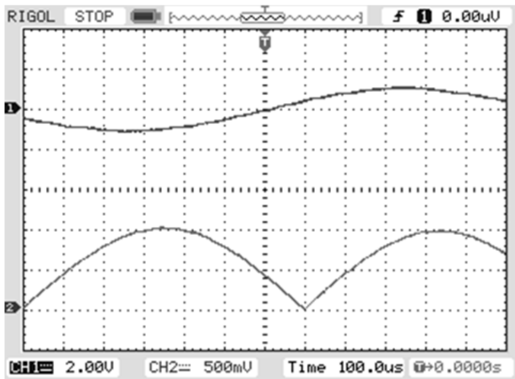
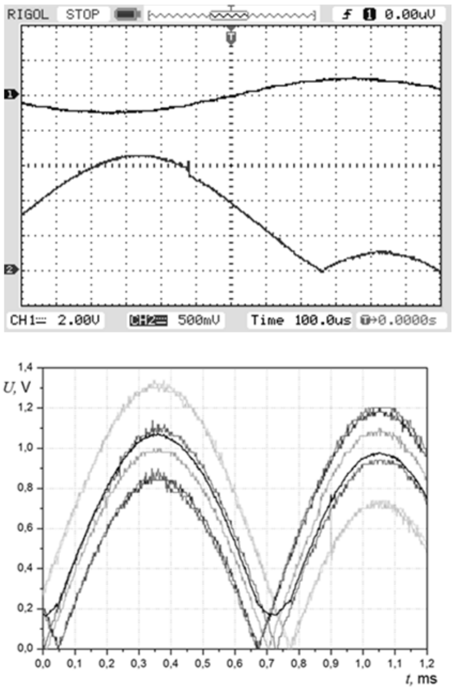


Fig. 7. Input signal $x(t)$ - 1 and the *CAAV* characteristic – 2 for B_1 = 25 kHz and M = 256; a) σ_1 = 0.45 V; b) σ_1 = 0.14 V; c) σ_1 = 0.04 V

Examples of output signals disturbed by low-pass white noise in the band B_2 = 55 Hz are shown in Figure 8. Behaviour 1 is the signal $x(t)$ with the period T = 1.4 ms. Figure 7a illustrates an example of a delayed signal $y(t)$ disturbed with a low-pass noise. Figure 7b represents implementations of delayed signals disturbed with the noise $N_2(0,\ 0.14)$ and B_2 = 55 Hz as well as the average value of the implementations (marked in black).

a)

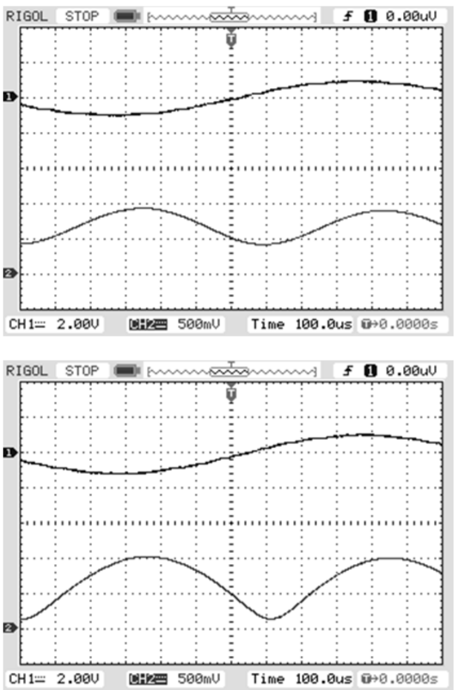


b)

Fig.8. Behaviours of signals $x(t)$ - 1 and the delayed signal $y(t)$ – 2; a) delayed signal disturbed by noise $N_2(0,\ 0.5)$; B_2= 55 Hz; b) examples of implementation of delayed signals disturbed by a noise on the level of -10 dB and the average value of the implementation (marked in black)

In figure 9a a *CAAV* characteristic was provided for σ_2 = 0.5V and M = 256 in Figure 9b the *CAAV* characteristic was determined with a smaller noise σ_2 = 0.14 V.

a)



b)

Fig.9. Input signal $x(t)$ - 1 and a *CAAV* characteristic – 2; a) σ_2 = 0.5 V; b) σ_2 = 0.14 V

Example: for the result of averaging provided in Figure 8b for the data: σ_n = 0.14 V; Y_m = 0.5 V; T = 1.4 ms; $\rho(T)$ = 0.9; M = 256; S/N = 6.39, the following was calculated: $\hat{\varphi}_{xy}$ = 28.29° and $\hat{\sigma}_{\hat{\varphi}_{xy}}$ = 1.13.

The example concerns relatively large values of correlation and a small value of S/N. The relative uncertainty of the phase shift evaluation equals 4 percent. In order to decrease it, the number M of averages needs to be increased.

Summary

The phase shift measurement model with an undisturbed input signal and a disturbance of the output signal with noise often occurs in tasks involving an active identification of dynamic industrial objects.

The proposed evaluation method of $\hat{\tau}_m$ and $\hat{\varphi}_{xy}$ may be particularly useful in ranges of infralow frequency due to the advantage of shortening the long time of analysis with the required accuracy in measuring the phase shift.

The application of the *CAAV* for sinusoidal signal disturbed with noise allows to reduces the variance of the time delays in relation to the variance when the signal crossing zero with the same S/N in the classical method. This time is proportional to the phase shift [10].

The evaluation of $\overline{w_2(\tau_m)}$ and $\hat{\tau}_m$ under conditions of a significant correlation of disturbing components is characterised by a large increase in variance for the minimising of which it is required to increase the number M of conditional averages.

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