Roman KASZYŃSKI, Adrian SZTANDERA, Katarzyna WIECHETEK

West Pomeranian University of Technology, Department of Systems, Signals and Electronic Engineering

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Filter network of dynamic nonstationary elements

Streszczenie. *W artykule przedstawiono koncepcję tworzenia szybkich filtrów niestacjonarnych z elementów I-go rzędu o zmiennych parametrach, które są elementami dolnoprzepustowymi i górnoprzepustowymi. Wprowadzone zmienne w czasie parametry w postaci funkcji wzmocnienia i funkcji czasowych pozwoliły wielokrotnie skrócić stany nieustalone, co spowodowało ich dużą przydatność w tworzeniu rozbudowanych struktur filtrów. Przeprowadzono badania symulacyjne struktur o wybranych, z góry założonych właściwościach częstotliwościowych. (**Filtracyjna sieć niestacjonarnych elementów dynamicznych**).*

Abstract. *In this paper a concept of creating fast nonstationary filters based on 1-st order high-pass and low-pass elements with time-varying parameters is presented. Implementing time-varying parameters in the form of gain function and time function enables shortening of the transient state, which results in their high utility in creation of complex filter structures. Computer simulation and research on these selected structures were conducted with previously established frequency characteristics.*

Słowa kluczowe: filtr niestacjonarny, charakterystyka częstotliwościowa, zmienne parametry, dynamika, analogowe przetwarzanie sygnałów, stan nieustalony, układy liniowe o zmiennych parametrach.
Keywords: nonstationary filter, frequency characteristic, time-varying parameters, dynamics, analog signal processing, transient behavior, linear time-varying systems properties

Introduction

Studies on time-varying structures were usually conducted with assumption that 2-nd order elements are basic structures, due to their great possibilities of variation of parameters and versatility. It is described by the equation:

$$\frac{d^2y(t)}{dt^2}+2\beta(t)\omega_0(t)\frac{dy(t)}{dt}+\omega_0^2(t)y(t)=k\cdot x(t) \tag{1}$$

One can easily notice that damping factor $\beta(t)$, responsible for oscillations and pulsation $\omega_0(t)$, affecting on passband and stopband in the frequency domain are variable parameters. According to the uncertainty principle, frequency characteristics are related to the duration of the transient state. By varying the parameters the transient state can be significantly reduced. In the preceding studies on 2-nd order elements the best results (the shortest settling times) were obtained for the final value $\lim_{t\to\infty}\beta(t)\approx 0.85\div 0.95.$

It is known that the value of the damping factor equal or greater than one results in lack of oscillations in the system and is equivalent with the same 2-nd order inertial system. This led to assumption that 1-st order inertial element is a basic structure. The time constant *T* and gain *k* was replaced with time constant function T(t) and gain function k(t). Serial connection of these two elements can be described by following set of equations:

$$\begin{aligned} &T_1(t)\cdot\frac{dy_1(t)}{dt}+y_1(t)=k_1(t)\cdot x(t)\\ &T_2(t)\cdot\frac{dy_2(t)}{dt}+y_2(t)=k_2(t)\cdot y_1(t) \end{aligned}, \tag{2}$$

where: x(t) - input signal, y2(t) - output signal.

Block diagram of the obtained connection is presented in Fig. 1.

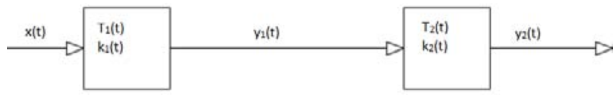

Fig.1. Block diagram of the filter described by the system of differential equations (2)

This system has slightly different properties from 2-nd order element described by equation (1) and the assumption $\lim_{t\to\infty}\beta(t)\approx 0.85\div 0.95$. It can be assumed that the 1-st order element can be used to create complex structures. As shown in the previous studies [1,3,6,9] single 1-st order elements with time-varying parameters have short settling times, which leads output signal to fast stationarity obtaining [2,5,7].

These elements with varying parameters can be fast and effective in the frequency domain if values of the function varying parameters set their value over time and after terminating the transient state have values resulting from the spectral assumptions of the filter. 1-st order elements with time-varying parameters have lower phase shift than 2-nd order structures. It is preferred feature due to the stability of the system created from these elements [1,4]. In 1-st order structures it is possible to use two outputs: one low-pass and one high-pass. In this way two different filtration of one signal at the same time by a single element with time-varying parameters can be obtained. It is described by a system of equations:

$$\begin{aligned} &T_{g_\infty}\cdot f_T^{-1}(t)\cdot\frac{dy(t)}{dt}+y(t)=k_\infty\cdot f_k(t)\cdot x(t)\\ &z(t)=T_g\cdot\frac{dy(t)}{dt} \end{aligned}, \tag{3}$$

where: $y(t)$ - low-pass output signal of the structure, $z(t)$ - high-pass output signal.

The element that enables fast high-pass and low-pass filtration of the same input signal simultaneously allows to create complex serial and parallel structures based on elements with time-varying parameters. Similar networks of elements with time-invariant parameters can be also created, however such structures are characterized by long transient states. In the complex structure composed of 1-st order elements with time-varying parameters it is possible to shape frequency characteristics by using weight functions. These structures can be used also for creating models of dynamic systems based on e.g. time characteristics or spectral properties.

Properties of elements with time-varying parameters

Functions varying parameters have a big impact on the speed of the element operation. Previous studies on duration of the transient state allowed to obtain preferred functions and their parameters values in relation to the limit of the passband or stopband, depending on which filter is used [5,10]. Precise determination of the shortest settling time for a particular filter structure is difficult. Varying parameters in time causes a significant problem: settling time appointed from the step response is insufficient. Combination of several nonstationary elements only increases difficulty of estimation of the duration of the transient state. In this case according to previous studies it is possible to use the slowest 1-st order element as decisive one. On the basis of the earlier research [3,4,8] the concept of varying parameters presented below was accepted. Functions $f_T^{-1}(t)$ and $f_k(t)$ in system of equation (3) can be written as follows:

(4) $$f_k(t) = f_k(t)\cdot[d_k - (d_k - 1)\cdot h_k(t)],$$

(5) $$f_T^{-1}(t) = f_T^{-1}(t)\cdot[d_T - (d_T - 1)\cdot h_T(t)].$$

Functions $f_T^{-1}(t)$ and $f_k(t)$ by varying parameters of the filter allow to create time constant function $T^{-1}(t) = T_\infty^{-1}(t)\cdot f_T^{-1}(t)$ and gain function $k(t) = k_\infty(t)\cdot f_k(t)$, where $d_T = \dfrac{T^{-1}(0)}{T^{-1}(\infty)}$ and $d_k = \dfrac{k(0)}{k(\infty)}$ are the multiplicity of their value change. Time constant function and gain function must achieve the values specified in the spectrum assumptions in time not longer than the duration of the transient state. It can be assured if element generating varying function has damping factor value between $0.71 < \beta < 1$

Fig. 2 shows an example of settling times $t_{\alpha 1}$ and $t_{\alpha\Omega}$ for the 1-st order filter with accuracy α = 0.05. Input signals are step response and sinusoidal signal with a stopband limit frequency.

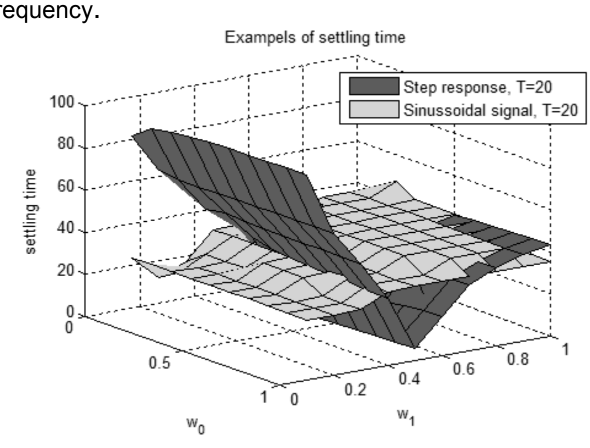

Fig.2. Example of settling times tα1 and tαΩ for step response and sinusoidal signal for the 1-st order filter with accuracy α = 0.05

One can easily see how values of settling times change: during the increase of one, the other is reduced.

Using a one function varying all of the filter parameters was considered. Instead of generated number of functions, which equals two times order of the filter, there is only one function. This action significantly simplifies the structure of filters. The research have shown that simplifying the structure does not affect much settling time but is important in the analysis of reliability and the possibility of errors deriving from elements drawbacks.

By assuming that 1st order elements with time-varying parameters are basic parts of more complex structures, filters with different characteristics can be created. It is possible because each of 1-st order element has low-pass and high-pass output. Time-varying parameters of the filter causes several times shorter transient state compared to the corresponding filter with invariant parameters. With simple, highly efficient elements involved in the process, creating a filter can be approached in a different ways.

The first method is to create small structures, each made up of several basic elements with different frequency characteristics (e.g. band-stop, band-pass, band-amplifying, stairs-stop and others). With a database of basic structures more complex filters can be created.

A second way is to connect the entire structure, which provides required frequency properties, from basic 1-st order elements without creating intermediate structures. In this method of creating filters a resemblance to the structure of neutral networks and multi-dimensional dynamical systems can be found. In synthesis of such filters an adequate selection of the weight function for connections between elements is an important issue.

Both methods of creating filters give an opportunity to obtain many real-time waveforms of the filtered signal in one filtration process. The course of the signal from the frequency band, which include useful information transmitted after damping noises can be obtained. Considering the filtrated signal as a random process, values of the following moments and other plots of filtrated signal components can be obtained. It should be mentioned that while creating structures only from 1-st order elements errors may be implemented in frequency characteristics, however they are deemed as acceptable. In filters created from 2-nd order elements these errors are not necessarily present. One example is a band-stop filter with a frequency for which the frequency characteristics module is equal zero. In presented filters zero cannot be obtained, but it can be any small value in the range of permissible error limits.

Basic filter structures

One of the main basic structure is filter formed by a parallel connection of a 1-st order low-pass element with time-varying parameters and a 1-st order high-pass element with time-varying parameters. By proper selection of functions varying parameters and especially values they are trying to reach, the stopband of the filter can be narrowed or widen. Fig. 3 shows the frequency characteristics.

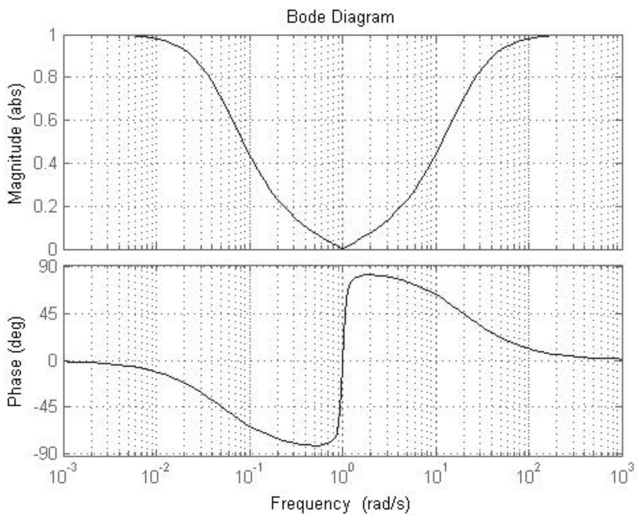

Fig.3. Frequency characteristic of the module and the phase of the 1st order band-stop filter with time-varying parameters.

One can see that minimum of the characteristics has value greater than zero and becomes smaller the more high-pass and low-pass parts are "moved apart". In order to obtain greater suppression in the stopband without its widening, two of the same 1-st order filters should be connected (with the same parameters values). Thereby a small module value in the stopband and twice as steep slopes of the frequency limits can be achieved. Fig.4 shows that by connecting in series two band-stop filters with different damping frequencies, the filter with two various suppression bands can be created.

The same frequency characteristics can be obtained by using systems with time-invariant parameters (stationary), but transient states of these systems are significantly longer than in systems with time-varying parameters. Fig.5 shows examples of signal filtration by systems with time-invariant and time-varying parameters.

Filters with time-varying and time-invariant parameters with the same frequency characteristics have different transient states. Filters with time-varying parameters have much shorter transient states than filters with time-invariant parameters and this feature determines their great practical usefulness.

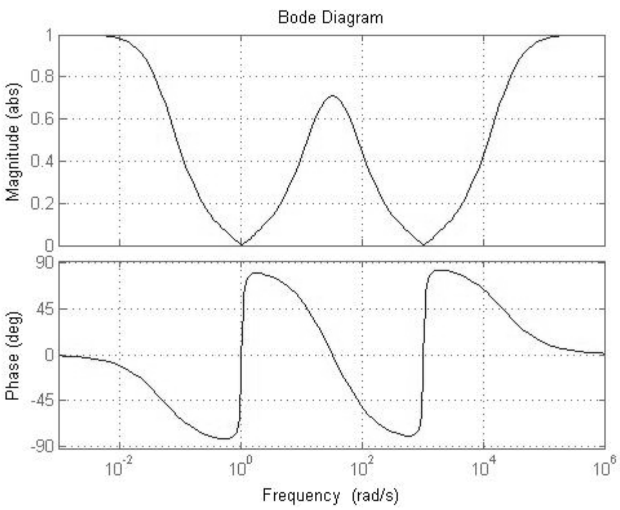

Fig.4. Frequency characteristic of the module and the phase of the 2-st order two-band-stop filter with time-varying parameters

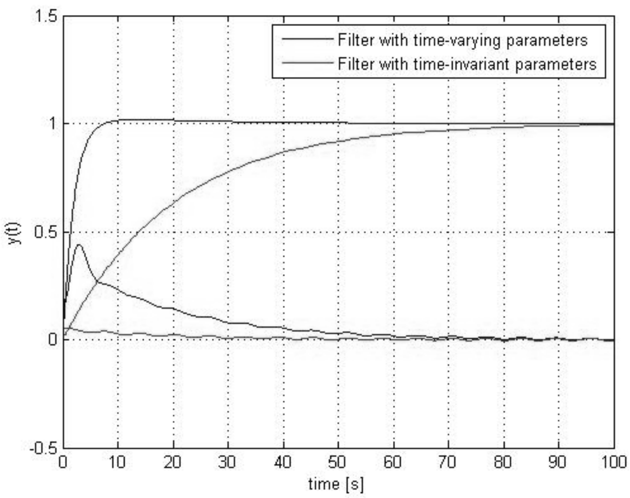

Fig.5. Step characteristic and other signals, resulting from filtration of sinusoidal signals with frequencies most suppressed in stopbands obtained by using systems with time-invariant and time-variable parameters

Band-pass and band-amplifying filters with time-varying parameters may be included as basic structures. The first one can be obtained by connecting in series low-pass and high-pass elements with appropriately selected final values of function varying parameters. Fig. 6 shows the characteristics of a two-band-pass filter resulting from the parallel connection of two different band-pass filters.

By parallel connection of the low-pass and the high-pass element a band-amplifying filter can be obtained. In this filter low and high frequencies are transmitted correctly while center frequencies in comparison to other are amplified.

In many measurement or signal processing systems this filter can be considered as a correction element. Fig. 7 shows an example of frequency characteristics.

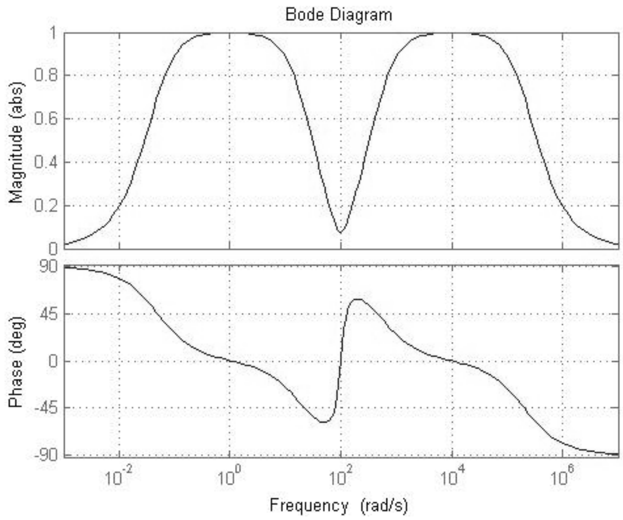

Fig.6. Example frequency characteristic of a two-band-pass filter

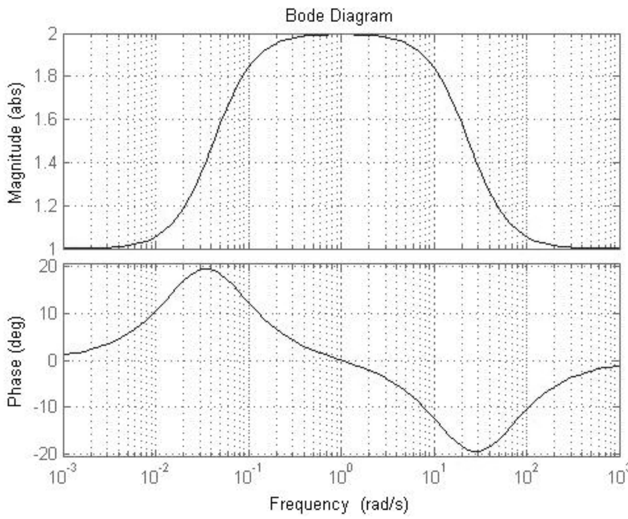

Fig.7. Example frequency characteristic of the module and the phase of the band-amplifying filter

With the use of this filters frequency characteristics, where signal components in the selected frequency band should be amplified, other dynamical systems can be corrected. For example, by using simple filters a complex structure with desired frequency characteristics can be formed. It should be underlined that occurrence of short transient states in these filters causes significantly shorter waiting time for the filtration result.

Filters complex structures

Second method of creating filters with complex structures is a series-parallel connection of single 1-st order elements with time-varying parameters. Each of these elements with time-varying parameters has an input, where filtrated signal is given, one or two outputs for function varying parameters implementation, output with high frequency signals suppression (low-pass filter output) and output with low frequency signals suppression (high-pass filter output). In every element there is a variation of two parameters: gain, which is replaced by gain function and time constant T replaced by time function. Fig. 8 shows an example of network structure. One can easily see that creating various structures is relatively simple, however obtaining specific properties may require more complex calculations.

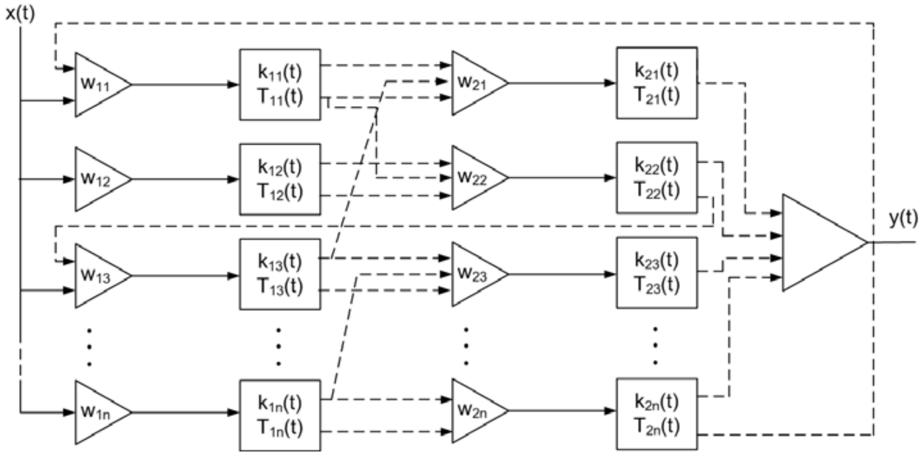

Fig.8. Example structure of nonstationary elements network

With such elements with different values, to which functions varying parameters aim, series-parallel structures with diverse frequency characteristics can be created.

Studies with 1st order elements network, where each of them has two outputs: one low-pass and one high-pass were conducted. The network can consist of n elements. Frequencies, for which the absolute value of the spectral transmittance of the low-pass output should be equal 0.05 are determined by following relation:

(6) $$n!\,.$$

In conducted studies network of eight elements was used. The following table shows frequencies of each element.

Table 1. Frequencies values of each element

n	Low-pass output Rad/s	High-pass output Rad/s
1	1	0.002
2	2	0.005
3	6	0.15
4	24	0.06
5	120	0.3
6	720	1,8
7	5040	12.6
8	40320	101

Table 2. Examples of settling times for step response and sinusoidal signal for the connected in series low-pass and high-pass elements

T	settling time for step response	settling time for sinusoidal signal	settling time for sinusoidal signal (amplitude = 0.5)
10	5.81	38.93	25.32
3.33	17.09	30.85	17.99
0.833	12.65	35.67	17.00
0.167	11.86	35.44	16.65
0.0278	11.74	35.31	16.51
0.0039	11.72	35.29	16.49
0.00049	11.72	35.29	16.49

Each component of the n-elements network has 2n+1 inputs (two outputs of each element and input signal). All of elements inputs have weights responsible for final shape of the frequency characteristic. That approach gives many possibilities in the frequency domain. The network is described by two weight matrix, inputs and outputs.

First matrix includes input weights of all network elements. The second one contains an output weights of each element, which are part of network output signal. Dimension of input matrix is m x n where m = 2n + 1. Index m specifies the number of the element, from which the signal is applied to the element n. It can be described precisely by following relation:

(7)
$$\begin{aligned}&m = 2n - 1 - (\text{ n - elements low - pass output}),\\&m = 2n \qquad - (\text{n - elements high - pass output }),\\&m = 2n + 1\ \ - \text{input signal } x(t),\end{aligned}$$

(8)
$$\begin{matrix} w_{11} & \cdots & w_{1n} \\ \vdots & \ddots & \vdots \\ w_{m1} & \cdots & w_{mn} \end{matrix}\,.$$

Dimension of output matrix is n x 2. Index n specifies the number of the element the output is from. First column represents low-pass output and the second one high-pass output:

(9)
$$\begin{matrix} w_{11} & \cdots & w_{1n} \\ \vdots & \ddots & \vdots \\ w_{m1} & \cdots & w_{mn} \end{matrix}\,.$$

Research on settling time for connected in series low-pass and high-pass elements was conducted. Assumptions that the basic element of the network is the slowest one with time function aiming T = 20 was adopted. The following results were obtained:

- settling time for step response: 4.99 s;
- settling time for sinusoidal signal: 35.28 s;
- settling time for sinusoidal signal (amplitude = 0.5): 16.47 s.

Example of the network configuration and the frequency characteristic is presented below. Matrix input weights:

$$
\begin{matrix}
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{matrix}
\tag{10}
$$

Matrix output weights:

$$
\begin{matrix}
0 & 0 \\
0 & 0 \\
1 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 1 \\
0 & 0
\end{matrix}
\tag{11}
$$

Obtained frequency characteristic:

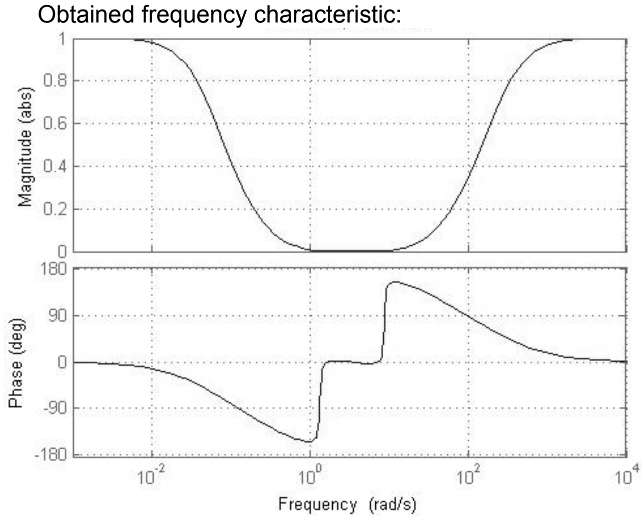

Fig.9. Frequency characteristic obtained for network configuration presented above

Conclusions

Conducted studies helped to determine that filters with time varying-parameters have significantly better dynamic properties in the time domain than filters with time-invariant parameters. Regardless of their frequency characteristics (low-pass or high-pass) it is important to estimate the duration of the elements transient state. In the specific cases, the obtained results should be referred to a stopband or a passband limit frequency of these elements. It appears that a decisive impact on the transient state have elements slightly dependent on the nature of the bandwidth (low-pass or high-pass or band-stop or other), dominant for the frequency properties.

Authors: *Roman Kaszyński Ph.D. D.Sc. Eng.* West Pomeranian University of Technology, Department of Systems, Signals and Electronic Engineering, Gen. Sikorskiego 37, 70-313 Szczecin, Poland, *E-mail:* roman.kaszynski@zut.edu.pl; *Adrian Sztandera Eng.* West Pomeranian University of Technology, Department of Systems, Signals and Electronic Engineering, Gen. Sikorskiego 37, 70-313 Szczecin, Poland *E-mail* sztanderaadrian@o2.pl: *Katarzyna Wiechetek Eng.* West Pomeranian University of Technology, Department of Systems, Signals and Electronic Engineering, Gen. Sikorskiego 37, 70-313 Szczecin, Poland, *E-mail:* wiechetekkatarzyna@gmail.com

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