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Assessment of uncertainty of determination of parameters of Jiles-Atherton model of hysteresis loops of isotropic materials

Streszczenie. *Metody określania parametrów modelu Jilesa-Athertona na drodze optymalizacji wieloparametrycznej są aktualnie intensywnie rozwijane, ze względu na możliwość praktycznego zastosowania tego modelu w technice. Jednak do tej pory nie przedstawiono wyników badań niepewności parametrów modelu wyznaczonych w ten sposób. W referacie przedstawiono wyniki badań niepewności wyznaczenia parametrów modelu Jilesa-Athertona dla pętli histerezy magnetycznej wysokoprzenikalnościowego ferrytu manganowo-cynkowego oraz stali martenzytycznej 3H13. Wykazano, ze niepewność wyznaczenia poszczególnych parametrów jest znacząco zróżnicowana i może, w niektórych przypadkach, przekroczyć 100%. Powinno to być brane pod uwagę w przypadku wykorzystania modelu Jilesa-Athertona do ilościowej analizy procesów magnesowania na potrzeby badań z zakresu fizyki ciała stałego. (**Ocena niepewności wyznaczenia parametrów modelu Jilesa-Athertona w modelowaniu pętli histerezy magnetycznej magnetyków izotropowych**).*

Abstract. *Recently developed methods of optimization-based determination of Jiles-Atherton model enable practical application of this model for technical purposes. However, uncertainty of determination of these parameters was not investigated before. This paper presents the results of investigation on assessment of uncertainty of determination of parameters of Jiles-Atherton model for high permeability Mn-Zn ferrite as well as for 3H13martensitic steel. It was indicated, that uncertainty of determination of parameters varies for different parameters and may exceed 100%. This information should be considered during solid state physics – oriented analyses based on the Jiles-Atherton model.*

Słowa kluczowe: model Jilesa-Athertona, stal martenzytyczna, ferryt manganowo-cynkowy.
Keywords: Jiles-Atherton model, martensitic steel, Mn-Zn ferrite.

Introduction

Since its introduction in 1984 [1], the Jiles-Atherton model [2] became one of the most popular methods of quantitative description of magnetic hysteresis loops. This model can be applied to both isotropic [2] and anisotropic [3, 4] magnetic materials as well as enable modelling of quasistatic [5] and dynamic [6] magnetic hysteresis loops. In addition, on the base of Jiles-Atherton model, the magneto-mechanical coupling may be explained [7]. This explanation covers both the magnetostrictive [8] and magnetoelastic [9] behaviour of soft magnetic materials.

The most important problem connected with the Jiles-Atherton model is determination of its parameters on the base of experimental hysteresis loops. Identification of parameters is carried out during the optimisation process [10]. Recently development methods [11, 12, 13] enable practical application of this model for technical purposes. However, it is not clear, how uncertainty of experimental measurements of magnetic hysteresis loop effect on uncertainty of determination of parameters of Jiles-Atherton model. This dependence can't be explained analytically, due to strong nonlinearities and feedback loop [14] occurring in the Jiles-Atherton model.

Presented paper is filling this gap. Influence of additive and multiplicative magnetic hysteresis loops measuring error was determined during the simulation process. This solution enables determination of propagation of uncertainty of measurements of magnetic hysteresis loops on uncertainty of determination of Jiles-Atherton model parameters.

Principles of Jiles-Atherton model

The Jiles-Atherton model [1] utilizes the conception of anhysteretic magnetization M_{anh}. This magnetization may be observed experimentally by performing demagnetization of magnetic material by exponentially decreasing sine wave magnetic field with addition of constant magnetizing field.

The Jiles-Atherton model utilizes not obvious assumption, that the anhysteretic magnetization in the ferromagnetic materials is similar to the magnetization of paramagnetic materials [2, 3]. For paramagnetic materials, anhysteretic magnetization M_{para} consider the Boltzmann distribution of magnetic domain directions [2], described by the equation:

(1) $$M_{para} = M_s \frac{\int_0^{\pi} e^{\frac{-E_m(\theta)}{k_B \cdot T}} \sin\theta \cdot \cos\theta \cdot d\theta}{\int_0^{\pi} e^{\frac{-E_m(\theta)}{k_B \cdot T}} \sin\theta \cdot d\theta},$$

where k_B is the Boltzmann constant, M_s is the saturation magnetization of a paramagnetic material and θ is the angle between the direction of the magnetizing field H and atomic magnetic moment m_{at}. Energy of the magnetic moment $E_m(\theta)$ is given by the following equation [2]:

(2) $$E_m(\theta) = -\mu_0 \cdot m_{at} \cdot H \cdot \cos\theta .$$

As it was indicated above, in the Jiles-Atherton model, for modelling anhysteretic magnetization of isotropic ferromagnetic materials the same Boltzmann distribution is used. However, the atomic magnetic moment m_{at} is substituted by the average domain magnetization m_d [2]:

(3) $$m_d = \frac{M_s}{N},$$

where N is the average domain density in the material.

Considering the fact, that integrals in the equation (1) have antiderivatives, the Langevin equation for anhysteretic magnetization M_{anh} of the ferromagnetic isotropic materials can be stated [2]:

(4) $$M_{anh} = M_s \left[\coth\left(\frac{H_e}{a}\right) - \left(\frac{a}{H_e}\right) \right],$$

where a describes the density of domains in the material [2]:

(5) $$a = \frac{N \cdot k_B \cdot T}{\mu_0 \cdot M_s}.$$

In addition, due to interdomain coupling α, accordingly to the Bloch model, the effective magnetizing field H_e is given by the following equation [1]:

(6) $$H_e = H + \alpha \cdot M \ .$$

It should be stressed, the simplified representation of the anhysteretic magnetization is valid only for isotropic materials. However, anhysteretic magnetization is very difficult to measure experimentally due to the problems of drift of fluxmeter during the demagnetization. For this reason, anhysteretic magnetization model was verified only for specific cases of anisotropic magnetic materials with narrow hysteresis loop.

The irreversible magnetization M_{irr} In the Jiles-Atherton model can be calculated from the following equation [2]:

(7) $$\frac{dM_{irr}}{dH} = \frac{M_{anh} - M_{irr}}{\delta \cdot k} \ ,$$

where parameter δ is equal 1 or -1 for increasing or decreasing of magnetizing field *H* respectively. Parameter *k* quantifies average energy required to break pining site and is considered as constant for all magnetic hysteresis loop. Reversible magnetization M_{rev} is given by the following equation [2]:

(8) $$M_{rev} = c \cdot (M_{anh} - M_{irr}) \ ,$$

where *c* describes magnetization reversibility. Total magnetization *M* is given as the sum of reversible magnetization M_{rev} and irreversible magnetization M_{irr} [2]:

(9) $$M = M_{rev} + M_{irr} \ .$$

After re-arrangements, the Jiles-Atherton model is given by the following ordinary differential equation [2]:

(10) $$\frac{dM}{dH} = \frac{1}{1+c} \frac{M_{anh}(H+\alpha M) - M}{\delta k - \alpha (M_{anh}(H+\alpha M) - M)} + \frac{c}{1+c} \frac{dM_{anh}(H+\alpha M)}{dH} \ ,$$

where M_{anh} is anhysteretic magnetization [1].

Description of parameters of Jiles-Atherton model is summarized in the table 1.

Table 1. Parameters of Jiles-Atherton model

Parameter	Unit	Description
a	A/m	Quantifies domain wall density
k	A/m	Quantifies energy required to break pinning site
c	-	Coefficient of reversibility
M_s	A/m	Saturation magnetization
α	-	Bloch coefficient, quantifying domain interactions
δ	-	Direction of changes of magnetizing field *H* (-1 for decrease, +1 for increase)

Important advantage of Jiles-Atherton model is the fact that it is given by the single, ordinary differential equation. However, solving of this equation is not trivial. As it was indicated previously [14], Runge-Kutta algorithm based methods with adaptive sampling should be used. Moreover, due to occurrences of fast, nonlinear changes in magnetization, this differential equation is especially computer resources consuming, in the case of very soft magnetic materials, such as amorphous alloys.

Determination of parameters

Problem of determination of Jiles-Atherton model's parameters on the base of experimental results is the most significant drawback of this model. Due to the fact, that only saturation magnetization M_s is directly connected with physical properties of magnetic material, the other parameters should be determined during the optimisation process [10]. Because the local minima occur on the target function for optimisation [15], deterministic gradient or gradient-free optimisation methods may be used only in the case, when at least the rough estimation of Jiles-Atherton model's parameters are known. In the other case, the soft-computing oriented methods, such as simulated annealing [16] and genetic algorithms [17] should be applied. Especially differential evolution based methods [13] seem to be the most effective in the case of determination of Jiles-Atherton model's parameters.

In the presented investigation, experimental magnetic hysteresis loops of ring-shaped samples made of isotropic magnetic materials were measured. Samples made of high permeability Mn-Zn ferrite, Ni ferrite and 3H13 martensitic steel were used. Measurements were carried out by digitally controlled hysteresis graph in the room temperature. To achieve quasi-static character of magnetic hysteresis loops, measurements were performed with frequency of magnetizing field equal 0.1 Hz. Schematic block diagram of the digitally controlled hysteresis graph is presented in the figure 1.

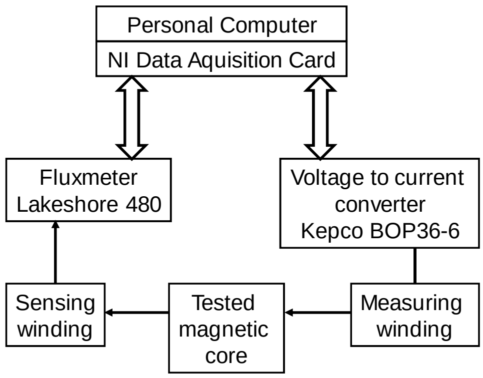

Fig.1. Schematic block diagram of the digitally controlled hysteresis graph for quasi-static measurements o *B(H)* magnetic hysteresis loops

Table 2. Parameters of Jiles-Atherton model

Parameter	Unit	
Mn-Zn ferrite		
a	A/m	7.46
k	A/m	1.60
c	-	0.595
M_s	A/m	270 994
α	-	$1.37\ 10^{-10}$
Ni ferrite		
a	A/m	231.7
k	A/m	425.6
c	-	0.975
M_s	A/m	131 746
α	-	$1.50\ 10^{-4}$
3H13 steel		
a	A/m	504
k	A/m	896
c	-	0.625
M_s	A/m	927 710
α	-	$2.13\ 10^{-3}$

Next, parameters of Jiles-Atherton model were determined in differential evolution-based optimisation process [13]. In the optimisation process, three loops of magnetic hysteresis of the material were considered simultaneously [18] to guarantee the correct results of modelling in the wide range of the amplitude of magnetizing field.

Identified parameters of Jiles-Atherton model for high permeability Mn-Zn ferrite, Ni ferrite and 3H13 martensitic steel are presented in the table 2, whereas results of modelling of magnetic hysteresis loops are presented in the figure 2.

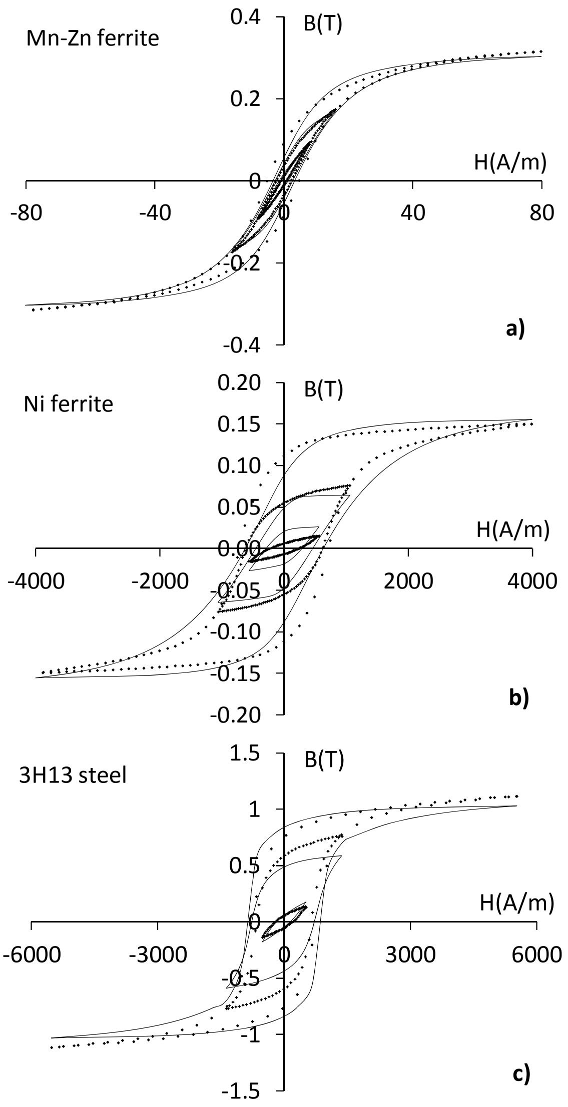

Fig.2. Magnetic B(H) hysteresis loops of: a) high permeability Mn-Zn ferrite, b) Ni ferrite, c) 3H13 martensitic steel. Dotted line – results of measurement, solid line – results of modelling

It should be stressed, that the results of modelling are in very good agreement with experimental results. This agreement is confirmed by the value of determination coefficient R^2, which exceeds 0.98 for all hysteresis loops.

Assessment of uncertainty

To estimate uncertainty propagation, additive and multiplicative error with standard distribution and standard deviation equal 1% was introduced to the results of magnetic hysteresis loops measurements. Then, parameters of Jiles-Atherton model were determined by Powell's optimisation algorithm [19]. Finally standard deviations of model's parameters were estimated.

Table 3 presents the results of propagation of additive and multiplicative measuring error with standard distribution and standard deviation equal 1% on Jiles-Atherton model's parameters. Simulation was performed 50 times to enable statistical analyse of the results. It can be seen, that this propagation significantly differs for different type of materials. Especially, the influence of measuring error on the Bloch coefficient, quantifying domain interactions, is very high for smaller values of this parameter, occurring in the case of high permeability Mn-Zn ferrite.

Presented assessment of uncertainty propagation indicates that multiplicative error has stronger influence on uncertainty of estimation of Jiles-Atherton model's parameters. However, uncertainty propagation strongly depends on value of Jiles-Atherton model's parameters. For this reason, uncertainty propagation should be analysed with use of the presented method for each set of Jiles-Atherton model's parameters.

Table 3. Parameters of Jiles-Atherton model

	Mn-Zn ferrite	
parameter	**additive**	**multiplicative**
a	2.7 %	3.1 %
k	5.6 %	6.4 %
c	0.2 %	3.0 %
M_s	0.4 %	1.3 %
α	101 %	107 %
Ni ferrite		
	additive	**multiplicative**
a	7.3 %	8.9 %
k	3.6 %	4.8 %
c	8.9 %	15 %
M_s	2.5 %	2.7 %
α	36 %	32 %
3H13 steel		
	additive	**multiplicative**
a	3.9 %	2.5 %
k	0.5 %	0.4 %
c	6.9 %	9.7 %
M_s	1.1 %	1.2 %
α	1.8 %	1.3 %

Conclusion

Analytical assessment of propagation of uncertainty of measurements of magnetic hysteresis loops on the parameters of Jiles-Atherton model is very difficult. For this reason presented method is based on numerical simulations.

Results presented in the paper are focused on assessment of uncertainty of determination of parameters of Jiles-Atherton model for high permeability Mn-Zn ferrite, Ni ferrite as well as for martensitic steel 3H13. It was indicated, that uncertainty of determination of parameters varies for different parameters and may exceed 100%.

This information should be considered during solid state physics – oriented analyses based on the Jiles-Atherton model. Moreover, due to the fact, that propagation of uncertainty is strongly dependent on model's parameters, module enabling such analyses is planned to be introduced to recently developed, open-source OCTAVE toolbox for modelling based on the Jiles-Atherton model.

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