

## Determination of power parameters of switched reluctance motor based on instantaneous values of phase voltages and currents

**Abstract.** An approach to determination of power efficiency of a switched reluctance motor (SRM) based on indirectly measured value of the torque is substantiated. A method for indirect determination of electromagnetic torque with the use of the results of measuring the instantaneous values of phase currents and voltages is presented. The most efficient and the factual angles of phase switching for the analyzed four-phase 8/6 SRM are determined. Comparison of SRM torque values obtained by calculation and by experiment is performed. Efficiency of the analyzed SRM is assessed on the basis of calculated values of the torque.

**Streszczenie.** W pracy wykazano, że podejście do wyznaczenia współczynnika sprawności silnika z przełączalną reluktancją w oparciu o niebezpośrednio mierzone wartości momentu jest uzasadnione. Przedstawiono metodę niebezpośredniego wyznaczenia momentu elektromagnetycznego poprzez pomiary chwilowych wartości prądu i napięcia fazowego. Wyznaczono najbardziej efektywne i faktyczne kąty przełączania fazy w czterofazowym silniku 8/6. Porównanie wartości momentu uzyskanego w obliczeniach i poprzez eksperyment wskazuje na to, że sprawność badanych silników może być oceniana w oparciu o obliczane wartości momentu. (Określenie parametrów mocy silnika z przełączalną reluktancją w oparciu o chwilowe wartości napięć i prądów fazowych)

**Key words:** switched reluctance motor, torque, power efficiency.

**Słowa kluczowe:** silnik z przełączalną reluktancją, momen, sprawność mocy

### Introduction

Development of controlled semiconductor commutating devices provided a possibility for creation of fundamentally new types of electric machines (EM) and their control systems. Switched reluctance motors (SRM) refer to such EMs.

Power effectiveness characterized by efficiency coefficient is an important parameter for SRM as for any electric machine. Accurate determination of efficiency allows provision of power-efficient operating conditions of SRM as a part of technological equipment. It is possible to determine efficiency by measurement of the consumed and output power or by consumed power and losses components [1], which cannot always be realized in practice because it is difficult and in some cases impossible to determine some parameters [2]. Due to simple design of SRM there is an alternative consisting in determination of its efficiency from time dependences for phase voltages and currents. Reliability of such approach is provided by accuracy of indirect determination of the torque.

Thus, the purpose of the paper consisted in obtaining calculation relations for determination of the torque based on time dependences for phase voltages and currents and assessment of reliability of such method.

### Theory

It is possible to determine the motor efficiency based on the torque in the following way:

$$(1) \quad \eta = \frac{P_2}{P_1} 100 = \frac{M_{rot} \omega}{P_1} 100,$$

Where  $P_1$  and  $P_2$  – consumed and mechanical power of the motor, respectively;  $M_{rot}$  – motor shaft torque, that can be determined from electromagnetic torque  $M_{em}$ :

$$(2) \quad M_{rot} = M_{em} - P_{mech} / \omega,$$

Where  $P_{mech}$  – mechanical losses of the motor;  $\omega$  – rotation frequency of the rotor.

*Conventional method for calculation of SRM electromagnetic torque*

It is shown in papers [3–4] that SRM electromagnetic torque  $M_{em}$  can be determined using relation of coenergy

$W_{coen}$  – a part of magnetic field energy used for mechanical work with rotor rotation angle  $\gamma$  as follows:

$$(3) \quad M_{em}(\gamma) = \frac{\partial W_{coen}}{\partial \gamma}, \text{ при } i = const,$$

where  $i$  – instantaneous value of the stator current. However, if a physical process of creation of SRM torque is analyzed, one may conclude that, during calculations based on (3) a number of specific features of the design and operational physical processes are omitted, which deteriorated the accuracy of its determination. First of all it includes:

1. Not taking into account the reach of the arm to which rotating force is applied; which is equal to the rotor radius.

2. Neglect of variation of instantaneous value  $M_{em}$  when the rotor poles move in relation to the stator poles.

In order to take into consideration the mentioned specific features the torque calculation is to include the following stages:

– determination of the force of SRM stator and rotor interaction from time dependences for phase voltages and currents;

– determination of the rotor turning effort and average value of SRM electromagnetic torque;

– determination of phase switching angle.

The obtained results are used to find SRM torque and efficiency respectively by (2) and (1) in any operating condition.

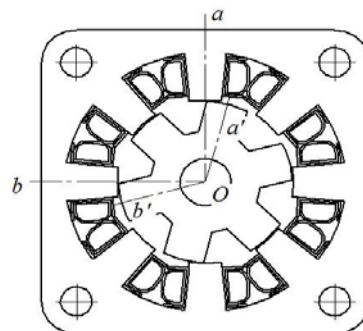


Fig. 1. Cross section of the analyzed SRM

All calculations mentioned below were carried out for an experimental model of a four-phase SRM with the number of poles in the stator and rotor respectively 8 and 6. Motor rated power – 20 W, maximum power – 32.2 W. Rotor diameter  $D_r = 5.46$  mm, air gap length  $\delta = 0.4$  mm, number of the stator winding turns  $w = 55$ . A cross section of the analyzed SRM is shown in Fig. 1.

#### Determination of the force of SRM stator and rotor interaction

Using Ampere's law it is possible to calculate SRM stator and rotor interaction force from the known relation:

$$(4) \quad F_{attr} = IBl,$$

where  $I$ ,  $B$ ,  $l$  – current, magnetic induction and magnetic circuit length, respectively.

During the calculation the rotor motion in relation to angles  $\gamma_a$  and  $\gamma_u$  was assessed. These angles correspond to coordinated (alignment of the rotor and stator interacting poles) and uncoordinated (alignment of the rotor pole and the stator slot) positions of rotor and stator. In this case movement of the rotor pole in relation to the stator pole (current is supplied to its winding) from  $\gamma_u$  to  $\gamma_a$ , corresponds to a positive torque. When the angle reaches  $\gamma_a$ , the torque is equal to zero and when the rotor movement exceeds  $\gamma_a$ , it corresponds to negative or braking torque if there is current in the winding.

As there are no windings and permanent magnets on SRM rotor and force interaction is performed due to stator magnetization, current  $I$  in the magnetic circuit can be presented as current  $I_r$  of the rotor and the rotor itself as a one-turn winding switched in accordance with the stator poles windings. In this case the rotor current will be

$$(5) \quad I_r = wI_w,$$

where  $w$  – number of winding turns;  $I_w$  – winding phase current determined by the results of measurement by a current sensor taking into account the way of windings connection.

Taking into consideration the direction of magnetic flux the rotor diameter  $D_r$  is considered to be  $l$  in (4). If remanent magnetization of the rotor is neglected, its influence on the stator may be ignored.

If leakage flux and air gap length are neglected due to their small value, magnetic induction  $B$  in formula (4) can be presented by the stator pole magnetic induction  $B_{sp}$ , determined from the phase flux linkage  $\psi_w$  as:

$$(6) \quad B_{sp} = \frac{\psi_w}{S_{sp}w},$$

where  $S_{sp}$  – cross-section area of the stator through which magnetic current flows.

Flux linkage  $\psi_w$  can be expressed from its time derivative  $t$ , connected with instantaneous values of phase current  $i_w$  and voltage  $u_w$  of the winding by a known relation

$$(7) \quad \frac{d\psi_w}{dt} = u_w - i_w R_w,$$

where  $R_w$  – winding resistance.

For real SRM the instantaneous values of currents and voltages are to be measured separately for every phase taking into account the design features of the windings. To avoid ambiguity in calculations of flux linkage or magnetic induction they are to be connected to one winding and one pole. In this case recalculation of the parameters for a phase consists in a simple correction for the number of windings in it.

Taking into consideration the above said the instantaneous value of winding flux linkage  $\psi_w$  is determined as:

$$(8) \quad \begin{aligned} \psi_w(n+1) &= \int_n^{n+1} \frac{d\psi_w}{dt} dt = \psi_w|_n^{n+1} + C = \\ &= \left( \frac{d\psi_w(n)}{dt} + \frac{d\psi_w(n+1)}{dt} \right) / 2 dt + d\psi_w(n), \\ &n = 1 \dots (m-1), \end{aligned}$$

where  $dt = 1/f_s$ ;  $f_s$  – frequency of sampling the sensors measuring channels;  $m$  – number of measured instantaneous values of phase values of current and voltage during one complete turn of the rotor. Assuming that  $\psi_w(1)$  corresponds to uncoordinated position of the rotor poles in relation to the stator poles, it will be obtained that in this case voltage is not supplied to the phase and current in it is equal to zero. Hence, value  $\psi_w(1)$  will be equal to zero. According to (8), every subsequent discrete value of flux linkage depends on the previous state of magnetic system.

According to (6) and (8), graph  $B_{sp}$  for one stator winding as a time function during a complete turn of the rotor is of the form shown in Fig. 2.

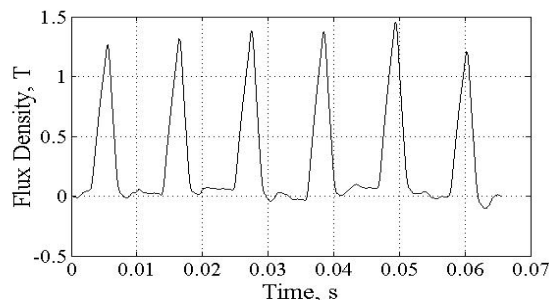


Fig. 2. Variation of magnetic induction in SRM stator pole during a complete turn of the rotor

Taking the above said into consideration formula (4) will be of the form:

$$(9) \quad \begin{aligned} F_{attr.w} &= wI_r(n)B_{sp}(n)D_r, \\ &n = 1 \dots m. \end{aligned}$$

#### Determination of SRM rotor turning effort

Fig. 3 demonstrates vectors of attractive force  $\vec{F}_{attr.w}$  and turning effort  $\vec{F}_{circ.w}$ , built in relation to central radial lines of stator and rotor poles.

The calculation problem consists in expression of the turning effort through attraction force taking into account relative position of stator and rotor interacting poles. In Fig. 3 section  $OA$  equals to rotor radius ( $R_r = D_r/2$ ), and  $OB$  – to the internal radius of stator bore ( $R_{si} = R_r + \delta$ ). Angle  $AOB$  characterizes interaction of the rotor with the stator pole during its transition between different adjacent states. It varies in the range from  $-360/(2N_s)$  to  $360/(2N_s)$ , where  $N_s$  – number of stator poles. For the analyzed SRM with configuration of poles 8/6 angle  $AOB$  is in the range from  $-22.5^\circ$  to  $22.5^\circ$ . In this case, if rotor poles are in an uncoordinated position in relation to stator poles ( $\gamma_u$ ), angle  $AOB$  equals to one of the limit values (in this case  $-22.5^\circ$  or  $22.5^\circ$ ). If rotor poles are in a coordinated position in relation to stator poles ( $\gamma_a$ ), angle  $AOB$  is equal to zero.

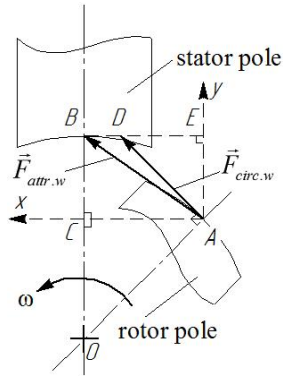


Fig. 3. Vectors of forces  $\vec{F}_{attr.w}$  and  $\vec{F}_{circ.w}$  in relation to position of stator and rotor poles

Attraction force vector  $\vec{F}_{attr.w}$  projections on axis x ( $F_{attr.w.x}$ ) and y ( $F_{attr.w.y}$ ) are:

$$(10) \quad F_{attr.w.x} = F_{attr.w} \cos \angle BAC ;$$

$$(11) \quad F_{attr.w.y} = F_{attr.w} \sin \angle BAC ,$$

Then, for different positions of rotor pole in relation to stator pole force  $F_{circ.w}$ , according to Fig. 3, is determined by the following expressions:

- if module of angle  $CAD$  is bigger than module of angle  $BAC$ , then

$$(12) \quad F_{circ.w} = F_{attr.w.y} s / \cos |\angle DAE| ;$$

- if module of angle  $CAD$  is smaller than module of angle  $BAC$ , then

$$(13) \quad F_{circ.w} = F_{attr.w.x} s / \cos |\angle CAD| ;$$

- if module of angle  $CAD$  is equal to module of angle  $BAC$ , then

$$(14) \quad F_{circ.w} = F_{attr.w} s ,$$

where  $s$  assumes values 1, 0 or  $-1$ , corresponding to positive, zero and negative torque.

Having determined force  $F_{circ.w}$  by (12–14) one can obtain instantaneous value of electromagnetic torque for one phase in the following way:

$$(15) \quad M_{em}(n) = F_{circ.w}(n) D_r 2 N_{wpp} s , \\ n = 1..m ,$$

where  $N_{wpp}$  – number of operating ports for one phase.

SRM average electromagnetic torque created by its all phases is determined via expression:

$$(16) \quad M_{emmean} = \left( \sum_{n=1}^m \sum_{N=1}^{N_{ph}} M_{em}(n, N) \right) / m ,$$

where  $N_{ph}$  – number of SRM phases.

#### Determination of phase switching angle

Determining SRM torque on the basis of instantaneous values of phase currents and voltages, it is necessary to take into consideration the rotor position corresponding to beginning of phase commutation, i.e. switching angle  $\gamma_{on}$ . This angle corresponds to angle  $AOB$  in Fig. 3, and force  $F_{attr.w}$  corresponds to it. As the rotor rotates, angle  $AOB$  will vary and calculated magnetic effort creating torque  $M_{em}$  will change with it.

Changing values of angle  $AOB$  within permissible limits, the authors obtained and analyzed the effort variation curves ( $F_{circ.w} = f(\angle AOB)$ ) during one commutation

stroke at different shifts of angle  $\gamma_{on}$  in relation to  $\gamma_u$ . A peculiar feature of their variation is shown in Fig. 4.

The shown results demonstrate that the curve in Fig. 4, b is characterized by lower values of electromagnetic efforts as compared with the curve in Fig. 4, a, and the curve in Fig. 4, c, when the rotor pole passes angle  $\gamma_a$ , may go to the domain of negative (braking) values. Thus, angle  $\gamma_{on} = \gamma_u$  is the most efficient phase commutation angle for obtaining the highest value of the torque.

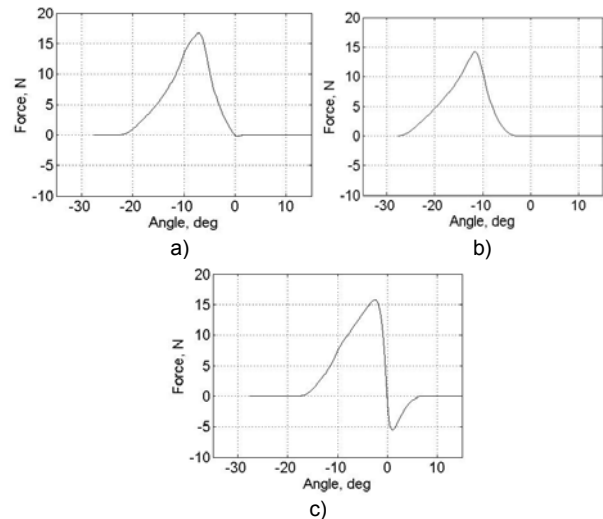


Fig. 4. Electromagnetic effort curves for different values of angles  $\gamma_{on}$ : a)  $\gamma_{on} = \gamma_u$ ; b)  $\gamma_{on} = \gamma_u - 5^\circ$ ; c)  $\gamma_{on} = \gamma_u + 5^\circ$

At the same time, according to existing practice [3], phase switching on takes place at angle position of the poles  $\gamma_{on} > \gamma_u$ . However, to determine electromagnetic torque curve it is necessary to know angle  $\gamma_{on}$  exactly. Let us consider determination of angle  $\gamma_{on}$  by example of SRM used in the research. This motor is characterized by fixed angle  $\gamma_{on}$  and is reversible. It allows assuming the value of the torque to be constant at rotor reversing for the determined level of phase voltage and load.

In Fig. 1 rays  $a$  and  $b$  are drawn across the middles of the stator poles; excitation voltage is supplied to their phases depending on determined direction of rotation. Rays  $a'$  and  $b'$  are drawn respectively across the middles of the rotor poles interacting with them.

Rays  $a$  and  $b$  make angle  $aOb$ , calculated as follows:

$$(17) \quad \angle aOb = 360 / N_s 2 = 360 / 8 \cdot 2 = 90^\circ ,$$

Rays  $a'$  and  $b'$  also make an angle calculated as:

$$(18) \quad \angle a'Ob' = 360 / N_r 2 = 360 / 6 \cdot 2 = 120^\circ ,$$

where  $N_r$  – number of rotor poles.

Taking into account the above said, the SRM torques in direct and reverse direction of rotation are the same under condition that angles  $aOa'$  and  $bOb'$  are equal to each other. Thus, phases switching angle in relation to angle  $\gamma_a$  can be determined as:

$$(19) \quad \gamma_{on} = -\angle aOa' = -\angle bOb' = -(\angle a'Ob' - \angle aOb) / 2 = -15^\circ .$$

The obtained results can be easily recalculated for SRM with other values of stator and rotor pole numbers based on relation

$$(20) \quad \gamma_{on} = -360 \frac{N_s - N_r}{N_s N_r}.$$

Fig. 5 shows a calculated curve of effort generating electromagnetic torque created by one stator pole during one pulse according to calculated angle  $\gamma_{on}$  of phase switching.

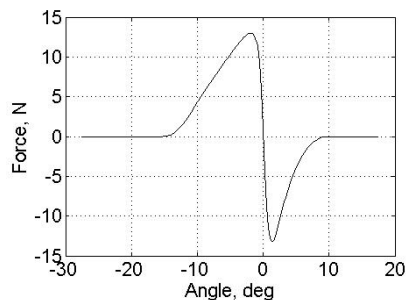


Fig. 5. Curve of effort created by one stator pole during one pulse

### Experimental research

The following experimental research was carried out to confirm reliability of the results of theoretic calculation.

The load of the analyzed SRM was set by means of a shunt excitation direct current motor (DCM). SRM load moment  $M_L$  was determined on the basis of DCM electromagnetic torque, calculated by the values of the keeper voltage and current, and also no-load loss. Electromagnetic torque created by DCM, was determined by measured values of DCM keeper current and voltage ( $I_{a,DCM}$  and  $U_{DCM}$ , respectively) in the following way:

$$(21) \quad M_{em,DCM} = (U_{DCM} I_{a,DCM} - I_{a,DCM}^2 R_a) / \omega,$$

where  $R_a$  – resistance of DCM keeper;  $\omega$  – angular velocity of shaft rotation.

Mechanical losses dependence on DCM rotor rotation frequency was determined by means of self-running-out method carried out in accordance with [5]. Besides, this method was used to determine SRM mechanical losses  $P_{mech}$  which enabled determination of its torque:

$$(22) \quad M_{SRM} = M_{em,mean} - P_{mech} / \omega.$$

The value of power on SRM shaft was calculated by value  $M_{SRM}$ :

$$(23) \quad P_{2,comp} = M_{SRM} \omega.$$

The research was carried out for three operating conditions of SRM, each of which corresponds to the set level of phases supply voltage, for six points of load, created by DCM. Power consumed by SRM was determined directly by results of measurement of instantaneous values of phase currents and voltages. The data of the current and voltage sensors at every SRM phase were read with the frequency of  $f_s = 40$  kHz at each measuring channel by means of data acquisition module L-CARD E14-440.

The essence of experimental research consisted in assessment of such parameters as power on shaft and efficiency of SRM obtained by calculation ( $P_{2,comp}, \eta_{comp}$ ) and experiment ( $P_{2,exp}, \eta_{exp}$ ). Characteristics  $\eta_{comp} = f(P_{2,comp})$  and  $\eta_{exp} = f(P_{2,exp})$  for the researched operating conditions of SRM are shown in Fig. 6.

In accordance with the obtained results the maximum error of determination of the calculated value of the torque

is 9.5%, which confirms sufficient reliability of the proposed method. In this case maximum efficiency of the analyzed SRM is 55.8%, and minimum one – 35.6%.

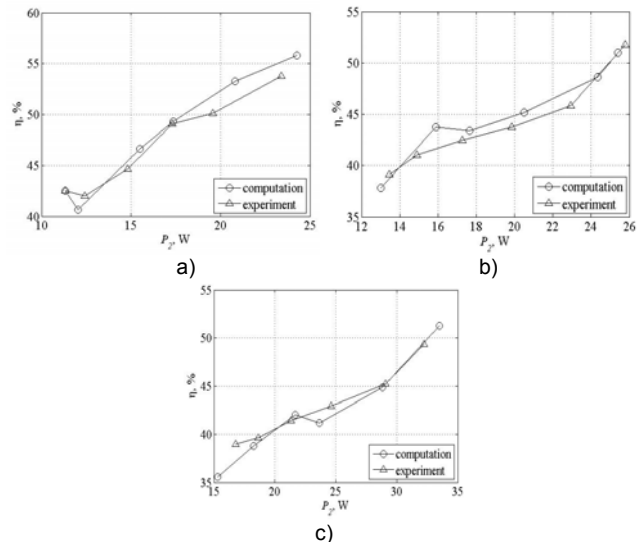


Fig. 6. SRM efficiency variation dependence on power on the shaft for researched operating conditions of SRM

### Conclusions

1. A method for SRM power efficiency assessment based on indirect determination of the torque has been substantiated.

2. A method for calculation of SRM electromagnetic torque based on instantaneous values of phase currents and voltages has been developed. The most efficient angle of phase commutation has been determined and recommendations as to its calculation have been formulated.

3. The results of comparison of the torque obtained by calculation and experiment have confirmed sufficient reliability of the proposed method.

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