Application of a fuzzy logic controller for a permanent magnet synchronous machine drive

Abstract. The article presents a model of the drive with a permanent magnet synchronous machine. In the drive control system was applied a fuzzy speed controller. The general formula of the rule base for the fuzzy controller with dual structure was presented. The simulation tests of the drive were carried out during start-up, changing load torque and reverse. The results of simulation research were presented as the time characteristics of the stator current, the rotor speed, the electromagnetic torque and the DC bus voltage.

Streszczenie. W artykule przedstawiono model napędu z silnikiem synchronicznym z magnesami trwałymi. W układzie sterowania napędem został zastosowany rozmyty regulator prędkości. Przedstawiono ogólną formułę bazy reguł dla regulatora rozmytego o podwójnej strukturze. Przeprowadzono badania symulacyjne napędu podczas rozruchu, zmiany momentu obciążenia oraz rewersu. Rezultaty badań przedstawiono w postaci przebiegów czasowych prędkości stojącej, prędkości wirnika, momentu elektromagnetycznego oraz napięcia stałego w obwodzie pośrednim. (Zastosowanie regulatora rozmytego w napędzie z silnikiem synchronicznym z magnesami trwałymi).

Keywords: electric drive, permanent magnet synchronous machine, fuzzy logic controller.

Słowa kluczowe: napęd elektryczny, silnik synchroniczny z magnesami trwałymi, regulator rozmyty.

Introduction

Permanent magnet synchronous motors (PMSM) are typically used for high-performance and high-efficiency motor drives in various industry branches and in transport. These motors are characterized by the simple mechanical structure, high reliability, a favourable rate of torque to the rotor’s moment of inertia, high energy efficiency and capability of sustained operation with high loads and small rotational speed. A higher energy efficiency as compared to that of induction motors results mainly from a different construction of the rotor. The most popular applications include drives of electric vehicles, lifts and conveyor belts [2, 12, 8].

High-performance motor control is characterized by smooth rotation over the entire speed range of the motor, full torque control at zero speed, and fast acceleration and deceleration. Permanent magnet synchronous motors are powered from the semiconductor converters, whose control algorithms include the angle between the rotor and the stator. The angle has therefore to be measured with maximal accuracy by means of an encoder [3, 7, 5].

The use of fuzzy controllers allows the search for new possibilities to modify the method of adjustment speed and torque. The fuzzy control algorithms are used in the control of the drives, because they allow to take into account the characteristics of the measuring transducers and the phenomenon of nonlinearity [6, 9, 10]. These algorithms also provide the improvement of quality indicators for the drive control system.

The model of the permanent magnet synchronous motor

The voltages of the stator windings of the permanent magnet synchronous motor are defined by the system of equations [2, 12]:

\[
\begin{bmatrix}
    v_a \\
v_b \\
v_c
\end{bmatrix} = 
\begin{bmatrix}
    R_s & 0 & 0 \\
    0 & R_s & 0 \\
    0 & 0 & R_s
\end{bmatrix} 
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix} + 
\begin{bmatrix}
dv_a \\
dv_b \\
dv_c
\end{bmatrix} = 
\begin{bmatrix}
    \frac{di_a}{dt} \\
    \frac{di_b}{dt} \\
    \frac{di_c}{dt}
\end{bmatrix}
\]

(1)

where: \(v_a, v_b, v_c\) – the individual phase voltages across the stator windings, \(R_s\) – the equivalent resistance of each stator winding, \(i_a, i_b, i_c\) – the currents flowing in the stator windings, \(dv_a/dt, dv_b/dt, dv_c/dt\) – the rates of change of magnetic flux in the stator windings.

The components of the total flux are defined as:

\[
\begin{bmatrix}
    \psi_a \\
    \psi_b \\
    \psi_c
\end{bmatrix} = 
\begin{bmatrix}
    L_{aa} & L_{ab} & L_{ac} \\
    L_{ba} & L_{bb} & L_{bc} \\
    L_{ca} & L_{cb} & L_{cc}
\end{bmatrix} 
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix} + 
\begin{bmatrix}
    \psi_{am} \\
    \psi_{bm} \\
    \psi_{cm}
\end{bmatrix}
\]

(2)

where: \(\psi_a, \psi_b, \psi_c\) – the total fluxes linking each stator winding, \(L_{aa}, L_{ab}, L_{ac}\) – the self-inductances of the stator windings, \(L_{ba}, L_{bb}, L_{bc}\) – the mutual inductances of the stator windings, \(\psi_{am}, \psi_{bm}, \psi_{cm}\) – the permanent magnet fluxes linking the stator windings.

The self-inductances describe the following equations:

\[
L_{aa} = L_s + L_m \cos(2\Theta_r)
\]

(3)

\[
L_{bb} = L_s + L_m \cos(2(\Theta_r - 2\pi / 3))
\]

\[
L_{cc} = L_s + L_m \cos(2(\Theta_r + 2\pi / 3))
\]

The mutual inductances of the stator windings are defined as:

\[
L_{ab} = -M_s - L_m \cos(2(\Theta_r + \pi / 6))
\]

(4)

\[
L_{bc} = -M_s - L_m \cos(2(\Theta_r + \pi / 6 + 2\pi / 3))
\]

\[
L_{ca} = -M_s - L_m \cos(2(\Theta_r + \pi / 6 + 2\pi / 3))
\]

where: \(M_s\) – the stator mutual inductance.

The linked motor flux is described by following equations:

\[
\begin{bmatrix}
    \psi_{am} \\
    \psi_{bm} \\
    \psi_{cm}
\end{bmatrix} = 
\begin{bmatrix}
    \psi_m \cos(\Theta_r) \\
    \psi_m \cos(\Theta_r - 2\pi / 3) \\
    \psi_m \cos(\Theta_r + 2\pi / 3)
\end{bmatrix}
\]

(5)

In order to transform the system of equations of the permanent magnet synchronous motor is used Park’s transformation [2, 12]:

\[
P = \begin{bmatrix}
    \cos(\Theta_r) & \cos(\Theta_r - 2\pi / 3) & \cos(\Theta_r + 2\pi / 3) \\
    -\sin(\Theta_r) & -\sin(\Theta_r - 2\pi / 3) & -\sin(\Theta_r + 2\pi / 3)
\end{bmatrix}
\]

\[
\begin{bmatrix}
    0.5 \\
    0.5 \\
    0.5
\end{bmatrix}
\]

(6)
\[ \theta_c = N \cdot \theta_r \]

where: \( \theta_r \) – the electrical angle, \( N \) – the number of pole pairs.

Applying Park's transformation to the system of equations of the permanent magnet synchronous motor produces the following equations:

\[ v_d = R_i i_d + L_d \frac{di_d}{dt} - N \omega i_q L_q \]

\[ v_q = R_i i_q + L_q \frac{di_q}{dt} + N \omega (i_d L_d + \psi_m) \]

\[ v_0 = R_i i_0 + L_0 \frac{di_0}{dt} \]

\[ T = \frac{3}{2} N (i_q (i_d L_d + \psi_m) - i_d i_q L_q) \]

where: \( v_d, v_q, v_0 \) – the d-axis, q-axis, and zero-sequence voltages, \( i_d, i_q, i_0 \) – the d-axis, q-axis, and zero-sequence currents, \( L_d \) – the stator d-axis inductance, \( L_q \) – the stator q-axis inductance, \( L_o \) – the stator zero-sequence inductance, \( \omega \) – the rotor rotational speed, \( N \) – the number of rotor permanent magnet pole pairs, \( T \) – the rotor torque.

The d-axis, q-axis, zero-sequence voltages and the d-axis, q-axis, zero-sequence currents are described by the relations:

\[
\begin{bmatrix}
  v_d \\
  v_q \\
  v_0
\end{bmatrix} = P \begin{bmatrix}
  v_a \\
  v_b \\
  v_c
\end{bmatrix}
\]

\[
\begin{bmatrix}
  i_d \\
  i_q \\
  i_0
\end{bmatrix} = P \begin{bmatrix}
  i_a \\
  i_b \\
  i_c
\end{bmatrix}
\]

The fuzzy controller of the drive control system

The fuzzy controller has been designed with the use of Fuzzy Logic Toolbox. The controller has the modules of linguistic input data \( e \) and \( ie \), and the modules of linguistic output data \( up \) and \( ui \). The linguistic input data include the triangular membership functions defining the error \( e \) and the integral error \( ie \). The functions are labelled as follows: \( NM \) – negative medium, \( NS \) – negative small, \( ZO \) – zero, \( PS \) – positive small, \( PM \) – positive medium. For the linguistic output variables \( ui \) and \( up \) were applied the Gaussian membership functions. The functions are distributed evenly in the standardised output interval.

The controller is characterized by the dual structure. The general base of the rules can be written down in as follows [11, 4]:

\[
\begin{array}{c}
R^{(1)}: IF (e \text{ is } LE^{(1)}) \quad \text{THEN (up is } LUP^{(1)}) \\
R^{(2)}: IF (e \text{ is } LE^{(2)}) \quad \text{THEN (up is } LUP^{(2)}) \\
R^{(3)}: IF (e \text{ is } LE^{(3)}) \quad \text{THEN (up is } LUP^{(3)}) \\
R^{(4)}: IF (e \text{ is } LE^{(4)}) \quad \text{THEN (up is } LUP^{(4)}) \\
R^{(5)}: IF (e \text{ is } LE^{(5)}) \quad \text{THEN (up is } LUP^{(5)}) \\
R^{(6)}: IF (e \text{ is } LE^{(6)}) \quad \text{THEN (up is } LUP^{(6)}) \\
R^{(7)}: IF (e \text{ is } LE^{(7)}) \quad \text{THEN (up is } LUP^{(7)}) \\
R^{(8)}: IF (e \text{ is } LE^{(8)}) \quad \text{THEN (up is } LUP^{(8)}) \\
R^{(9)}: IF (e \text{ is } LE^{(9)}) \quad \text{THEN (up is } LUP^{(9)}) \\
R^{(10)}: IF (e \text{ is } LE^{(10)}) \quad \text{THEN (up is } LUP^{(10)}) \\
\end{array}
\]

where: \( e, ie \) - the linguistic input variables: of the error and of the error integral, \( up, ui \) - the linguistic output variables of the trajectories: the proportional one and the integrating one, \( LE^{(1)}, ..., LE^{(10)} \) - the linguistic values of the error and the error integral, \( LUP^{(1)}, ..., LUP^{(10)} \) - the linguistic values of the output variables of the trajectories: the proportional one and the integrating one, \( w_p, w_i \) - are the weight factors.

The rule base for the fuzzy controller was initially defined by means of the standard Mac Vicar-Whelan table, which was subsequently modified taking into account the input and output parameters of the membership functions defining the linguistic variables [7, 1].

In the defuzzification process was applied the Center of Sums method (CoS). This method is similar to the Center of Area (CoA) method, but much faster. It also provides continuity and participation in the inference process all active rules. Common areas of the membership functions are included in the defuzzification process more than once using the Center of Sums method. For this method, the scalar value of the fuzzy controller is determined using the formula [11]:

\[
\mu_N = \frac{\sum_{i=1}^{m} \sum_{k=1}^{n} \mu_{CLU(k)}(u_i)}{\sum_{i=1}^{m} \sum_{k=1}^{n} \mu_{CLU(k)}}
\]

where: \( u_N \) – the normalized output linguistic variable, \( \mu_k \) - the function of output fuzzy set, \( \mu_{CLU(k)} \) - the compressed membership function for each \( k \)-th rule

The simulation model of the drive with permanent magnet synchronous motor was presented in Fig. 1 and the control system of the PMSM drive was shown in Fig. 2.
The results of simulation experiments
The simulation experiments of the drive with the permanent magnet synchronous motor fed by the three-phase inverter were performed at the start-up, changing the load torque and the reverse.

The basic parameters of the PMSM: $P_N=2.2\,\text{kW}$, $R_s=0.2\,\Omega$, $L_d=0.0082\,\text{mH}$, $L_q=0.0082\,\text{mH}$, $\varphi_m=0.170\,\text{Vs}$, $J=0.09\,\text{kg}\cdot\text{m}^2$, $F=0.005\,\text{Nms}$, $p=4$.

The selected results in the form of time characteristics of the stator current, the rotor speed, the electromagnetic torque and the DC bus voltage are presented in Fig. 3-6.

In the electrical drive, an indicator is often applied which is calculated as an integral of the product of time and the absolute value of the control deviation. This indicator is determined using the formula [2]:

\[
I_{ITAE} = \int_0^\infty t \cdot |e(t)| \, dt = \int_{t_0}^{t_f} t \cdot |e(t)| \, dt
\]

where: $e(t)$ - the control deviation, $t_f$ - the time of ending the control process, $t$ - the time.

The integral quality indicator of the rotor speed can be defined in the following formula:

\[
I_n = \int_0^\infty t \cdot |e_n(t)| \, dt = \int_{t_0}^{t_f} t \cdot |e_n(t)| \, dt = \int_{t_0}^{t_f} t \cdot |n_R - n(t)| \, dt
\]

where: $n(t)$ - rotor speed, $n_R$ - prescribed rotor speed, $e_n(t)$ - rotor speed error.
The results of the simulation investigations of the drive with the fuzzy controller and the conventional controller were compared. This comparison allowed to verify the proposed control strategy for the speed control of the drive. The selected results of research in the form of integral quality indicators are presented in Table 1.

Table 1. Quality indicators for the PMSM drive model with the fuzzy controller and the conventional controller

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Fuzzy control system</th>
<th>Conventional control system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load torque $T_e$ [Nm]</td>
<td>Integral quality indicator $I_q$</td>
<td>Integral quality indicator $I_q$</td>
</tr>
<tr>
<td>2</td>
<td>3.71</td>
<td>3.82</td>
</tr>
<tr>
<td>4</td>
<td>3.85</td>
<td>4.03</td>
</tr>
<tr>
<td>6</td>
<td>4.01</td>
<td>4.35</td>
</tr>
<tr>
<td>8</td>
<td>4.12</td>
<td>4.53</td>
</tr>
<tr>
<td>10</td>
<td>4.25</td>
<td>4.62</td>
</tr>
</tbody>
</table>

Conclusions
The time characteristics of the stator current, the rotor speed, the electromagnetic torque, as well as quality indicators of the rotor speed confirm the usefulness and validity of the fuzzy controller in the drive control system. The tuning method enables creating a knowledge base and determining the mutual position of input and output membership functions of the linguistic variables of the fuzzy controller used in the drive control system of the permanent magnet synchronous motors. The comparative analysis of the drive system with the fuzzy controller and the one with the conventional controller demonstrates that integral quality indicators are also smaller for the fuzzy controller, however, the correlation with the load torque values is significantly smaller.

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