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Topological Methods to Determine Damages of Flood Embankments

***Abstract**. This paper presents a method of testing flood embankment. There was used a specially built laboratory model to determine the moisture level of flood embankments. The finite element method was used to solve the forward problem. The proposed algorithm was initialized by using one step methods and topological sensitivity analysis. There was solved the inverse problem in order to visualize moisture inside objects. There was made possible to change topology during the optimization. The level set method and the Gauss-Newton method have been applied very successfully in many areas of the scientific modelling. Topological algorithms were based on shape sensitivity include the boundary design of the elastic interface. These algorithms are a relatively new procedure to overcome this problem.*

***Streszczenie.** Artykuł przedstawia metodę badania wału przeciwpowodziowego. Został zbudowany specjalny model laboratoryjny wału w celu określenia poziomu wilgotności. Do rozwiązania zagadnienia prostego została wykorzystana metoda elementów skończonych. Proponowany algorytm inicjowany jest metodą jednokrokową i rozwiązywany topologiczną analizą wrażliwościową. Rozwiązano zagadnienie odwrotne w celu wizualizacji wilgoci wewnątrz obiektów, poprzez zmianę topologii podczas procesu optymalizacji. Metody zbiorów poziomicowych i Gaussa-Newtona stosuje się z dużym powodzeniem w wielu dziedzinach modelowania naukowego. Metody topologiczne opierają się na analizie wrażliwościowej dostosowując kształt brzegu elastycznego interfejsu. Algorytmy te są relatywnie nowymi rozwiązaniami dla tego typu problemu.* (**Metody topologiczne do określania uszkodzeń w wałach przeciwpowodziowych**).

Keywords: Electrical Impedance Tomography, Finite Element Method, Inverse Problem
Słowa kluczowe: elektryczna tomografia impedancyjna, metoda elementów skończonych, zagadnienie odwrotne

Introduction

This paper presents the new method examining the flood embankment dampness by electrical impedance tomography (EIT) [2,8,9,11]. Numerical methods of the shape and the topology optimization were based on topological algorithms. Discussed techniques can be applied to the solution of inverse problems in electrical impedance tomography [1,3,4,13]. There were implemented algorithms to identify unknown conductivities (dampness). The purpose of the presented methods is obtaining the image reconstruction by the proposed solution. Numerical methods of the shape and the topology optimization were based on Gauss-Newton method and the level set function.

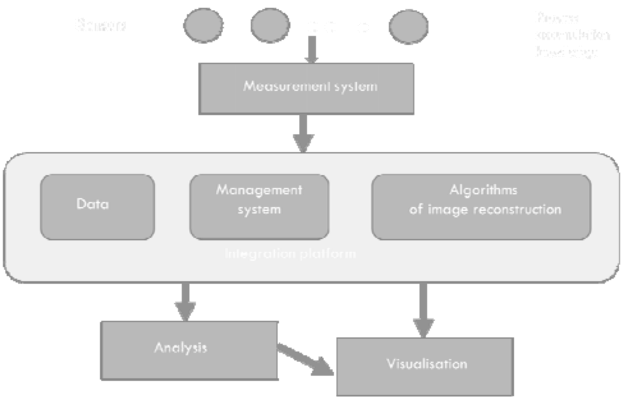

Fig. 1. The architecture of the flood embankment system

There were implemented algorithms to identify unknown conductivities (dampness) [10]. Numerical methods of the shape and the topology optimization were based on the level set representation and the Gauss-Newton method. Level set methods have been applied very successfully in many areas of the scientific modelling, for example in propagating fronts and interfaces [5-7,12]. Therefore, there were used to study shape optimization problems. Instead of using the physically driven velocity, the level set method typically moves the surfaces by the gradient flow of an energy functional. The architecture of the flood embankment system is presented in Figure 1.

Electrical impedance tomography

Electrical impedance tomography image reconstruction problem is an ill-posed inverse problem. In EIT, electrical voltages are injected into an object using a set of electrodes attached on the surface of the object and the potentials are measured. The conductivity of the object is reconstructed based on the known voltages and measured potentials. Electrical impedance tomography reconstruction requires accurate modeling. EIT is an imaging modality in which the conductivity distribution of an examined object is estimated from measurements of electrical voltages and electrode potentials at the boundary. To achieve quantitative information of the conductivity change, it would be preferable to use a non-linear model in the solution of the difference imaging. Non-linear difference imaging approaches for reconstruction of changes in a target conductivity from EIT measurements. The proposed algorithm is evaluated both with simulated measurements and real data. The image reconstruction is a highly ill-posed inverse problem. The forward problem in EIT is described by Laplace's equation:

(1) $$\nabla \cdot (\gamma \nabla u) = 0$$

where: γ denotes conductivity, symbol *u* represents electrical potential.

The level set method is a numerical technique which can follow the evolution of interfaces (Fig. 2). These interfaces can develop sharp corners, break apart and merge together. The level set function relies on the shape derivative, while the topological gradient method is depended on the material derivative. The topological method is based on the differentiability of solutions to variational inequalities with respect to the coefficients of the governing differential operator. For the minimization problem iterative coupling of the level set method and the topological gradient method have been proposed. Both methods can be cast into the framework of alternate directions descent algorithms. A Gauss-Newton method is deployed to the regularized tangential movement problem. The electrodes move tangentially to the domain at each iteration and so do not in general lie on the boundary of the domain after each iteration.

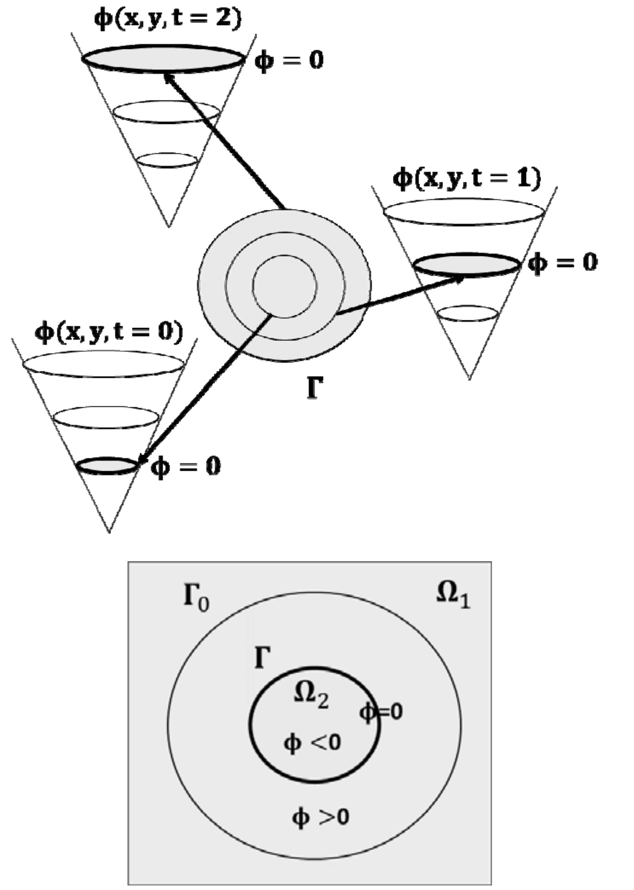

Fig. 2. The idea of the level set function

The motion is seen as the convection of values (levels) from the function ϕ with the velocity field $\vec{v}$. Such a process is described by the Hamilton-Jacobi equation:

(2) $$\frac{\partial \phi}{\partial t} + \vec{v} \cdot \nabla \phi = 0.$$

Here, $\vec{v}$ is the desired velocity on the interface, and is arbitrary elsewhere. Actually, only the normal component of $\vec{v}$ is needed ($v_n \equiv \vec{v} \cdot \vec{n} \equiv \vec{v} \cdot \nabla \phi / |\nabla \phi|$), so (2) becomes:

(3) $$\frac{\partial \phi}{\partial t} + v_n \left|\nabla \phi\right| = 0.$$

We can update the level set function ϕ by solving discretized version of the Hamilton-Jacobi equation:

(4) $$\frac{\phi^{k+1} - \phi^k}{\Delta t} + v_n^k \left|\nabla \phi^k\right| = 0.$$

Transforming above equation we get:

(5) $$\phi^{k+1} = \phi^k - v_n^k \left|\nabla \phi^k\right| \Delta t.$$

The gradient of the level set function in the k-th time step ($\left|\nabla \phi^k\right|$) has been calculated by the essentially non-oscillatory (ENO) polynomial interpolation scheme. The stability of received solution is achieved by Courant-Friedreichs-Lewy condition (CFL condition):

(6) $$\Delta t < \frac{\min(\Delta x, \Delta y)}{\max(|\vec{v}|)}.$$

Inequality (6) is satisfied by choosing the CFL number α:

(7) $$\Delta t \frac{\max(|\vec{v}|)}{\min(\Delta x, \Delta y)} = \alpha,$$

where: 0 < α < 1. The optimum value equals 0.9.

The calculated velocity must be extended off the interface to the whole domain. This process is called the extension of velocity and is based on the solution of the additional partial differential equation:

(8) $$\frac{\partial v_n}{\partial t} + S(\phi) \frac{\nabla \phi}{\left|\nabla \phi\right|} \cdot \nabla v_n = 0,$$

where S(ϕ) is defined as following:

(9) $$S(\phi) = \frac{\phi}{\sqrt{\phi^2 + \varepsilon^2}}.$$

In (9) |ε| << 1. Additionally, we need extend velocity to neighbourhood of the interface, by defining velocity along normal direction.

Reinitialization is necessary when flat or steep regions complicate the determination of the zero contour. The level set function ϕ is signed distance function if at given time for every point:

(10) $$\left|\nabla \phi\right| = 1.$$

Reinitialization is based on replacing ϕ by another function that has the same zero level set, but satisfies condition (10). This process is described by following partial differential equation:

(11) $$\frac{\partial \phi}{\partial t} + S(\phi)\,(|\nabla \phi| - 1) = 0.$$

Differential equation (11) is solved until a steady state is achieved. Similar to the velocity extension a first order upwind scheme for the spatial dimension and forward Euler time discretization is used.

The topological method is based on so-called conical differentiability of solutions to variational inequalities with respect to the coefficients of the governing differential operator. It is required that the metric projection in the energy space. Such property is sufficient to obtain the directional differentiability of solutions to the variational inequality with respect to the boundary variations and the changes in the topology. A useful concept for calculating derivatives for cost functional is the so-called material and shape derivative of states u. In the application of inverse problems, these states typically are the solutions of partial differential equations which model the probing fields and which depend in one way or another on the shape.

Let λ be the adjoint function satisfying:

(12) $$-\Delta\lambda = u - u_m.$$

The material derivative $\dot{u}(x)$ is given by:

(13) $$\dot{u}(\vec{r}) \equiv \lim_{t \to 0} \frac{u_t(\vec{r} + t\vec{v}(\vec{r})) - u(\vec{r})}{t},$$

where $(x, y) \in \Omega_t$. The shape derivative is following:

(14) $$u'(\vec{r}) \equiv \lim_{t \to 0} \frac{u_t(\vec{r}) - u(\vec{r})}{t} = \dot{u}(\vec{r}) - \vec{v}(\vec{r}) \cdot \nabla u(\vec{r}).$$

The steepest descent direction $\vec{v}$ is given by:

(15) $$\vec{v} = -(\nabla u \cdot \nabla \lambda)\,\vec{n}.$$

In next step the level set function is updated:

(16) $$\phi^{k+1} = \phi^k - \left(\nabla u^k \cdot \nabla \lambda^k\right)\left|\nabla \phi^k\right| \Delta t.$$

For the minimization problem, iterative coupling of the level set method and the topological gradient method have been proposed. Both methods are gradient-type algorithms, and the coupled approach can be cast into the framework of alternate directions descent algorithms. One step methods and topological algorithms were used to solve this problem.

The proposed algorithm is iterative method, structured as follows (Fig. 3):

- calculate one step Gauss-Newton method,

- initialization the zero level set function,
- use the finite element method to solve the Laplace's equation,
- compute the difference of the obtained solution with the observed data,
- solve the Poisson's equation,
- calculate velocity,
- update the level set function,
- reinitialize the level set function.

The level set function relies on the shape derivative, while the topological gradient method is based on the material derivative.

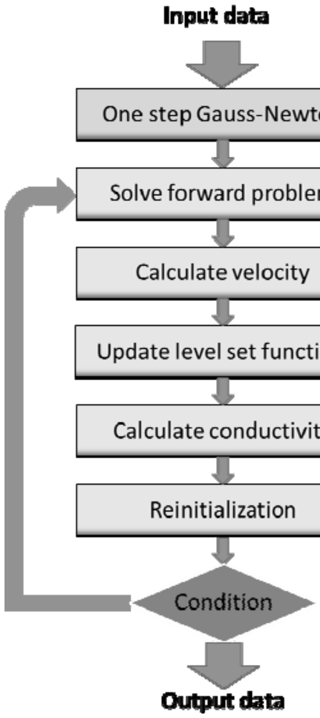

Fig. 3. The scheme of the algorithm to minimize the objective function

Model

There was prepared a special model of the flood embankment (Fig. 4). Figure 5 presents the laboratory measurement system.

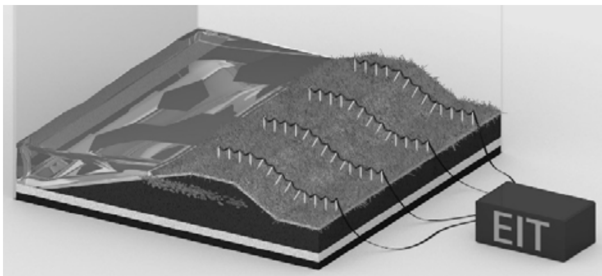

Fig.4. The geometrical laboratory models of the flood embankment

Results

The Gauss-Newton method and the level set method were used with the optimization approach. Numerical algorithms of shape and topology optimization were based on the level set representation and the shape differentiation. There were made topology changes possible during the optimization process. These approaches are based on shape sensitivity include the boundary design of the elastic interface.

The image reconstruction obtained by the Gauss Newton Level Set Method with simulation data is shown in Figure 6. Figure 7 and 9 present the geometrical models of the investigated flood embankment with 16 electrodes and the image reconstruction by the Gauss-Newton and the level set methods. Figure 8, 10 and 12 present the objective function for the tested models. Figure 11 shows the image reconstruction with real data taken 15 minutes after flooding.

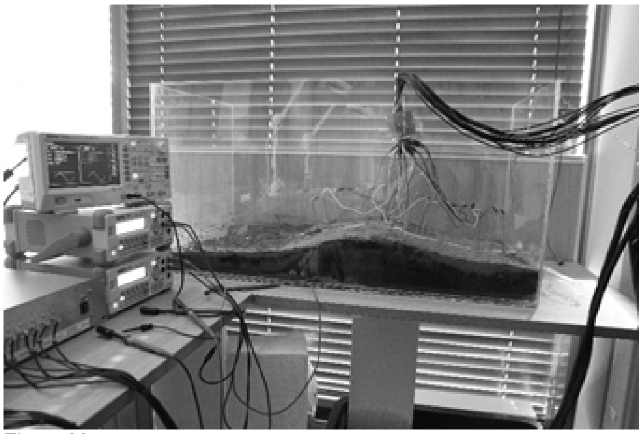

Fig. 5. Measurement system

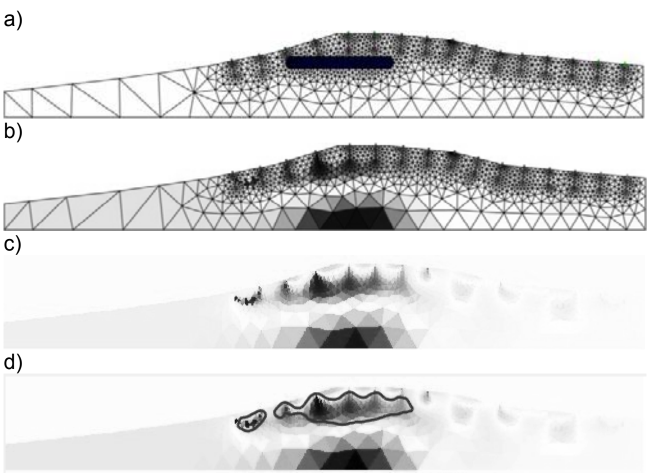

Fig. 6. The image reconstruction obtained by the Gauss Newton Level Set Method with simulation data: a) the model, b) the image reconstruction, c) the image without mesh, d) the level set method component

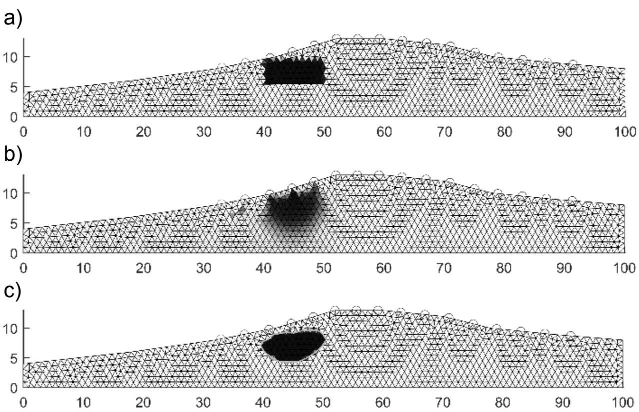

Fig. 7. The geometrical model I of the investigated flood embankment with 16 electrodes: a) the initial model, b) the image reconstruction by Gauss-Newton method, c) the image reconstruction by the level set method

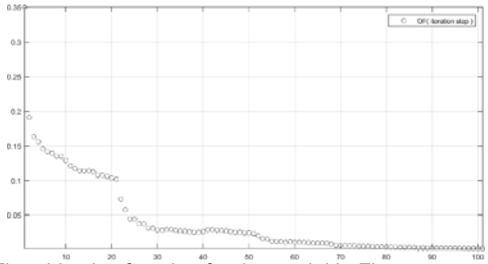

Fig. 8. The objective function for the model in Figure 7

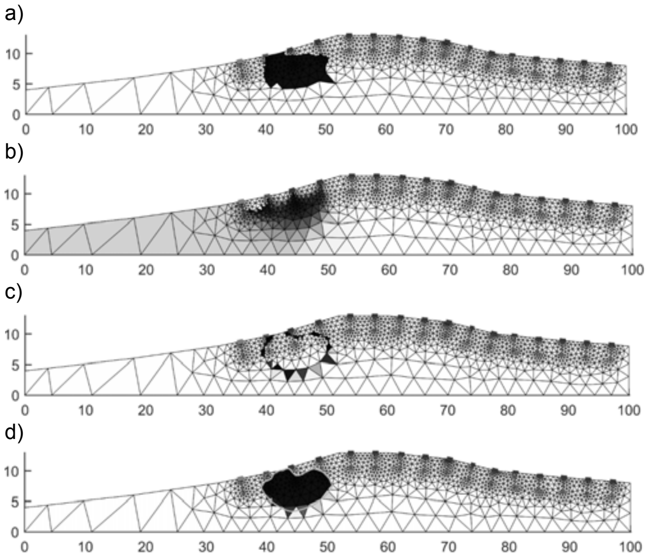

Fig. 9. The geometrical model II of the investigated flood embankment with 16 electrodes: a) the initial model, b) the image reconstruction by the Gauss-Newton method, c) the image reconstruction by the level set method

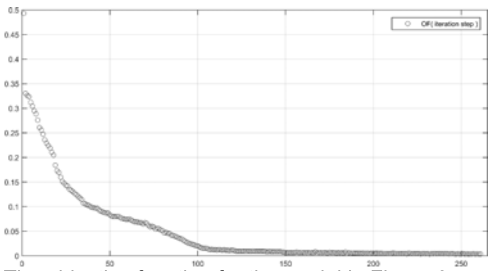

Fig. 10. The objective function for the model in Figure 9

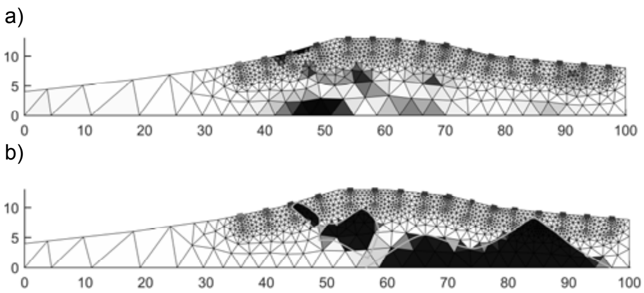

Fig. 11. The image reconstruction with real data obtained by: a) the Gauss Newton method, b) the Gauss Newton method with Level Set Method (data taken 15 minutes after flooding)

Summary

The proposed method was verified by simulations and real measurements by the laboratory model. The Gauss-Newton method and the level set function have been shown to be successful to identify the unknown boundary shapes. The presented method determines the moisture of the test model. These methods have been applied successfully. Applying the line measurement model is enough effective to solve the inverse problem in the moisture flood embankment.

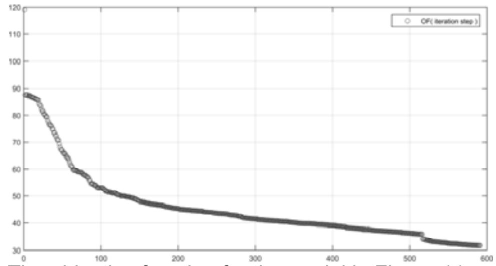

Fig. 12. The objective function for the model in Figure 11

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