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doi:10.15199/48.2016.12.50

Disturbance observer based control of active suspension system with uncertain parameters

Streszczenie. *Artykuł przedstawia układ sterowania pojedynczej osi aktywnego zawieszenia pojazdu z zastosowaniem obserwatora zakłóceń (DO). Obserwator przeciwdziała zakłóceniom poprzez kompensację w torze głównym obliczaną na podstawie sygnału sterującego i sprzężenia stanu z nominalnym modelem obiektu. Zaproponowany sposób sterowania został zweryfikowany eksperymentalnie.*
*(**Sterowanie układem aktywnego zawieszenia z niepewnymi parametrami z zastosowaniem obserwatora zakłóceń**).*

Abstract. *The paper deals with application of the disturbance-observer (DO) based control to a quarter car active suspension system with uncertain parameters.The DO counteracts disturbances by feedforward compensation computed on the basis of the control input, the state feedback and the nominal system model. The proposed control scheme is verified experimentally on a mechatronic laboratory model.*

Słowa kluczowe: aktywne zawieszenie, LQR, obserwator zakłóceń.
Keywords: active suspension, LQR, disturbance observer.

Introduction

A vehicle suspension plays a crucial role in isolating passengers from vibrations generated by road surface roughness and improving vehicle handling and safety by keeping tires in uninterrupted contact with the road. Unfortunately, requirements concerning ride comfort and vehicle handling (formulated e.g. in terms of maximum vertical acceleration and the suspension stiffness and stroke) are generally conflicting: ride comfort requires smaller suspension damping and longer stroke, better handling and stability requires higher stiffness, shorter stroke and small dynamic deflection of the tire. Therefore, successful designing or setting a universal passive suspension is difficult.

Active suspension systems have become a popular research topic in recent years due to their great potential to handle the trade-offs between the conflicting requirements. They are based on electro-hydraulic actuators, placed parallel to passive suspension elements between the vehicle body and the wheel axle, and controlled directly to generate a desired control force to add or dissipate energy from the suspension system. The main factors why active suspension systems have not been widely used in vehicle production are: cost, high energy demand and complex control. More commonly implemented solution are semi-active suspensions which employ dampers, whose force is commanded indirectly through a controlled change of the damping (e.g. magnetorheological dampers use magnetic field of varying intensity to change viscosity of fluid with tiny ferrous particles 20-50 microns in diameter [1]). Control of a semi-active suspension is relatively simple.

Over the past two decades the active suspension system has become a test bench for a wide range of control algorithms. Research has shown that a linear optimal control LQR provides a relatively easy and efficient way to design a controller that can improve both ride comfort and handling performance [2,3]. In the LQR framework the model parameters are assumed to be known and an optimal state feedback gain that minimizes a quadratic cost function is obtained. However, a suspension system typically contains parameters that are inherently uncertain, first of all the sprung (body) mass that depends on the vehicle load. The load-dependent control proposed in [4] is an example of multi-objective control schemes that allow to achieve a compromise between several performance requirements and preserve good results in the presence of parameter variations. One recently popular control technique, well suited for dealing with the active suspension, is the disturbance observer/estimator based control [5,6,7]. In general, factors such as uncertain parameters, nonlinearities, modeling errors and external disturbances, can be considered as disturbances that have to be rejected by the control. The advantage of the DO approach is that it deals with disturbances by active feedforward compensation control (based on disturbance estimation) rather than by feedback control [8,9].

This paper presents an application of a control system consisting of a DO combined with an LQR state feedback controller for damping vibrations in a linear quarter car active suspension system under road roughness (external disturbance) and uncertain parameters (internal disturbance). Theoretical results are verified in laboratory experiments.

Model of a quarter car active suspension

The two-degree-of-freedom quarter car active suspension system is shown in Figure 1.

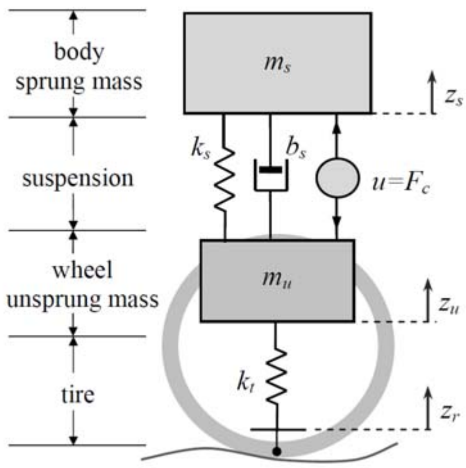

Fig.1. Quarter car active suspension system model

It consist of sprung mass m_s, representing the vehicle body and load, and unsprung mass m_u, representing the wheel and its associated parts; the passive parts of the suspension: the spring and the damper, are assumed to be linear, so they are described by constant spring and damping coefficients k_s and b_s respectively. The tire elasticity is also linear, represented by spring coefficient k_t, the tire damping is neglected. $F_c(t)$ represents the active suspension actuator control force. Variables $z_s(t)$ and $z_u(t)$ are vertical displacements of the sprung and unsprung masses, $z_r(t)$ is the road level disturbance input. The dynamics of the active suspension system is described as:

$$(1)\qquad \begin{cases} m_s\ddot{z}_s = -k_s(z_s - z_u) - b_s(\dot{z}_s - \dot{z}_u) + F_c \\ m_u\ddot{z}_u = k_s(z_s - z_u) + b_s(\dot{z}_s - \dot{z}_u) - k_u(z_u - z_r) - F_c \end{cases}$$

To present the active suspension system in the state space form we define the following state and measured output variables:

$$(2)\qquad \begin{aligned} \mathbf{x} &= [x_1,\ x_2,\ x_3,\ x_4]^T = [z_s - z_u,\ \dot{z}_s,\ z_u,\ \dot{z}_u]^T \\ \mathbf{y} &= [y_1,\ y_2]^T = [z_s - z_u,\ \ddot{z}_s]^T \end{aligned}$$

The state space equations of the system take the form:

$$(3)\qquad \begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{B}_d d \\ \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}u \end{cases}$$

with control input $u=F_c$, external disturbance $d=z_r$, and the following system matrices:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & -\frac{b_s}{m_s} & 0 & \frac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{b_s}{m_u} & -\frac{k_t}{m_u} & -\frac{b_s}{m_u} \end{bmatrix},\quad \mathbf{B} = \begin{bmatrix} 0 \\ \frac{1}{m_s} \\ 0 \\ -\frac{1}{m_u} \end{bmatrix},\quad \mathbf{B}_d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_t}{m_u} \end{bmatrix},$$

$$(4)\qquad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{k_s}{m_s} & -\frac{b_s}{m_s} & 0 & \frac{b_s}{m_s} \end{bmatrix},\quad \mathbf{D} = \begin{bmatrix} 0 \\ \frac{1}{m_s} \end{bmatrix}$$

The system is controllable form the defined input u and observable form the defined output $\mathbf{y}$.

Disturbance observer [5]

Disturbance rejection is one of the key objectives in control design. Popular control approaches, like robust control, adaptive control or sliding mode control (SMC), are based on rejection of disturbances by feedback control, which is referred to as the passive antidisturbance control.

The active antidisturbance control counteract disturbances directly by faster feedforward compensation based on disturbance estimates. A disturbance observer can be used when application of a sensor to measure the disturbance is impossible or unreasonable.

The disturbance $d(t)$ in the active suspension state space model (3) is an *unmatched* disturbance since the input channels for control $u(t)$ and the disturbance are different (corresponding input matrices in (3) are not equal: $\mathbf{B}\neq\mathbf{B}_d$), which requires a more general approach.

Let us assume that the nominal plant matrices ($\mathbf{A}_n$, $\mathbf{B}_n$, $\mathbf{C}_n$, $\mathbf{D}_n$) are known, but the true matrices ($\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$) are uncertain. We define *lumped* disturbances $\mathbf{d}_{lx}$ and $\mathbf{d}_{ly}$ acting on the plant states and outputs:

$$(5)\qquad \begin{cases} \mathbf{d}_{lx} = (\mathbf{A} - \mathbf{A}_n)\mathbf{x} + (\mathbf{B} - \mathbf{B}_n)u + \mathbf{B}_d d \\ \mathbf{d}_{ly} = (\mathbf{C} - \mathbf{C}_n)\mathbf{x} + (\mathbf{D} - \mathbf{D}_n)u \end{cases}$$

The lumped disturbances include external disturbances and internal disturbances from the plant parameter uncertainties and other reasons. The equations of the plant using the lumped disturbances take the following form:

$$(6)\qquad \begin{cases} \dot{\mathbf{x}} = \mathbf{A}_n\mathbf{x} + \mathbf{B}_n u + \mathbf{d}_{lx} \\ \mathbf{y} = \mathbf{C}_n\mathbf{x} + \mathbf{D}_n u + \mathbf{d}_{ly} \end{cases}$$

We will make the following assumptions:

(i) Disturbance $\mathbf{d}_{lx}$ varies slowly relative to the DO dynamics and tends to a constant steady state: $\lim_{t\to\infty}\dot{\mathbf{d}}_{lx} = \mathbf{0}$

(ii) State variables $\mathbf{x}(t)$ are available (both the DO and the LQR controller require the state feedback).

Now we define a linear DO for estimating disturbance $\mathbf{d}_{lx}$ in the following form:

$$(7)\qquad \begin{cases} \dot{\mathbf{p}} = \mathbf{A}_n\mathbf{x} + \mathbf{B}_n u + \hat{\mathbf{d}}_{lx} \\ \hat{\mathbf{d}}_{lx} = \mathbf{L}(\mathbf{x} - \mathbf{p}) \end{cases}$$

where $\mathbf{L}$ is the DO gain matrix to be designed. The disturbance estimation error and its derivative:

$$(8)\qquad \mathbf{e}_{dlx} = \mathbf{d}_{lx} - \hat{\mathbf{d}}_{lx}$$

$$(9)\qquad \dot{\mathbf{e}}_{dlx} = \dot{\mathbf{d}}_{lx} - \mathbf{L}(\dot{\mathbf{x}} - \dot{\mathbf{p}}) = -\mathbf{L}\mathbf{e}_{dlx} + \dot{\mathbf{d}}_{lx}$$

Thus, if the DO gain matrix $\mathbf{L}$ is Hurwitz and assumption (i) is satisfied, the DO estimation error decays asymptotically to zero. The coefficients of $\mathbf{L}$ should be chosen so that the observer response is much faster than the main closed loop response and the disturbance variations.

The estimate of disturbance $\mathbf{d}_{ly}$ can be obtained directly from the algebraic nominal output equation:

$$(10)\qquad \hat{\mathbf{d}}_{ly} = \mathbf{y} - \mathbf{C}_n\mathbf{x} - \mathbf{D}_n u$$

Clearly, estimation error $\mathbf{e}_{dly}=\mathbf{d}_{ly}-\hat{\mathbf{d}}_{ly}=\mathbf{0}$. Note that estimation of disturbance $\mathbf{d}_{ly}$ requires the output feedback.

In general, the unmatched disturbances cannot be attenuated from the state equations. However, it is possible to remove the disturbances from the output in steady-state (i.e. assure that $\lim_{t\to\infty}\mathbf{y}=\mathbf{0}$) using the following composite control law:

$$(11)\qquad u = -\mathbf{K}\mathbf{x} - \mathbf{K}_d\hat{\mathbf{d}}_l$$

where $\hat{\mathbf{d}}_l = [\hat{\mathbf{d}}_{lx},\ \hat{\mathbf{d}}_{ly}]^T$ and $-\mathbf{K}\mathbf{x}$ is the standard state (e.g. LQR) feedback, by the following choice of the lumped disturbance estimate gain $\mathbf{K}_d = [\mathbf{K}_{dx}, \mathbf{K}_{dy}]$:

$$(12)\qquad \begin{aligned} \mathbf{K}_{dx} &= \left(\mathbf{H}\mathbf{B}_n - \mathbf{D}_n\right)^{-1}\mathbf{H} \\ \left(\mathbf{H}\mathbf{B}_n - \mathbf{D}_n\right)\mathbf{K}_{dy} &= -\mathbf{I} \end{aligned}$$

where $\mathbf{H}=(\mathbf{C}_n-\mathbf{D}_n\mathbf{K})(\mathbf{A}_n-\mathbf{B}_n\mathbf{K})^{-1}$.

The diagram of the control system with the main state feedback and the lumped disturbance compensation is shown in Figure 2. The internal structure of the proposed DO is presented in Figure 3.

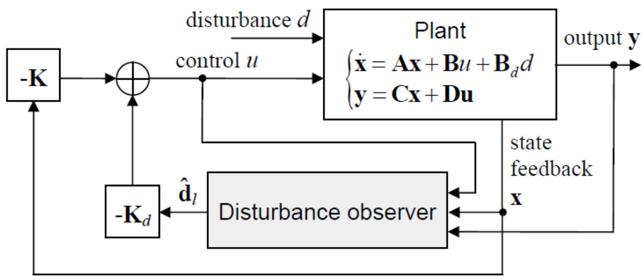

Fig.2. Diagram of the control system with the state feedback and compensation of the lumped disturbance

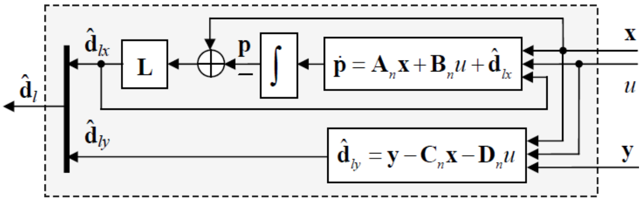

Fig.3. Internal structure of the disturbance observer for estimation of unmatched disturbances

Laboratory setup

The effectiveness of the DO based control presented in Figures 2 and 3 was verified experimentally using a laboratory setup with a mechatronic model of the quarter car active suspension shown in Figure 4.

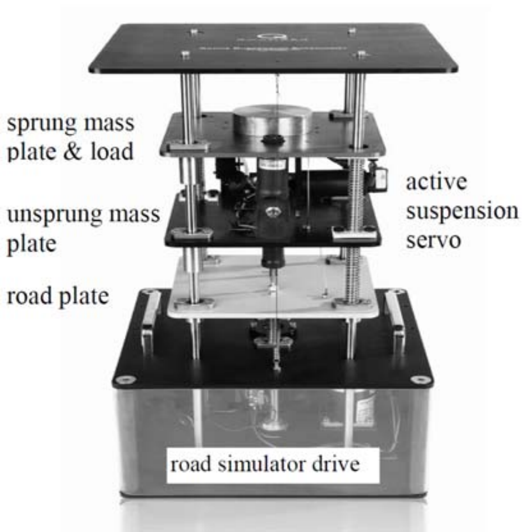

Fig.4. Mechatronic model of the quarter car active suspension

Metal plates, representing the sprung mass, the unsprung mass and the road, are interconnected with pairs of springs and linear bearings working as low friction dampers. Mechanical parameters of the active suspension system are given in Table 1.The active suspension actuator is a high efficiency, low inductance DC servomotor with fast dynamic response (the control force bandwidth is 50 Hz). It is supplied (as well as the road simulator DC servomotor) from a linear PWM current amplifier. The setup is equipped with three high resolution encoders (4000/4096 counts per full rotation) that allow direct measurements of the suspension deflection and displacements of the sprung mass and the road level. Vertical velocities can be calculated as differences between successive displacement readings. An accelerometer capable of measuring both static and dynamic vertical acceleration up to 10G is mounted on the sprung mass plate. Thus, all the defined state and output variables are available for control.

The DO based control algorithm was implemented as a Simulink block diagram on a PC computer with a multichannel data acquisition card and run in real time with sampling frequency f_s=1000 Hz.

Table 1. Parameters of the active suspension experimental setup

Parameter	Value	Units
m_s	2.45	kg
m_u	1	kg
k_s	900	N/m
b_s	7.5	Ns/m
k_t	2500	N/m
b_t	5	Ns/m
rattle space	0.038	m

Results of experiments

The presented laboratory experiments show the effects of the DO based control in the presence of both external (road roughness) and internal disturbance. The internal disturbance was an uncertain value of the sprung mass: the nominal mass used to design the composite control law (11) was m_{sn}=2.45 kg, while the true mass was m_s=2.95 kg.

The state feedback gain was designed using the LQR method. Minimization of the cost functional:

$$J_{LQR} = \int_0^\infty \left(\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{R}u^2\right)dt \tag{13}$$

with weighting matrices:

$$\mathbf{Q} = \mathrm{diag}\begin{bmatrix}500 & 50 & 5 & 0.1\end{bmatrix}, \quad \mathbf{R} = [0.1] \tag{14}$$

yielded the optimal feedback gain matrix:

$$\mathbf{K} = \begin{bmatrix}16.51 & 64.99 & \text{-}65.58 & 3.440\end{bmatrix} \tag{15}$$

The choice of the DO gain matrix:

$$\mathbf{L} = \mathrm{diag}\begin{bmatrix}50 & 50 & 50 & 50\end{bmatrix} \tag{16}$$

assures that the DO dynamics is much faster than that of the LQR closed loop and than variations of the external disturbance. Calculation of the DO compensation gains using formulas (13) for the nominal plant parameters (Table 1) and the feedback gain (15) gives:

$$\mathbf{K}_{dx} = \begin{bmatrix}72.49 & 2.514 & 68.43 & 0.0262\end{bmatrix}, \quad \mathbf{K}_{dy} = \begin{bmatrix}916.5 & 0\end{bmatrix} \tag{17}$$

Figure 5 shows the sprung mass displacement z_s (vehicle body) response to the road level z_r (external disturbance) in the form of a steep bump with flat top (z_r tends to a constant). The bump step-like profile is defined by maximum velocity $\dot{z}_{rmax}$=0.2 m/s and acceleration $\ddot{z}_{rmax}$=10 m/s^2. In this case the active suspension parameters are exactly known. The graphs show good effectiveness of the LQR control in attenuating weakly damped oscillations of the open-loop suspension system. Adding the DO improves the control quality even more.

Figure 6 presents the vertical displacements z_u, z_s in response to the road bump under uncertain sprung mass (the LQR controller and the DO are designed for the nominal mass m_{sn}). Note that – in contrast to the LQR control only – the DO based control removes the disturbance effect at the system output: the suspension deflection y_1=z_s-z_u is reduced to zero in the steady state. However, the steady state displacements z_u, z_s are still not equal to z_r (it would require an integral control). Another advantage is that adding the DO causes a slight reduction of the top sprung mass acceleration.

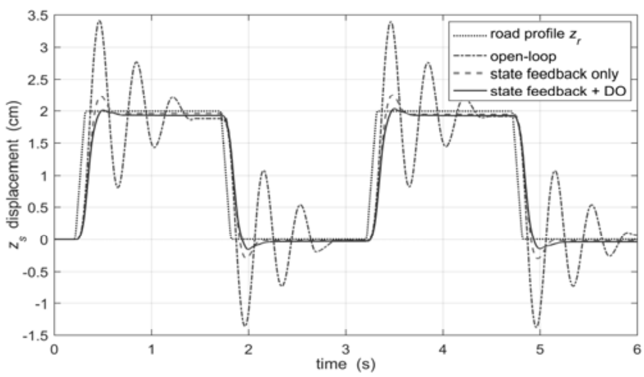

Fig. 5. Sprung mass level z_s response to road level z_r bump for: a) open-loop suspension system (no active damping), b) closed-loop with LQR controller, c) closed-loop with LQR controller and DO

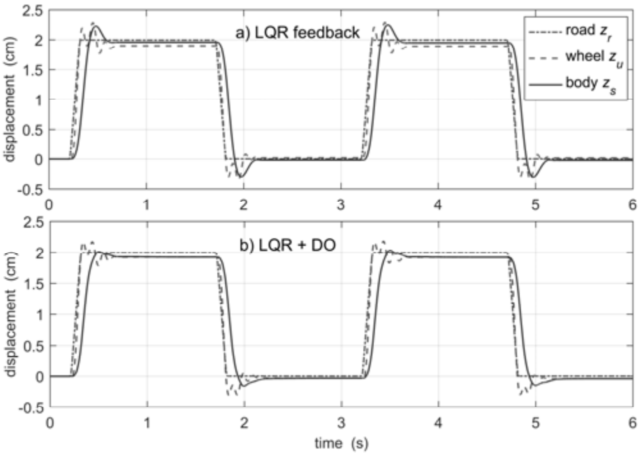

Fig. 6. Quarter car wheel (unsprung mass) and body (sprung mass) displacements z_u, z_s in response to the road bump under uncertain sprung mass m_s: a) LQR control, b) LQR control and DO.

Comparison of theoretical frequency responses from the road level disturbance z_r to the suspension deflection z_s-z_u in Figure 7 confirms that the closed-loop with the DO is a differentiator for low frequency disturbances, in particular it reduces the steady-state suspension deflection to zero.

Figure 8 presents the waveforms of the voltage control signal, equivalent to actuator force F_c and corresponding to the active suspension behavior shown in Figures 6 and 7. The pure LQR control maintains a constant control in the steady state for a nonzero road level disturbance (it results in a nonzero suspension deflection). The advantage of the DO is that it produces at the same time a control component of the opposite sign, which reduces the total control effort almost to zero. The operation of the DO is illustrated in Figure 9, which presents two nonzero components of lumped disturbance estimate $\hat{\mathbf{d}}_{lx}$ versus time (in the considered conditions component $\hat{\mathbf{d}}_{ly}$=0). The internal disturbance (resulting from not exact sprung mass m_s) is estimated as d_{l2} (because m_s occurs only in the second row of matrix $\mathbf{A}$, see equations (4)), the external disturbance (road level) is estimated as d_{l4} (because the only nonzero coefficient of $\mathbf{B}_d$ is in the 4th row).

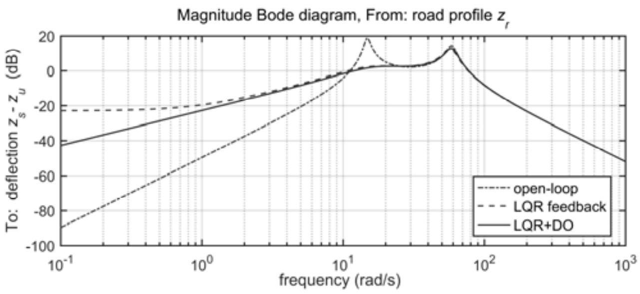

Fig. 7. Theoretical magnitude frequency responses from road level disturbance z_r to suspension deflection z_s-z_u

Concluding remarks

- The DO can be added as an extension of the main control loop. It estimates and counteracts both external and internal disturbances so the control enhances performance of the system and is more robust. However, effectiveness of the DO is limited to low frequency disturbances. The experimental results showed that the DO based control cancels the steady state deflection of the active suspension occurring for the pure LQR control and consequently reduces the control effort under a constant disturbance. It can also improve dynamic properties of the control system.
- Further research should go towards more general nonlinear model of the active suspension and a nonlinear DO, integration of the DO with (extended) state observer and applying more advanced control algorithms.

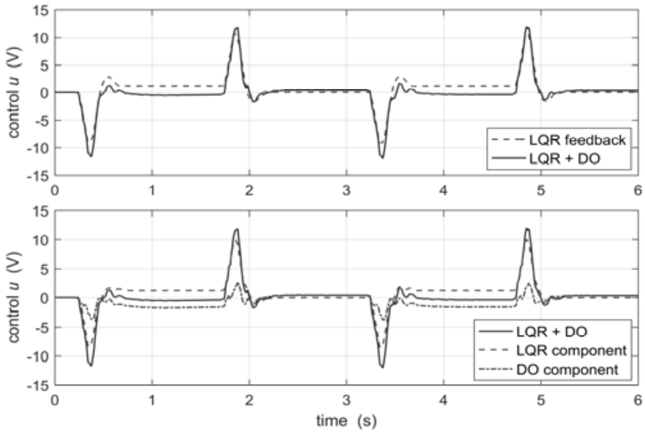

Fig. 8. Waveforms of the voltage control signal, equivalent to the suspension actuator force F_c in Newtons, corresponding to the active suspension behavior shown in Fig. 7 and 8

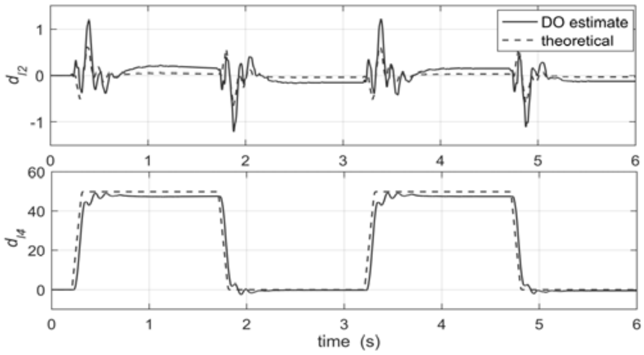

Fig. 9. Waveforms of nonzero components of the DO lumped disturbance estimate $\hat{\mathbf{d}}_{lx}$

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