Laaredj GHAOUTI[1], Mohamed BOURAHLA[1], Nadir BOUCHETATA[1]

University of sciences and the technology of Oran -Mohamed Boudiaf - (USTO-MB) BP 1505 EL M' naouer Oran, Algeria

doi:10.15199/48.2016.12.64

Sensorless field oriented control of PMSM based on the extended KALMAN filter observer

***Abstract**. in this paper, a study of sensorless field oriented control applied to surface-mount permanent-magnet synchronous motor is presented, the motor has been described in a stationary two-axes reference frame (α,β), to overcome the uncertainties the internal, measurement noises, the non zero initial rotor position and the dynamic nonlinearity of PMSM, the extended kalman filter (EKF) is used as a robust observer. The speed and rotor position estimation of PMSM drive are obtained through an EKF algorithm, by simply measurement of the stator line voltages and currents. Good state estimation with EKF observer is proved by the simulation results.*

Streszczenie.** Artykuł poświęcony jest bezczujnikowemu sterowaniu silnikiem synchronicznym z magnesami trwałymi. Jako odporny obserwator jest użyty filtr Kalmana. Prędkość I pozycja wirnika są określane na podstawie algorytmu filtru prze pomiar napięcia I prądu stojanas.* ***Bezczujnikowe sterowsanie silnikiem synchronicznym bazujące na filtrze Kalmana

Keywords: Permanent magnet synchronous motor (PMSM), Field Oriented Control (FOC), Sensorless Speed Control, observer, observability, state estimator, Lie derivative, Extended Kalman Filter (EKF).
Słowa kluczowe: silnik synchroniczny, sterowanie bezczujnikowe, filtr Kalmana

Introduction

Permanent Magnet Synchronous Machines (PMSM) are successfully used in different domains for their numerous advantages over other kinds of traditional motors such as DC motors or induction motors. These advantages include a simple structure, small size, high speed, high power density, high efficiency and large torque to inertia ratio, etc [1][4]. Despite of the advantageous features, high-performance PMSM drive characterized by the fast and precise speed response is still a difficult task, since the control performance is greatly affected by the uncertain motor parameters, load variation and nonlinear dynamical behaviour with strong coupling between the rotor speed and the stator currents [8][10]. Vector control technique is the most popular control strategy for the high performance control of PMSM in the synchronous frame, and three proportional–integral (PI) algorithms are conventionally adopted as speed and current controllers, one for the outer speed control loop and the other two for the inner current loops[2][11].

In vector control of a PMSM the rotor position must be known instantaneously. This can be achieved by using a position sensor. The cost of mechanical sensors, the difficulty to place them, and the lack of reliability of the motor encourage researchers to avoid their use [3][6]. There are different solutions to evaluate the mechanical variables of the motor, three different categories can be distinguished [3][9]:

- Techniques based on the machine's physical properties,
- Back-EMF estimation based techniques,
- State observers and extended Kalman filter (EKF).

The EKF algorithm is a suboptimal recursive estimation algorithm for nonlinear systems. It processes all available measurements, to provide a quick and accurate estimate of the state variables, and also achieves a rapid convergence. This is done using the following factors:

- A knowledge of the system and measurement device dynamics.
- The statistical description of the system noises, disturbances, measurement errors, and uncertainties in the system model.
- Any available information about the initial conditions of the state variables.
- However the EKF algorithm is computationally intensive and all of the steps involved require vector and matrix operations.

PMSM model for vector control

In the *dq* axis coordinates, rotating synchronously with the rotor, the voltage, flux linkage, torque and mechanical equations of the PMSM are [5][7]:

$$\text{(1)}\quad \begin{cases} v_d = R_s \cdot i_d + L_d \cdot \dfrac{d\, i_d}{dt} - L_q \cdot \omega \cdot i_q \\ v_q = R_s \cdot i_q + L_q \cdot \dfrac{d\, i_q}{dt} + L_d \cdot \omega \cdot i_d + \psi_r \cdot \omega \end{cases}$$

$$\text{(2)}\quad \begin{cases} \psi_d = L_d \cdot i_d + \psi_r \\ \psi_q = L_q \cdot i_q \end{cases}$$

$$\text{(3)}\quad T = \frac{3}{2} \cdot p \cdot \left(\psi_r \cdot i_q + \left(L_d - L_q\right) \cdot i_d \cdot i_q\right)$$

$$\text{(4)}\quad J\frac{d}{dt}\Omega + f\Omega = \frac{3}{2} \cdot p \cdot \left(\psi_r \cdot i_q + \left(L_d - L_q\right)\, i_d \cdot i_q\right) - T_r$$

where, i_d, i_q, d- and q-axis components of armature current, v_d, v_q, d- and q-axis components of terminal voltage, ψ_d, ψ_q, d- and q-axis components of flux linkage, L_d, L_q, d- and q-axis components of armature self inductance, ψ_r flux linkage due to permanent magnet, R_s armature resistance, ω and Ω are the rotor electrical angle speed and the rotor mechanical speed respectively, p number of pole pairs, *J* rotor inertia, *f* viscous damping coefficient, *T* output torque and T_r resistance torque.

Vector control

In FOC, the measured currents are firstly transformed into the d and q axes. The field orientation consists in setting $i_d = 0$. *Fig. 1*. Then the torque of the PMSM is produced only by the constant flux (ψ_r) the permanent magnet and the torque generating current i_q, as well as of the PMSM. A completely linear system results, and linear control theory applies [6].

Extended Kalman Filter observer designing

The EKF is a suboptimal estimator of dynamic nonlinear systems states. To apply this algorithm, the nonlinear state motor nonlinear state equations are written in the following form [7][12]:

$$\text{(5)}\quad \begin{cases} \dfrac{dx}{dt} = f\left(x(t), u(t), t\right) + w(t) \\ y(t_k) = h\left(x(t_k)\right) + v(t_k) \end{cases}$$

where *x(t)* and *y(t)* are, the state and the measurement vectors, respectively. *w(t)* and $v(t_k)$ are zero-mean white Gaussian noises with covariance *Q(t)* and *R(t)*, respectively, and independent from the system state *x(t)* and $t_k=kT_e$. The noise *w(t)* takes into account the system disturbances and model inaccuracies, while $v(t_k)$ represents the measurement noise. The initial state vector $x(t_0)$ is considered as a Gaussian random vector with mean value x_0 and covariance matrix P_0, and *u(t)* is the deterministic input vector.

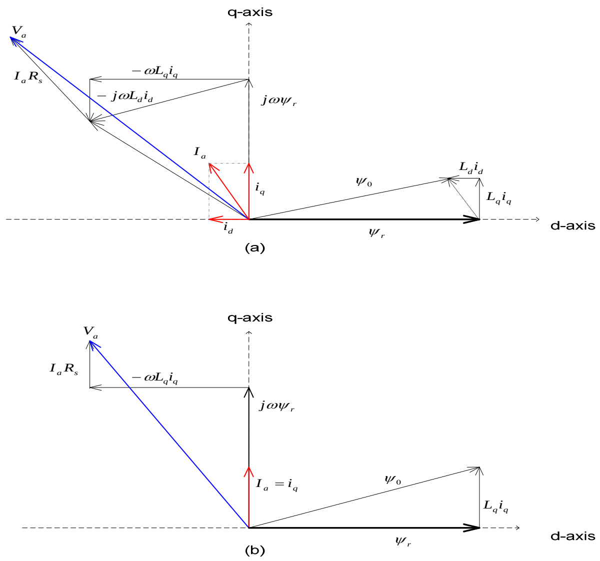


Fig. 1: Basic vector diagram of a PMSM; (a) general (b) modified with i_d=0

In order to get the system equations in the most suitable form, the motor has been described in a stationary two-axes reference frame (*α,β*). This choice yields to a model in which the derivatives of the motor currents are linearly related to both currents and applied voltages. By assuming:

$$x(t)=\begin{bmatrix} i_\alpha & i_\beta & \Omega & \theta_e \end{bmatrix}^T,\quad u(t)=\begin{bmatrix} v_\alpha & v_\beta \end{bmatrix}^T \text{ and } y(t)=\begin{bmatrix} i_\alpha & i_\beta \end{bmatrix}^T$$

The system matrices *f(x(t),u(t))* and *h(x(t))* result [4]

$$\text{(6)}\quad \begin{cases} f(x(t),u(t))=\begin{pmatrix} \dfrac{-R_s}{L_s}\cdot i_\alpha-\dfrac{p\cdot\psi_r}{L_s}\cdot\sin(\theta_e)\cdot\Omega+\dfrac{1}{L_s}v_\alpha \\ \dfrac{-R_s}{L_s}\cdot i_\beta+\dfrac{p\cdot\psi_r}{L_s}\cdot\cos(\theta_e)\cdot\Omega+\dfrac{1}{L_s}v_\beta \\ \lambda\cdot\sin(\theta_e)\cdot i_\alpha-\lambda\cdot\cos(\theta_e)\cdot i_\beta-\dfrac{f}{J}\cdot\Omega-\dfrac{1}{J}\cdot T_l \\ p\cdot\Omega \end{pmatrix} \\ h(x(t))==\begin{bmatrix} i_\alpha & i_\beta \end{bmatrix}^T \end{cases}$$

With: $\quad \lambda=\dfrac{-3\cdot p\cdot\psi_r}{2\cdot J}\quad$ and $\quad L_s=L_d=L_q$

For this system, which is clearly nonlinear, an EKF is derived. First of all, let us define the Jacobian matrices.

$$\text{(7)}\quad F(x(t),u(t))=\left(\frac{\partial f}{\partial x}\right)_{x=\hat{x}(t)}=\begin{bmatrix} F_{1.1} & 0 & F_{1.3} & F_{1.4} \\ 0 & F_{2.2} & F_{2.3} & F_{2.4} \\ F_{3.1} & F_{3.2} & F_{3.3} & F_{3.4} \\ 0 & 0 & F_{4.3} & 0 \end{bmatrix}$$

With:

$$F_{1.1}=\frac{-R_s}{L_s}\ ;\quad F_{1.3}=\frac{p\cdot\psi_r}{L_s}\cdot\sin(\theta_e(t))\ ;\quad F_{1.4}=\frac{p\cdot\psi_r}{L_s}.\Omega(t).\cos(\theta_e(t))\ ;$$

$$F_{2.2}=\frac{-R_s}{L_s}\ ;\quad F_{2.3}=-\frac{p\cdot\psi_r}{L_s}\cdot\cos(\theta_e(t))\ ;\quad F_{2.4}=\frac{p\cdot\psi_r}{L_s}.\Omega(t).\sin(\theta_e(t))$$

$$F_{3.1}=\frac{-3\cdot p\cdot\psi_r}{2\cdot J}\cdot\sin(\theta_e(t))\ ;F_{3.2}=\frac{3\cdot p\cdot\psi_r}{2\cdot p\cdot J}\cdot\cos(\theta_e(t))\ ;\quad F_{3.3}=-\frac{f}{J}\ ;$$

$$F_{3.4}=\frac{-3\cdot p\cdot\psi_r}{2\cdot J}\cdot\left(i_\alpha(t).\cos(\theta_e(t))+\cdot i_\beta(t).\sin(\theta_e(t))\right)\ ;\quad F_{4.3}=p\ ;$$

$$\text{(8)}\quad H(t)=\left(\frac{\partial h}{\partial x}\right)_{x=\hat{x}(t)}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

The state estimate $\hat{x}(t)$ generated by the EKF is a minimum variance estimate of $x(t)$, and is computed in a recursive manner. The filter has a predictor-corrector structure as follows:

Step 1: Prediction (from $t_{k/k}$ to $t_{k+1/k}$)

The state estimate $\hat{x}(t)$ and the corresponding covariance matrix $P_{k+1/k}$ are propagated from time (t_k) to time (t_{k+1}), based on the previous values, the system dynamics, and the previous control inputs and errors of the actual system. This is done by numerical integration of the following equations:

$$\text{(9)}\quad \hat{x}_{k+1/k}=\hat{x}_{k/k}+\int_{kTe}^{(k+1)Te} f\left(x(t/t_k),u(t),t\right)dt$$

$$\text{(10)}\quad P_{k+1/k}=\Phi(t_{k+1},t_k)\cdot P_{k/k}\cdot\Phi(t_{k+1},t_k)^T+Q_k$$

Starting from the initial conditions, where

$$\text{(11)}\quad \Phi(t_{k+1},t_k)=e^{\left(\left(\frac{\partial f}{\partial x}\right)_{x=\hat{x}(t)}\right)\cdot(t_{k+1}-t_k)}\quad \text{and}\quad t_{k+1}=(k+1)T_e$$

Step 2: Filtering (from *$t_{k+1/k}$ to $t_{k+1/k+1}$*)

By comparing the measurement vector, y_k, to the predicted one, $\hat{y}_{k+1/k}=H\cdot\hat{x}_{k+1/k}$, a correction factor is obtained and used to update the state vector. The filter gain matrix is defined as.

$$\text{(12)}\quad K_{k+1}=P_{k+1/k}\cdot H^T\cdot\left[H\cdot P_{k+1/k}\cdot H^T+R_{k+1}\right]^{-1}$$

The estimated state vector and the covariance matrix are

$$\text{(13)}\quad \begin{aligned} \hat{x}_{k+1/k+1}&=\hat{x}_{k+1/k}+K_{k+1}\cdot\left(Y_k-H.\hat{x}_{k+1/k}\right) \\ P_{k+1/k+1}&=\left(I-K_{k+1}\cdot H\right)\cdot P_{k+1/k} \end{aligned}$$

Where $\hat{x}_{k+1/k+1}$ represent the estimate state vector.

Matrices $P_{0/0}$, Q and R were selected as follow:

$$P_{0/0}=diag\begin{bmatrix} 0.1 & 0.1 & 50 & 5 \end{bmatrix},\ Q=diag\begin{bmatrix} 1 & 1 & 10 & 0.1 \end{bmatrix}$$
$$R=diag\begin{bmatrix} 0.1 & 0.1 \end{bmatrix}$$

Observability study of PMSM model

To estimate the state of a process must be observable. In the case where the process is nonlinear writing the following state equation.

$$\text{(14)}\quad \begin{cases} \dot{x}(t)=f\left(x(t),u(t)\right) \\ y(t)=h\left(x(t)\right) \end{cases}$$

Where $x\in\Re^n, u\in\Re^m and\ \ y\in\Re^p$ are the states, input and output vectors, respectively.

$f:\Re^n\times\Re^m\to\Re^n$ and $h:\Re^n\to\Re^p$ are smooth maps.

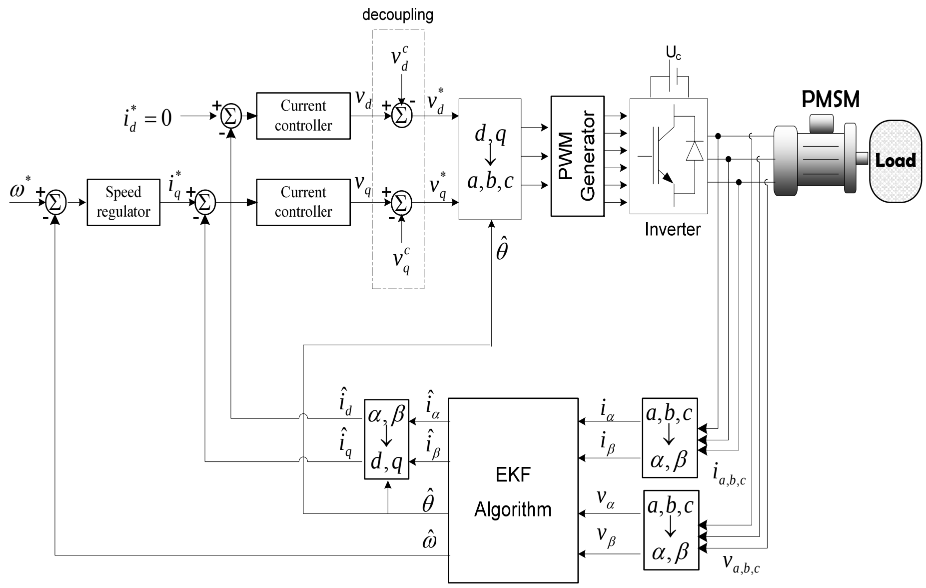


Fig.2 The block diagram of the proposed sensorless vector control for PMSM.

A system under the form of (14) is said to be locally observable at a point x_0 if all state x can be instantaneously distinguished by a judicious choice of input u on a neighborhood U of x_0. The system is locally observable if there exist integers $k_1 \geq k_2 \geq \cdots k_p \; with \sum_{i=1}^{p} k_i = n$ and a neighborhood U of x_0 such that:

$$\dim\left\{ L_f^j \,/\, i = 1,...,p;\; j = 0,...,k_i \right\}(x) = n$$

For all $x \in U$ Recall that the Lie derivative of h along the vector field f is defined as $L_f = \frac{\partial h}{\partial x} f$

Either the space '*O*' called observation space defined by the smallest vector space containing $h_1,..., h_p$ and all their successive Lie derivatives:

(15) $$O = span\left\{ L_f^j h(x); i = 1,\cdots,p;\; j = 0,1,2\cdots \right\}$$

With: $L_f^j h\left(L_f^{j-1} h\right),\; j \geq 2$ and $L_f^0 h = h$

Where: $L_f h(x) = \sum_{i=1}^{n} f_i(x) \cdot \frac{\partial h(x)}{\partial (x_i)}$

For the model of the PMSM described by equation (6) we obtain:

$$L_f^0 \frac{\partial h_1(x)}{\partial x} = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \quad , \quad L_f^0 \frac{\partial h_2(x)}{\partial x} = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}$$

$$L_f^1 \frac{\partial h_1(x)}{\partial x} = \frac{\partial h_1(x)}{\partial x} \cdot \frac{\partial f(x)}{\partial x} + f^T \frac{\partial}{\partial x}\left(\frac{\partial h_1(x)}{\partial x}\right)^T =$$

$$\begin{pmatrix} -\frac{R_s}{L_s} & 0 & \frac{p \cdot \psi_r}{L_s} \cdot \sin(\theta_e) & \frac{p \cdot \psi_r}{L_s} \cdot \Omega \cdot \cos(\theta_e) \end{pmatrix}$$

$$L_f^1 \frac{\partial h_1(x)}{\partial x} = \frac{\partial h_1(x)}{\partial x} \cdot \frac{\partial f(x)}{\partial x} + f^T \frac{\partial}{\partial x}\left(\frac{\partial h_1(x)}{\partial x}\right)^T =$$

$$\begin{pmatrix} -\frac{R_s}{L_s} & 0 & \frac{p \cdot \psi_r}{L_s} \cdot \sin(\theta_e) & \frac{p \cdot \psi_r}{L_s} \cdot \Omega \cdot \cos(\theta_e) \end{pmatrix}$$

The observability matrix which is generated by $L_f^0 dh_1, L_f^0 dh_2, L_f^1 dh_1 \; and \; L_f^1 dh_2$ given by:

(16) $$OB = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{R_s}{L_s} & 0 & \frac{p \cdot \psi_r}{L_s} \cdot \sin(\theta_e) & \frac{p \cdot \psi_r}{L_s} \cdot \Omega \cdot \cos(\theta_e) \\ 0 & -\frac{R_s}{L_s} & -\frac{p \cdot \psi_r}{L_s} \cdot \cos(\theta_e) & \frac{p \cdot \psi_r}{L_s} \cdot \Omega \cdot \sin(\theta_e) \end{pmatrix}$$

With:

$$\Delta(OB) = \frac{p^2 \cdot \psi_r^2 \cdot \Omega \cdot \left(\sin^2(\theta_e) + \cos^2(\theta_e)\right)}{L_s^2}$$

Is full rank if and only if $\Omega \neq 0$. Hence, one can conclude that system (6) is locally observable for all Ω values except at the point $\Omega = 0$. Indeed, if $\Omega = 0$ in the system (6), electrical variables are decoupled mechanical variables and we can't reconstruct the mechanical variables from only the electrical measurements.

Generally, the unobservable points (or sets) are the singular points (or sets) that should be avoided in the observer design. This is the reason why a thorough analysis of the observability of the PMSM is important since it helps explain why the sensorless operation performs poorly in low-speed environment.

Table 1. Parameters of the PMSM

Symbol	Parameters	Value
L_s [mH]	Stator inductance	2,2 mH
R_s [Ω]	Stator resistance	0,8
J [kg.m^2]	Rotor inertia	0,74*10^{-3}
f [N.m.sec/rad]	Frictional constant	2,6*10^{-3}
ψ_r [Wb]	Rotor magnetic flux	0.133
p	Number of pole pairs	4

Simulation Results

The simulation results using the MATLAB-SIMULINK software package and the motor parameters listed in Table1. A zero-mean white Gaussian noises is injected into the voltages and currents measured (fig.3) with a load torque of 7.4 N.m is applied at time t = 0.1s.

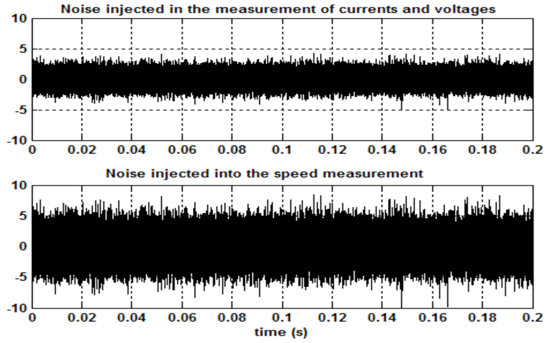
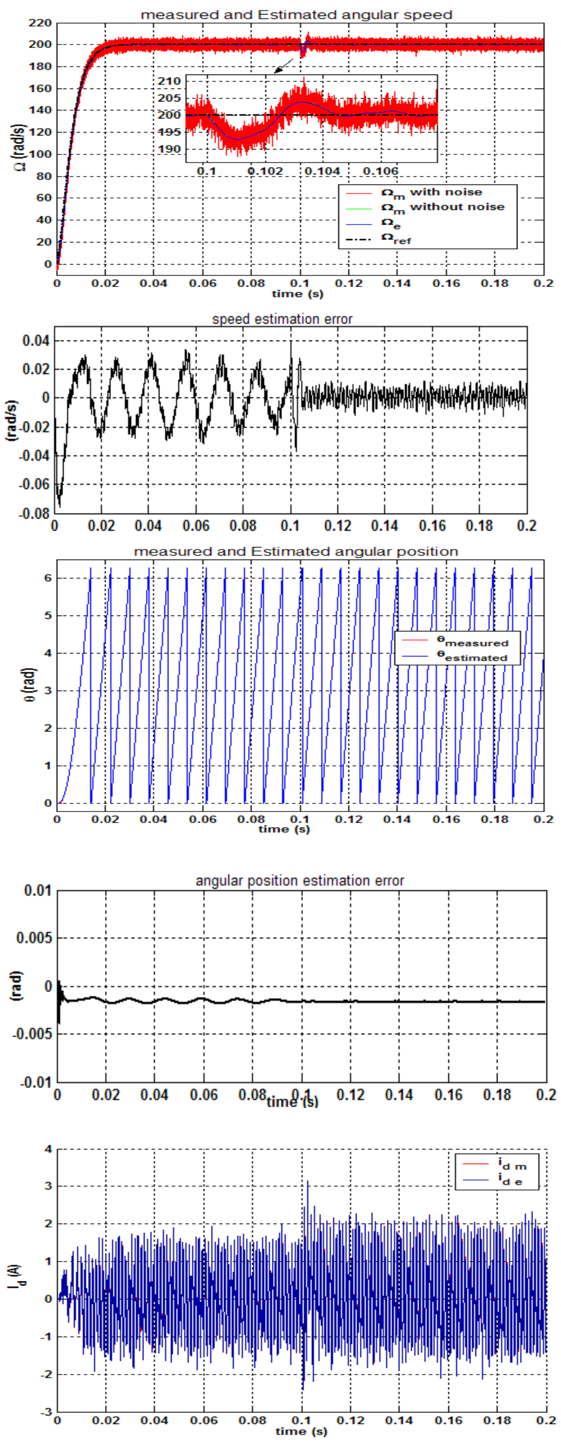


Fig.3: the zero-mean white Gaussian noises

Test 1: control with noisy measurements and estimated speed



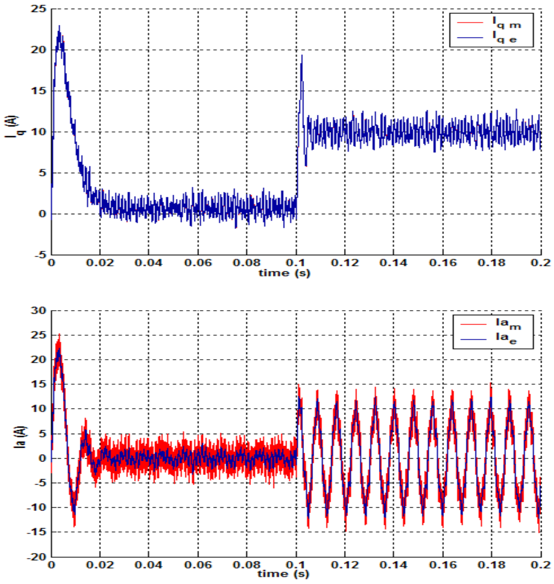
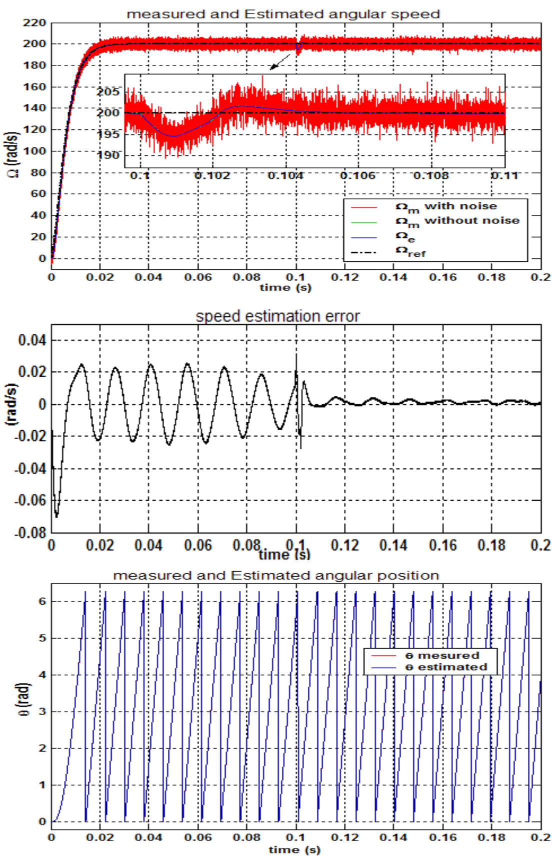


Fig.4 : control with noisy measurements

In fig. 4 where the control is effected by noisy measurements, it is observed that achieved the objective of the field-oriented control (motor speed Ω_m well follow the speed command Ω_{ref} and the direct current id is hovering around zero, the EKF gives good estimates of the speed and position with rejection of noises). Fig.5 shows that the control with estimated variables improves responses (speed, position and current) and also the estimates.



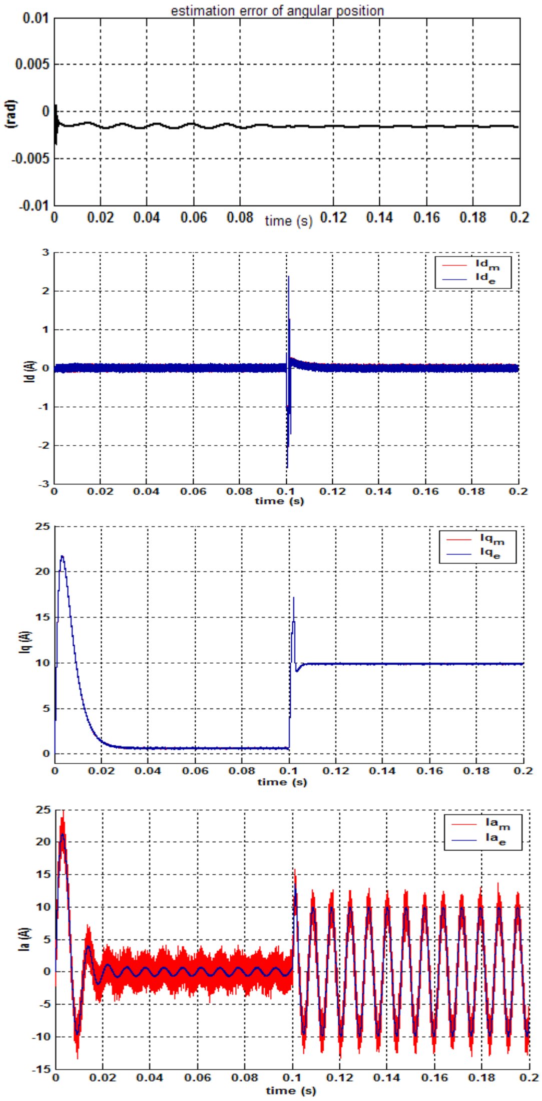


Fig.5 : control with estimated speed

Test 2: Robustness testing (variation of R_s and j)

For robustness testing, the stator resistance is increased by 20% in fig.6 and in fig.7 the rotor inertia is increased by 10%, it is noted that the estimation errors (speed and position) are not important and rapidly converges to zero.

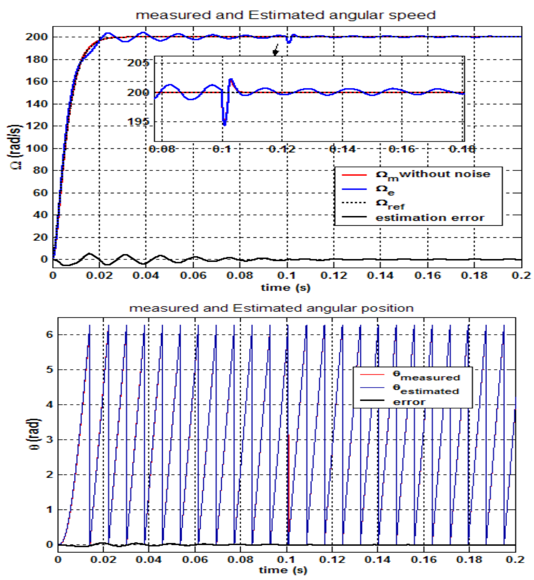


Fig.6 variation of the resistance R_s (+ 20%)

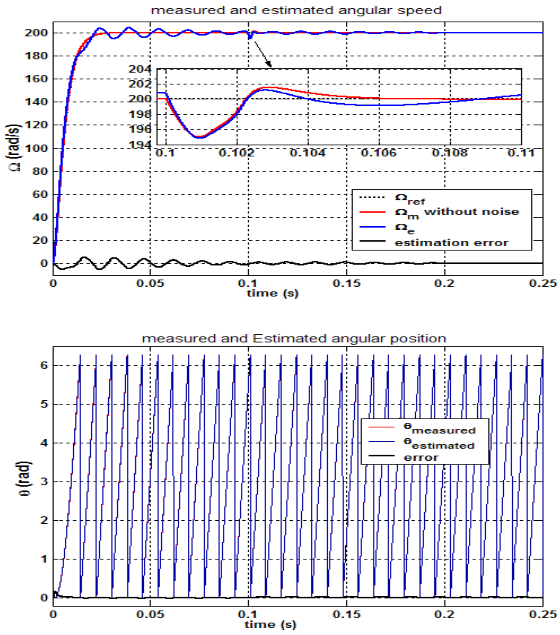


Fig.7 variation of the moment of inertia j (+ 10%)

Test 3: Measured and estimated speed and position

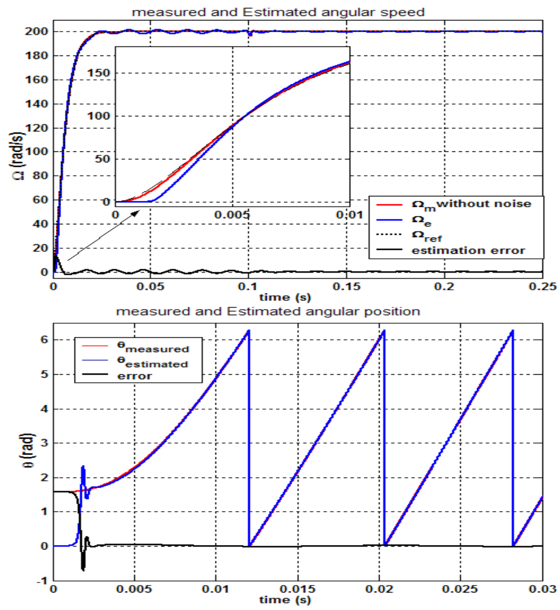


Fig.8 nonzero initial angular position (θ_0 =π/2 rad)

Fig.8 shows measured and estimated state variables (speed and position) with a non zero initial angular position (θ_0 = π/2 rad), at t = 0s the measured position is 1.57rad and the estimated is zero but after a short time the estimation error tends to zero.

Conclusion

In this paper a sensorless field oriented speed control of PMSM using an extended Kalman filter observer is presented, has been tested with simulation under various conditions. The nonlinear observability of the PMSM model in the stationary two-axes reference frame (α,β) is verified before applying the EKF for estimating the the state vector. Simulation results show good estimate of the state vector by the EKF observer also its robustness against noise, non zero initial position and process uncertainties. The major disadvantage of this method is its execution time. Indeed, the sensorless control based on the EKF observer requires computing time relatively high compared to other methods.

Authors: *dr L.GHAOUTI, University of Sciences and the Technology of Oran -Mohamed Boudiaf - BP 1505 Al Mnaouar, 31000 Oran, ALGERIA, E-mail: glaaredj@yahoo.fr; laaredj.ghaouti@univ-usto.dz dr M.BOURAHLA, University of Sciences and the Technology of Oran -Mohamed Boudiaf - BP 1505 Al Mnaouar, 31000 Oran, ALGERIA, E-mail: Bourah3@yahoo.fr; dr N.BOUCHETATA, University Sciences of and the Technology of Oran-Mohamed Boudiaf - BP 1505 Al Mnaouar, 31000 Oran, ALGERIA, E-mail: Nadil2004@yahoo.fr;*

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