Giennadij CZAWKA, Marek GARBARUK

Bialystok University of Technology, POLAND

doi:10.15199/48.2016.02.04

Analysis of power parameters of two-port antenna using a new scattering matrix

***Abstract**: The paper presents results of a computer simulation of the power parameters' characteristics of a two-port antenna with the use of a new scattering matrix normalized to a complex impedance matrix of coupled generators. It is shown that there is an **infinite number** of the scattering matrices for the given two-port complex load (or the network of generators) and all of these describe **the same** power distribution in the given networks.*

***Streszczenie**: Artykuł przedstawia wyniki symulacji komputerowej charakterystyk parametrów energetycznych dwuwrotnikowej anteny z zastosowaniem nowej macierzy rozproszenia normalizowanej do pełnej macierzy impedancji generatorów. Pokazano, że istnieje **nieskończona** ilość takich macierzy rozproszenia dla danego dwuwrotnikowego obciążenia (lub macierzy impedancji generatorów), określających **ten sam** rozkład mocy w danym układzie. (**Analiza parametrów energetycznych dwuwrotnikowej anteny z zastosowaniem nowej macierzy rozproszenia**).*

Keywords: two-port network, new scattering matrix, power parameters.
Słowa kluczowe: układ dwuwrotnikowy, nowa macierz rozproszenia, parametry energetyczne.

Introduction

A common problems in antenna designing is an issue of a broadband matching of antenna arrays with a set of generators. This problem can be executed by using a new scattering matrix, presented in the paper.

The new scattering matrix is constructed for the given non-diagonal output impedance matrix of the antenna feeding network $\mathbf{Z}_o$ and a corresponding equivalent structure (Fig.1). The bases of the theory concerning the scattering matrix normalized to *n*-port complex load network is described in [1]. Using this matrix for a multiport broadband matching problem for the arbitrary antenna array is presented in [3-5].

The paper presents results of a computer simulation of the power parameter's characteristics of a two-port antenna with the use of a new scattering matrix normalized to a complex impedance matrix of coupled generators. This matrix is based on eigenvalues and eigenvectors of the multiport matrices. It is shown that there is an infinite number of the scattering matrices for the given two-port complex load (or the network of generators) and all of these describe the same power distribution in given networks.

Power parameters of a two-port network

As shown in [1, 3-5], incident $\mathbf{a}$ and reflected $\mathbf{b}$ waves and a complex normalized scattering matrix $\mathbf{S}$ of the multiport network normalized to the impedance matrix $\mathbf{Z}_o$ of *n*-port complex source network N_o are given by:

$$\text{(1)} \qquad \begin{cases} 2\,\mathbf{H}\,\mathbf{a} = \mathbf{U} + \mathbf{Z}_o\mathbf{I}, \\ 2\,\mathbf{H}^+\mathbf{b} = \mathbf{U} - \mathbf{Z}_o^+\mathbf{I}, \end{cases} \quad \mathbf{b} = \mathbf{S}\,\mathbf{a}\,.$$

Therefore, total average power absorbed by two-port N is given by [2]:

$$\text{(2)} \qquad P_N = \mathrm{Re}(\mathbf{U}^+\mathbf{I}) = \mathbf{a}^+\mathbf{a} - \mathbf{b}^+\mathbf{b} = P_{\max} - P_{ref}\,,$$

where [5]:

$$\text{(3)} \qquad P_{\max} = \mathbf{a}^+\mathbf{a} = 0.25\,\mathbf{E}^+\mathbf{R}_o^{-1}\mathbf{E}\,,$$

$$\text{(4)} \qquad P_{ref} = \mathbf{b}^+\mathbf{b} = 0.25\,\mathbf{I}^+(\mathbf{Z}^+ - \mathbf{Z}_o)\,R_o^{-1}(\mathbf{Z} - Z_o^+)\,\mathbf{I}\,.$$

Let's consider an example of a two-port antenna and its corresponding excitation network (Fig.1).

The structure of a two-port generator impedance Z_o is *symmetrical*, but a two-port load is an *arbitrary* network (it may be non-symmetrical) (Fig.1).

Then impedance matrices of the whole two-port structure are:

$$\text{(5)} \qquad \mathbf{Z}_o = \begin{bmatrix} z_{o\,11} & z_{o\,12} \\ z_{o\,12} & z_{o\,11} \end{bmatrix},\ \mathbf{Z} = \begin{bmatrix} z_{11} & z_{12} \\ z_{12} & z_{22} \end{bmatrix}.$$

Matrix $\mathbf{Z}_o$ of the symmetrical two-port and its real part $\mathbf{R}_o$ have the same orthogonal expansion:

$$\text{(6)} \qquad \mathbf{Z}_o = \mathbf{V}\,\{z_{o\,1,2}\}\,\mathbf{V}\,,$$

$$\text{(7)} \qquad \mathbf{R}_o = \mathrm{Re}\,\mathbf{Z}_o = \begin{bmatrix} r_{o\,11} & r_{o\,12} \\ r_{o\,12} & r_{o\,11} \end{bmatrix} = \mathbf{V}\,\{r_{o\,1,2}\}\,\mathbf{V}\,,$$

where $\mathbf{V}$ is an orthogonal real eigenvector matrix:

$$\text{(8)} \qquad \mathbf{V} = \begin{bmatrix} \mathbf{V}_1 & \mathbf{V}_2 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix},$$

$z_{o1,2}$ and $r_{o1,2}$ are eigenvalues connected with elements' values of corresponding matrices:

$$\text{(9)} \qquad z_{o\,1,2} = z_{o\,11} \pm z_{o\,12}\,, \quad r_{o\,1,2} = r_{o\,11} \pm r_{o\,12}\,.$$

Then a factor matrix $\mathbf{H}_o$ has the following structure:

$$\text{(10)} \quad \mathbf{H}_o = \mathbf{V}\{\sqrt{r_{o\,i}}\}\,\mathbf{W} = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{r_{o\,1}} & \sqrt{r_{o\,2}} \\ \sqrt{r_{o\,1}} & -\sqrt{r_{o\,2}} \end{bmatrix} \mathbf{W}\,,$$

where $\mathbf{W}$ is an arbitrary complex unitary matrix.

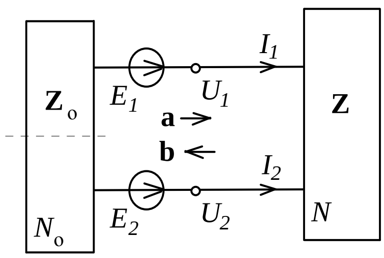

Fig.1. Two-port antenna and generators

Table 1. Complex normalization of scattering matrix for different matrices $\mathbf{W}$

$\mathbf{W}=\mathbf{1}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\mathbf{W}=\mathbf{V}=\frac{1}{\sqrt{2}}\begin{bmatrix}1&1\\1&-1\end{bmatrix}$
$\mathbf{H}_{\text{o}}=\frac{1}{\sqrt{2}}\begin{bmatrix}\sqrt{r_{\text{o}1}}&\sqrt{r_{\text{o}2}}\\ \sqrt{r_{\text{o}1}}&-\sqrt{r_{\text{o}2}}\end{bmatrix},\ a_{1,2}=\frac{E_1\pm E_2}{2\sqrt{2\,r_{\text{o}1,2}}}$ $b_{1,2}=\frac{1}{2\sqrt{2\,r_{\text{o}1,2}}}\left[(U_1\pm U_2)-z^*_{\text{o}1,2}(I_1\pm I_2)\right]$	$a_{1,2}=\frac{E_1+E_2}{4\sqrt{r_{\text{o}1}}}\pm\frac{E_1-E_2}{4\sqrt{r_{\text{o}2}}}\ ,\ \mathbf{H}_{\text{o}}=\frac{1}{2}\begin{bmatrix}\sqrt{r_{\text{o}1}}+\sqrt{r_{\text{o}2}}&\sqrt{r_{\text{o}1}}-\sqrt{r_{\text{o}2}}\\ \sqrt{r_{\text{o}1}}-\sqrt{r_{\text{o}2}}&\sqrt{r_{\text{o}1}}+\sqrt{r_{\text{o}2}}\end{bmatrix}$ $b_{1,2}=\frac{(U_1+U_2)-z^*_{\text{o}1,2}(I_1+I_2)}{4\sqrt{r_{\text{o}1}}}\pm\frac{(U_1+U_2)-z^*_{\text{o}1,2}(I_1+I_2)}{4\sqrt{r_{\text{o}1}}}$
$\mathbf{U}=\left\{1/\sqrt{r_{\text{o}i}}\right\}\mathbf{V}\left[\{z^*_{\text{o}i}\}\,\mathbf{a}+\{z_{\text{o}i}\}\,\mathbf{b}\right]$ $\mathbf{I}=\left\{1/\sqrt{r_{\text{o}i}}\right\}\mathbf{V}\,(\mathbf{a}-\mathbf{b})$	$\mathbf{U}=\left\{1/\sqrt{r_{\text{o}i}}\right\}(\mathbf{Z}^*_{\text{o}}\,\mathbf{a}+\mathbf{Z}_{\text{o}}\,\mathbf{b})$ $\mathbf{I}=\left\{1/\sqrt{r_{\text{o}i}}\right\}(\mathbf{a}+\mathbf{b})$

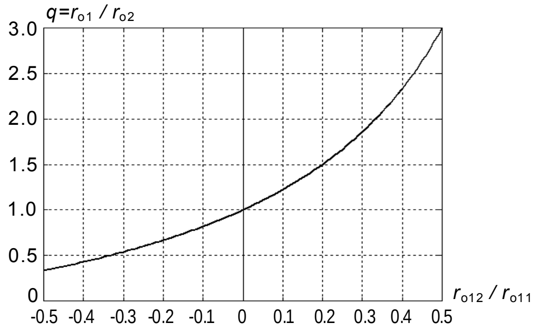

Fig.2. Ratio of eigenvalues of a real matrix $\mathbf{R}_{\text{o}}$ of two coupled generators

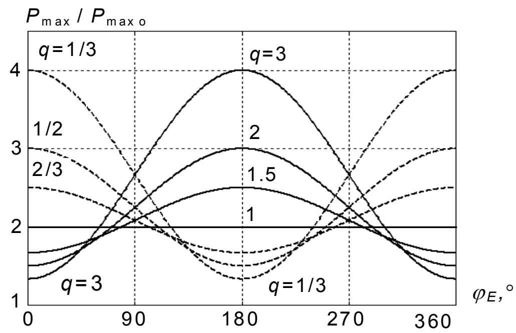

Fig.3. Normalized maximal power of two coupled generators

Then there is an **infinite number** of the matrices $\mathbf{H}_{\text{o}}$, incident wave vectors $\mathbf{a}$ and matrices $\mathbf{S}$ for the given two-port network $\mathbf{Z}_{\text{o}}$ where:

$$\mathbf{a}=0.5\,\mathbf{H}_{\text{o}}^{-1}\mathbf{E}\ ,\quad \mathbf{b}=\mathbf{S}\,\mathbf{a}\ . \tag{11}$$

According to (3,4) all these scattering matrices describe the same power distribution in the network because a hermitian form (3,4) is used for power calculation and matrices W and W^+ disappear.

As an example in Table 1 there are formulas for H_o, incident and reflected waves, voltage and current vectors for two special forms of matrix W: W=1 and W=V. We can see different formulas of all parameters for these selected matrices W. When we consider a unit matrix W=1, matrices H_o and S are non-symmetrical but when W=V these matrices are symmetrical.

Further, for **W=1** a condition a_2=0 (zeroing of a_2 wave) defines an in-phase excitation: E_1=E_2 and a_1 can be calculated as: $a_1=E_1/\sqrt{2r_{\text{o}1}}$. Oppositely, a condition a_1=0 results in an anti-phase excitation: E_1=–E_2 and $a_2=E_1/\sqrt{2r_{\text{o}2}}$.

When **W=V**, the condition a_2=0 results in the following expressions:

$$a_1=\frac{E_1}{\sqrt{r_{\text{o}1}}+\sqrt{r_{\text{o}2}}}\ ,\ E_2=E_1\frac{\sqrt{r_{\text{o}1}}-\sqrt{r_{\text{o}2}}}{\sqrt{r_{\text{o}1}}+\sqrt{r_{\text{o}2}}}\ . \tag{12}$$

Using formulas defining $a_{1,2}$ and $b_{1,2}$ (Table 1) we can calculate elements of scattering matrix **S**.

A normalized power absorbed by network *N* (Fig.1):

$$P/P_{\max}=\frac{|a_1|^2+|a_2|^2-|b_1|^2-|b_2|^2}{|a_1|^2+|a_2|^2}\ , \tag{13}$$

where $P_{\max}$ is maximal power of two generators:

$$P_{\max}=\frac{|E_1+E_2|^2}{8\,r_{\text{o}1}}+\frac{|E_1-E_2|^2}{8\,r_{\text{o}2}}\ . \tag{14}$$

In spite of changing of incident and reflected waves it may be proved that *power does not depend* on the matrix W because a hermitian form is used for power calculation and, as a result of further transformation of formulas, both matrices W and W^+ disappear.

It is easy to show that for diagonal matrix Z_o (z_{12}=0) and $|E_1|$=$|E_2|$ maximal power of generators equals:

$$P_{\max}=2\,P_{\max\text{o}}=\frac{|E_1|^2}{2\,r_{\text{o}11}}\ , \tag{15}$$

where: $P_{\max\text{ o}}$ – maximal power of one generator.

If matrix Z_o is non-diagonal and $|E_1|$=$|E_2|$ then maximal power of generators depends on coupling of generators (value of z_{12}):

$$P_{\max}=P_{\max\text{o}}\frac{q+1}{2q}\left[\left(1+\cos\varphi_E\right)+q\left(1-\cos\varphi_E\right)\right], \tag{16}$$

where q – a ratio of eigenvalues of a real matrix $\mathbf{R}_{\text{o}}$, φ_E - a phase difference of the generators.

The ratio of eigenvalues of a real matrix $\mathbf{R}_{\text{o}}$ can be expressed by:

$$q=\frac{r_{\text{o}1}}{r_{\text{o}2}}=\frac{1+r_{\text{o}12}/r_{\text{o}11}}{1-r_{\text{o}12}/r_{\text{o}11}}\ ,\quad E_2=E_1\,e^{j\varphi_E}\ . \tag{17}$$

Dependence of the eigenvalues ratio of a real matrix $\mathbf{R}_{\text{o}}$ of two coupled generators is shown in Fig.2. When generators are uncoupled ($r_{\text{o}12}$=0) both eigenvalues $r_{\text{o}1}$ and $r_{\text{o}2}$ have the same value (q=1).

Fig.3. shows a ratio $P_{max}/P_{max\,o}$ as a function of a phase difference of the generators φ_E for different values q (a different level of coupling between generators). If there is no coupling between generators: $P_{max}=2\cdot P_{max\,o}$. For a larger coupling a difference between eigenvalues is higher and maximal power may be *several times larger* than power of two uncoupled generators.

Maximal normalized power is received when $\varphi_E=k\pi$. Then for the in-phase excitation:

$$\varphi_E=0\,,\quad q<1,\quad \left(\frac{P_{max}}{P_{max\,o}}\right)_{max}=\frac{q+1}{q}=\frac{r_{o1}+r_{o2}}{r_{o1}}\;; \tag{18}$$

and for the anti-phase excitation:

$$\varphi_E=\pi\,,\quad q>1,\quad \left(\frac{P_{max}}{P_{max\,o}}\right)_{max}=q+1=\frac{r_{o1}+r_{o2}}{r_{o2}}\,. \tag{19}$$

It means that when φ_E=0 and q<1 (r_{o12}<0) maximum normalized power of two coupled generators is $(q+1)/q$ times larger than power of one generator. Analogously when $\varphi_E=\pi$ and q>1 (r_{o12}>0) maximum normalized power of two coupled generators is (q+1) times larger than power of one separate generator.

Power parameters of two-port antenna

Fig.4 shows, as an example, elements of a generator impedance matrix $\mathbf{Z}_o$ of a two-port antenna array [3,4]. The matrix $\mathbf{Z}_o$ of this structure has double symmetry – corresponding diagonal and non-diagonal elements have the same frequency characteristics.

Frequency characteristic of the eigenvalues ratio r_{o1}/r_{o2} calculated using (9) for presented impedance matrix $\mathbf{Z}_o$ are shown in Fig.5. A value of the ratio q is larger than 1 almost in the whole considered frequency range. The ratio q equals 1 at about 5.3GHz, where a real part of non-diagonal element r_{o12} equals zero.

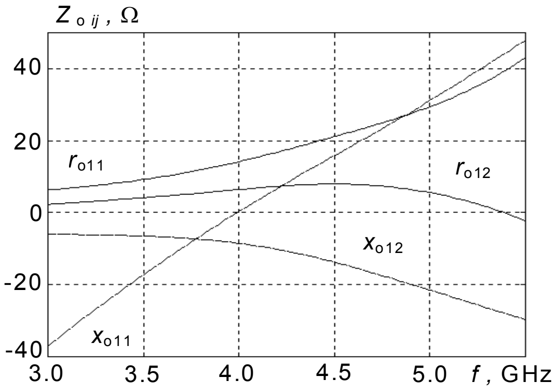

Fig.4. Elements of generator impedance matrix

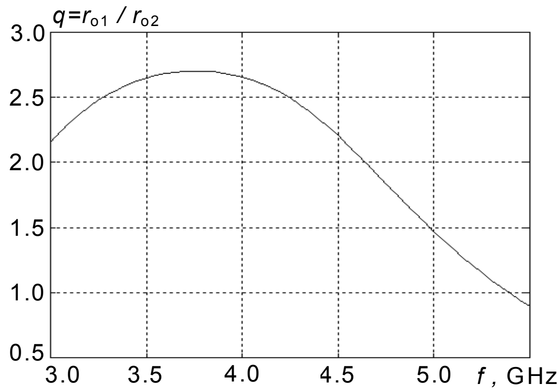

Fig.5. Eigenvalues ratio of real part of impedance matrix

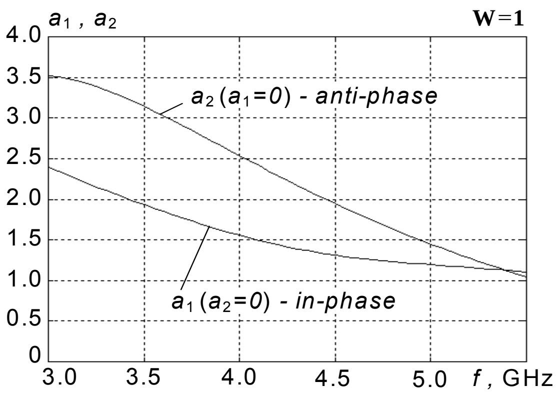

Fig.6. Incident waves for in-phase and anti-phase excitation, **W=1**

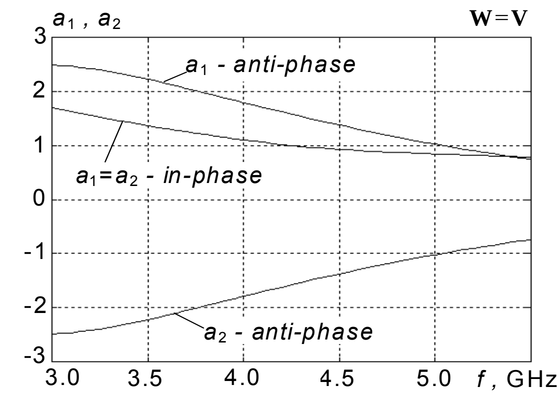

Fig.7. Incident waves for in-phase and anti-phase excitation, **W=V**

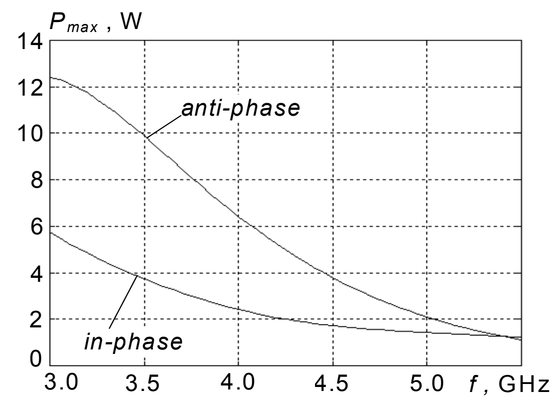

Fig.8. Maximal power for in-phase and anti-phase excitation

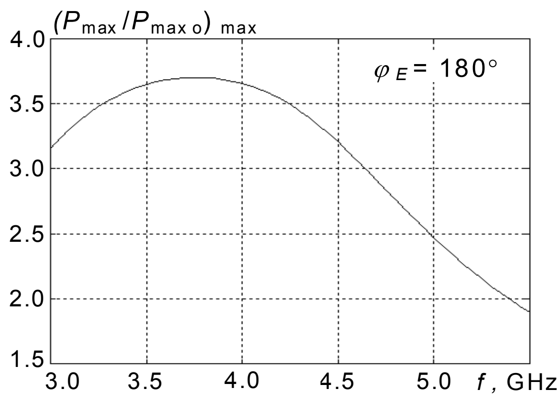

Fig.9 Normalized maximal power of the structure

Characteristics of incident waves for analyzed structure of the 2-port antenna were calculated for two cases of the matrix **W**. As it was discussed in the paper, **W=1** and **W=V**. In both cases it was assumed that amplitudes of both generators are equal: $|E_1|=|E_2|$=10V.

Fig.6. shows frequency characteristics of incident waves a_1 and a_2 in the first case of a unit matrix **W=1**. Two types of the antenna array excitation were considered: the in-phase excitation (φ_E=0°) and the anti-phase excitation (φ_E=180°). In both cases of the excitation only one incident wave has a non-zero value. When φ_E=0° (in-phase excitation) a non-zero wave is a_1 but when φ_E=180° (anti-phase excitation) a non-zero wave is a_2. There is a relationship at the same time, that level of a_2 is larger than a_1 in corresponding cases. It is because the fact, that in almost whole analyzed frequency band the eigenvalue r_{o2} has value less than r_{o1}, as a difference of real parts of proper elements of the impedance matrix $\mathbf{Z}_o$ (9).

Corresponding frequency characteristics of incident waves a_1, a_2 in case of another form of matrix **W=V** were presented in Fig.7. In this situation for the in-phase excitation (φ_E=0°) it can be seen that both values of incident waves are the same: $a_1=a_2$. Moreover, for anti-phase excitation (φ_E=180°) incident waves have non-zero, opposite values: $a_1=-a_2$.

Fig.8 shows frequency characteristics of maximal power in both cases of the in-phase and the anti-phase excitation, for given values of generators' amplitudes $|E_1|=|E_2|$=10V. As it was discussed before, this characteristic does not depend on the form of the matrix **W** but it depends only on the excitation form. Fig.9 presents frequency characteristic of maximal normalized power for the anti-phase excitation (φ_E=180°) calculated for the given two-port antenna structure. In this case the shape of this characteristic is the same as the ratio q shown in Fig.5, because both parameters are related according to (19). In a major frequency range this maximal normalized power is larger than 2 and achieves the value **3.7** for the maximal value of the ratio q, calculated for a real matrix $\mathbf{R}_o$ of analyzed two-port antenna structure.

Conclusion

The complex normalized scattering matrix introduced in the paper may be used for a solution of the multiport matching problem and maximization of power transmission in multiport antenna array excited by the set of coupled generators. The considered mathematical model of new complex normalized scattering matrices can be also used for the evaluation of the broadband properties of arbitrary passive multiport loads, including antenna arrays.

The two-port antenna was used as the example of the application of presented method using the complex normalized scattering matrix. It was proved that this structure may be described using an infinity number of new scattering matrices which define the same power distribution in the whole network.

This work was supported by the Rector's project S/WE/1/2015 in Bialystok University of Technology.

***Authors**: prof. dr hab. inż. Giennadij Czawka, Politechnika Białostocka, Wydział Elektryczny, ul. Wiejska 45D, 15-351 Białystok, E-mail: g.czawka@pb.edu.pl; dr inż. Marek Garbaruk, Politechnika Białostocka, Wydział Elektryczny, ul. Wiejska 45D, 15-351 Białystok, E-mail: m.garbaruk@pb.edu.pl.*

REFERENCES

[1] Rohrer A.R., The Scattering Matrix: Normalized to Complex n-Port Load Networks, *IEEE Trans. on Circuit Theory*, vol. CT-12, June 1965, 223-230
[2] Newcomb R.W., Linear Multiport Synthesis, *McGraw Hill Book Co.*, New York, 1966
[3] Chavka G., Analysis of antenna array using new scattering matrix, *19th Intern. Wrocław Symposium and Exhibition on Electromagnetic Compatibility EMC*, Wrocław, June 11-13, 2008, 310-315
[4] Czawka G., Nowa macierz rozproszenia wielowrotnikowych układów antenowych, *Przegląd Telekomunikacyjny i Wiadomości Telekomunikacyjne*, nr 6, 2014, 381-384
[5] Chavka G., Scattering matrices of multiport radio devices, published in this issue