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Short-time Fourier transform of ultrawideband signals

***Abstract**. The paper presents theoretical bases concerning the usage of a short-time Fourier transform for the analysis of UWB pulses. In the article there was shown the analysis of Fourier spectrograms of a base UWB wavelet interfered with other signals of different forms: UWB wavelets, an UWB Gaussian pulse and a continuous sine signal. 2D and 3D spectrograms of signals were calculated and compared to the results obtained using a normal Fourier transform.*

***Streszczenie.** W artykule omówiono wykorzystanie krótkoczasowej transformaty Fouriera do analizy sygnałów ultraszerokopasmowych UWB. Jako przykład rozpatrzono impuls falkowy UWB zakłócony innymi sygnałami o różnych formach: impulsami UWB oraz ciągłym sygnałem sinusoidalnym. Przedstawiono porównanie wyników analizy sygnałów uzyskanych za pomocą transformaty STFT i klasycznego przekształcenia Fouriera. (**Transformata STFT sygnałów ultraszerokopasmowych**).*

Keywords: UWB signals, short-time Fourier transform, Fourier spectrograms.
Słowa kluczowe: sygnały UWB, krótkoczasowa transformata Fouriera STFT, spektrogramy Fouriera.

Introduction

Ultra-wideband (UWB) technology is characterized by many interesting and unique features that allow to use it in many areas of everyday life and in modern engineering [1-3]. New applications and functions of ultra-wideband impulse radio communication also require new approach and latest techniques of analysis of ultra short UWB pulses transmitted in radio systems. Using a short-time Fourier transform (STFT) seems to be a promising technique of the analysis of the ultra-wideband signals [4-7].

The paper describes theoretical bases of a short-time Fourier transform and its usage in the analysis of nanoseconds UWB pulses. The paper presents the analysis of a superposition of a base ultra-wideband wavelet and a few additional interfering signals. The analysis takes into account the European UWB band 6-8.5GHz [8].

Short-time Fourier transform – Fourier spectrograms

A short-time Fourier transform for a given signal $s(\tau)$ is given by [6]:

$$\dot{S}_d(\omega,t)=\int_{-\infty}^{\infty} s(\tau)w(\tau-t)\exp(-j\omega\,\tau)d\tau \tag{1}$$

where: $\dot{S}_d(\omega,t)$ is a complex spectral function representing the magnitude and the phase of the analyzed signal over time and frequency domain, $w(\tau)$ is a window function. In numerical calculation, in discrete time, the STFT transform can be obtained by using the following formula:

$$\dot{S}_d(\omega,m)=\sum_{n=0}^{N-1} s(n)w(n-m)\exp(-j\omega n) \tag{2}$$

where n and m describe discrete time, whereas N denotes a total number of samples in a considered time period. Assuming the above, a Fourier spectrogram is an energy density spectrum of STFT:

$$P_d(\omega,t)=\left|\dot{S}_d(\omega,t)\right|^2 . \tag{3}$$

The formula (3) denotes that the result of a short-time Fourier transform is a spectrum of a multiplication of the given signal and a window function. It is not a spectrum of a signal itself as it is in an ordinary Fourier transform. Thus, for a suitable STFT analysis, proper parameters of a window function have to be determined. Two basic parameters of windows' spectra are used to evaluate their properties in the STFT analysis: the width of the main lobe and the maximum level of side lobes in the spectrum that is calculated by a basic Fourier transform. There are many window types developed for the use in the STFT calculation [6,7,10].

One of the most common window function is a sine function described by:

$$w_S(t)=sin\left(\frac{\pi t}{t_0}\right), \tag{4}$$

$$w_S(n)=sin\left(\frac{\pi n}{N-1}\right), \tag{5}$$

where t_0 is a window length, N is a corresponding parameter used in a discrete time. The shape of the sine window and its normalized spectrum are presented in Fig.1. In this case an example value of t_0=1ns was used.

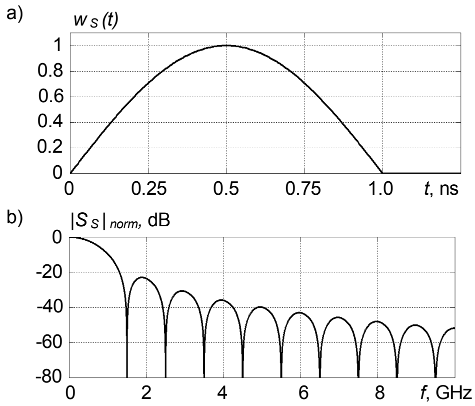

Fig.1. An example of sine window function: waveform (a), normalized spectrum (b)

It is commonly known that the narrowest main lobe characterizes a rectangle-shaped window and the first spectrum zero is at $1/t_0$ frequency, but the level of the highest side lobe is about -13dB. In case of a sine window it can be observed that the width of the main lobe is slightly wider but the level of the first side lobe, compared to a rectangular one, is lower and equals -22dB. The characteristics and formulas used to describe other types of windows are presented in [7,10].

It was shown in [7,10] that other types of window shapes are characterized by a lower side lobes level and a wider main lobe of their spectrum. This significantly changes the results of the Fourier spectrograms calculations and, thus, it can contribute to a more efficient usage in practical solutions.

Numerical analysis

A numerical analysis carried out in the paper concerns a base UWB wavelet expressed in the time domain by:

$$s_{\mathrm{wav}}(t) = \exp\left[-A\left(\frac{t-t_0}{\tau}\right)^2 \right] \cos\left[2\pi f_0 (t-t_0)\right] \quad (1)$$

where: A – amplitude of a wavelet, τ – describes signal duration, f_0 – frequency of a sine wave, t_0 – describes pulse allocation in time. For the basic wavelet analyzed in the paper it was assumed: A=1, τ=1.05ns, f_0=6.5GHz (Fig.2). The basic wavelet's center was positioned at t_0=10ns in the time scale. It was carried out the Fourier spectrogram analysis of the basic wavelet signal and three other interfering signals of different forms.

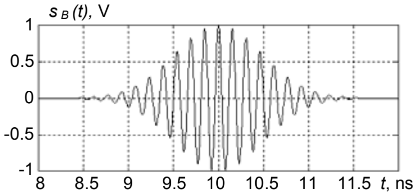

Fig.2. A waveform of a basic UWB wavelet A=1, τ=1.05ns, f_0=6.5GHz

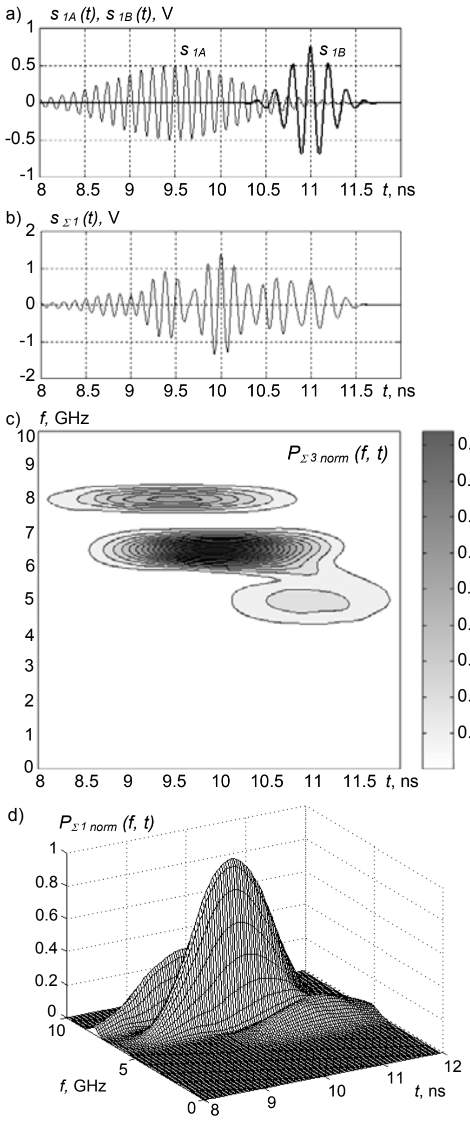

Fig.3. Interfering wavelets $s_{1A}(t)$: A=0.5, τ=1.5ns, f_0=8GHz and $s_{1B}(t)$: A=0.75, τ=0.5ns, f_0=5GHz (a), the sum signal $s_{\Sigma 1}(t)$ (b), 2D spectrogram (c), 3D spectrogram (d)

In each case considered in the paper 2D and 3D Fourier spectrograms were calculated using a sine window function. The duration of a sine window function was assumed as 3ns. That value was adapted to the length of the basic UWB wavelet. There were three cases examined and the results are shown in Fig.3-5.

The first examined case is a superposition of the basic wavelet $s_B(t)$ and two other interfering wavelets described as $s_{1A}(t)$ and $s_{1B}(t)$ in Fig.3,a. Parameters of interfering signals are: for $s_{1A}(t)$ - A=0.5, τ=1.5ns, f_0=8GHz, t_0=9.5ns and for $s_{1B}(t)$ - A=0.75, τ=0.5ns, f_0=5GHz, t_0=11ns. The resulting signal $s_{\Sigma 1}(t)$ is visible in Fig.3,b. Normalized Fourier spectrograms in 2D and 3D presentation are shown in Fig.3,c,d. Despite the imposition of wavelets at one another at the same time it can be seen that, depending on amplitudes, they are perfectly distinguishable on the time-frequency plane of spectrograms.

The second example is the analysis of a disturbance in the form of a continuous sine signal $s_2(t)$ with an amplitude A=0.5 and a frequency f=8GHz. The results are shown in Fig.4 in the same order as in the previous example. In this case each signal can be perfectly distinguished. A Fourier spectrogram of a continuous sine, which is a non-pulse signal in this performance, presents an almost uniform ridge parallel to the time axis (Fig.4,c,d).

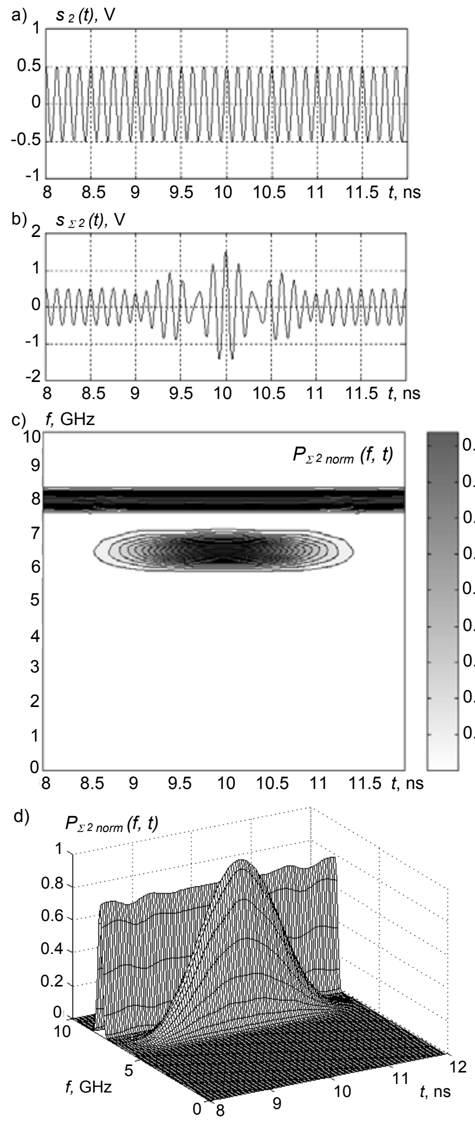

Fig.4. Interfering sine wave $s_2(t)$: A=0.5, f=8GHz (a), the sum signal $s_{\Sigma 2}(t)$ (b), 2D spectrogram (c), 3D spectrogram (d)

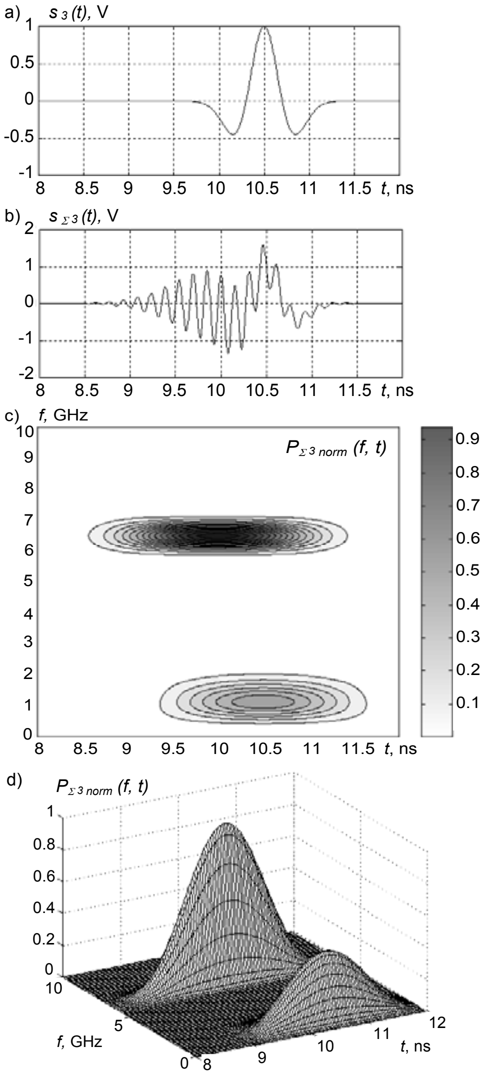

Fig.5. Interfering UWB Gaussian pulse $s_3(t)$: A=1, τ=1/5ns (a), the sum signal $s_{\Sigma 3}(t)$ (b), 2D spectrogram (c), 3D spectrogram (d)

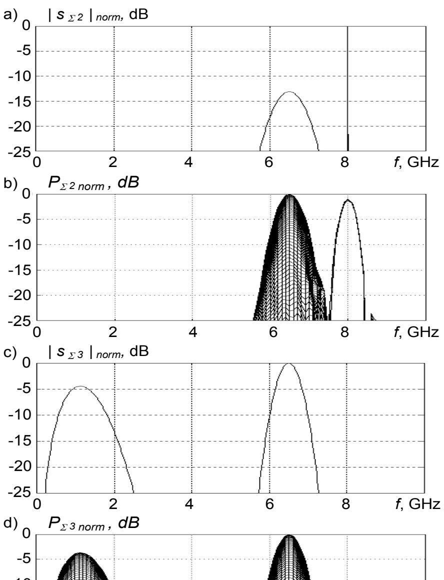

Fig.6. A normalized spectrum and a Fourier spectrogram projection for the sum signal $s_{\Sigma 2}(t)$ (a,b) and $s_{\Sigma 3}(t)$ (c,d)

The last example is the superposition of the analyzed basic wavelet $s_B(t)$ and an UWB Gaussian pulse $s_3(t)$ [9] with parameters A=1, τ=1/5ns (Fig.5,a). As in the previous cases, both signals are perfectly distinguishable on the plane of Fourier spectrograms.

Fig.6 illustrates the difference between results obtained using Fourier spectrograms analysis of UWB pulses in comparison to a normal Fourier transform. Fig.6,a,c present the normalized spectrum of the superposition signals $s_{\Sigma 2}(t)$ and $s_{\Sigma 3}(t)$ correspondingly, obtained by using normal Fourier transform. Fig.6,b,d show a projection of 3D Fourier spectrograms over a frequency axis for corresponding signals.

It can be observed that for the pulse-shaped signals the differences between these two presentations are almost invisible. On the other hand, this difference is enormous when compared to the results for a wavelet pulse interfered with a continuous sine wave (Fig.6,a,b).

Conclusions

A short-time Fourier transform and Fourier spectrograms can be very helpful in analyzing and distinguishing impulse signals, including UWB, and signals imposed on them which can be regarded as interference or disturbances. STFT offers more capabilities than a normal Fourier analysis. However, it requires slightly more complex calculations and larger computing power. STFT analysis also requires more attention in choosing parameters of window functions – their shapes and durations.

REFERENCES

[1] Allen et al B., Ultra-Wideband Antennas and Propagation for Communications, Radar and Imaging, *John Wiley & Sons*, 2007

[2] Matin M.A. (editor), Ultra Wideband Communications Novel Trends - Antennas and Propagation, *Intech Open Access Publisher*, 2011

[3] Hejdari G., WiMedia UWB, *John Wiley & Sons*, 2008

[4] Cohen L., Time-frequency analysis, *Prentice-Hall PTR*, 1995

[5] Zou W., Yang Ch., Wang W., Zhou Z., A Novel Method to Analyze UWB Signal Algorithm, *International Symposium on Microwave, Antenna, Propagation and EMC Technologies for Wireless Communications*, Hangzhou, China, 2007, 856–859

[6] Lazorenko O.V., Czernogor L.F., Ultrawideband signals i processes, *Kharkiv National University*, 2009

[7] Czawka G., Garbaruk M., Litwińczuk N., An Analysis of Fourier Spectrograms of Different Ultra-wideband Signals for Radio Systems, *20th International Conference on Microwaves, Radar and Wireless Communications: MIKON'2014*, Gdańsk, Poland, June 16-18, 2014, 576-580

[8] Commission decision on allowing the use of the radio spectrum for equipment using UWB technology in a harmonised manner in the Community, *decision of European Parliament*, Brussels, 21.02.2007

[9] Garbaruk M., Time-frequency characteristics of ultra-wideband signals for radio systems, *XVI International Conference MIKON'2006*, Kraków, Poland, May 22-24, 2006, 419-422

[10] Garbaruk M., Litwińczuk N., Krótkoczasowa transformata Fouriera impulsów ultraszerokopasmowych UWB, *Przegląd Elektrotechniczny*, 91 (2015), nr 1, 95-98

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