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Applying a modified Voronin arc model for simulating processes in gliding arc plasma generators

***Abstract**. The paper presents modified variants of the Voronin model of electric arc. In these modified variants, free changes in the arc diameter caused by current alternations are taken into account. Current alternations, in turn, are due to variable forcing voltage and diverging electrodes, on which the variable length arc is gliding. To approximate the arc diameter, two different functions are applied, allowing for the arc diameter expansion at the point of the current passing through zero. Simulations of the processes occurring in the circuit with a voltage source and a gliding arc (GA) plasma generator indicate that the mathematical model developed adequately represents the current and voltage variation in the elongated electric arc.*

***Streszczenie.** W artykule przedstawiono zmodyfikowane warianty modelu Woronina łuku elektrycznego. Uwzględnia on swobodne zmiany średnicy kolumny łuku wywołane zmianami natężenia prądu. Z kolei zmiany prądu są spowodowane działającym zmiennym wymuszeniem napięciowym i rozbieżnością elektrod, po których ślizgając się porusza się łuk o zmiennej długości. Wykorzystano dwie różne funkcje aproksymujące średnicę kolumny, w których uwzględniono ekspansję przekroju łuku w okolicach przejścia prądu przez wartość zerową. Drogą symulacji procesów w obwodzie ze źródłem napięciowym i plazmotronem gliding arc wykazano zdolność opracowanego modelu matematycznego do odwzorowywania przebiegów prądu i napięcia w rozciąganym łuku elektrycznym. (**O wykorzystaniu zmodyfikowanego modelu Woronina łuku elektrycznego do symulowania procesów w plazmotronach gliding arc**).*

Keywords: plasma reactor, gliding arc, arc model, Voronin model.
Słowa kluczowe: reaktor plazmowy, gliding arc, model łuku, model Woronina.

Introduction

When mathematical models of the electric arc are developed, the functions applied in them are typically assumed to have constant values. Such is the case with the time constant and dissipated power in the Mayr model and the arc column voltage in the Cassie model. This kind of an approach impacts on the possibility of applying these models for representing the processes occurring in arcs of variable length or diameter. The aim of the modifications [1] is then to weaken the rigid assumptions and to treat the relevant parameters as variable rather than constant. This can lead to satisfactory approximation of results but renders the physical interpretation of the arc phenomena difficult. Significant problems arise in the modelling of AC arcs in high-power plasma generators. This is especially true of GA discharges, due to the speed and diversity of processes occurring when thermal and non-thermal plasma is generated.

In publications [2-5] integral forms of the Schwarz-Avdonin models were applied for representing the electrical processes in elongated arcs. With regard to the modified variants of the Mayr and Cassie models, two equations were obtained, one of which is preferred for modelling weak-current arcs and the other for modelling strong-current arcs. A similar approach was taken in [6], where the differential Novikov-Shellhase and Cassie-Berger models were used for providing an account of GA processes in plasma generators of various power. It was also established that differential models are equivalent to the corresponding models employing integral equations.

In hybrid models, it is possible to take into account the non-linear character of static characteristics and the damping factor function. Modifications of such models with a variable-length arc were described in [1]. With these models being highly complex, it is difficult to identify the parameters or interpret the physical processes. Besides, they do not allow for individual or joint external influences on the arc length or diameter. Such a possibility is afforded by the Voronin model, which naturally accounts for the damping factor function, dissipated power and static characteristics as dependent on both the arc diameter and length.

The Voronin model of the variable-size arc

The mathematical model of the variable-size gliding arc discussed in this section was developed by A.A. Voronin [7] as a theoretical foundation for simulating processes occurring in electrical apparatuses. The model allows for external influences on the arc length and diameter, causing the arc to fade. The model employs the following simplifying assumptions:

1. The arc column is cylindrical with equal cross-section throughout its axis;
2. Plasma is homogeneous both lengthwise and crosswise;
3. Heat is dissipated only through the arc lateral surface;
4. Arc length and cross-sections are the only parameters that can vary in time.

The model is based on the simplified equation of the arc heat balance

$$\frac{dQ}{dt} = P_{el} - P_{dys} = u_{kol} i - P_{dys} \tag{1}$$

where: Q – plasma enthalpy, J; P_{el} – electric power supplied to the arc, W; P_{dys} – thermal power dissipated from the arc column, W. In this general form of the equation there is no distinction into different forms of heat dissipation, such as plasma thermal conduction, convection or radiation. The following symbols are subsequently introduced:

$$Q = q_V V = q_V l S \tag{2}$$

$$g = \frac{g_V S}{l} \tag{3}$$

$$P_{dysS} = p_S S_b = p_S l \sqrt{4\pi S} \tag{4}$$

where: q_V – volumetric density of enthalpy, J/m^3; g_V – conductance of the arc per a unit length, S/m; l – arc length, m; p_S – density of power dissipated through the arc lateral surface, W/m^2; S – area of the arc cross-section, m^2; S_b – area of the arc lateral surface, $S_b = l\sqrt{4\pi S}$, m^2.

Since the arc conductivity is a function of plasma enthalpy, it is possible to adopt Mayr's assumption [7]

(5) $$g_V(i) = K_g \cdot \exp\left(\frac{q_V(\sigma(i))}{q_M}\right)$$

where: q_M – constant reference factor, J/m^3; σ - plasma conductivity, S/m; K_g – constant approximation factor, S/m. Then, the arc model with variable dimensions $S(t)$ and $l(t)$ has the general conductance form [7]

(6) $$\frac{1}{g}\frac{dg}{dt} = \frac{p_S}{q_M}\frac{\sqrt{4\pi}}{\sqrt{S}}\left(\frac{u_{kol}i}{p_S l\sqrt{4\pi S}} - 1\right) + \\ -\frac{1}{l}\frac{dl}{dt}\left(1 + \ln\frac{g\,l}{K_g S}\right) + \frac{1}{S}\frac{dS}{dt}\left(1 - \ln\frac{g\,l}{K_g S}\right)$$

The three parameters p_S, q_M, K_g are obtained experimentally and assumed to be constant. The quantity u_{kol} is a voltage drop on the arc. The generalised model employs:
- the damping function

(7) $$\theta_S(S) = \frac{q_M}{p_S}\sqrt{\frac{S}{4\pi}},\ \mathrm{s}$$

- dissipated power function

(8) $$P_{dys}(S,l) = p_S l\sqrt{4\pi S},\ \mathrm{W}$$

so that the following notation can be applied

(9) $$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_S(S)}\left(\frac{u_{kol}i}{P_{dys}(S,l)} - 1\right) + \\ -\frac{1}{l}\frac{dl}{dt}\left(1 + \ln\frac{g\,l}{K_g S}\right) + \frac{1}{S}\frac{dS}{dt}\left(1 - \ln\frac{g\,l}{K_g S}\right)$$

Special cases of these models with constant or slowly changing dimensions $l(t)$ and $S(t)$ were discussed in [8, 9], where it was demonstrated that these are in fact equivalent to the Mayr or Cassie models or to their modified versions, which possibilities are limited in representing real processes in electrotechnological devices. Models with constant arc column diameter and with variable arc length are also inadequate for the majority of AC arc devices. Models assuming constant arc length and variable diameter are, on the other hand, not in use for controlling power of devices.

The modified Voronin arc model for simulating processes in a plasma GA generator

Let us assume that the arc cross-section is a function of the current $S(i)$. The external forcing has impact only on current i and the arc length $l(t)$. Taking into account the axially-symmetrical shape of the arc column with the diameter d, the equation can be represented as

(10) $$\frac{1}{g}\frac{dg}{dt} = \frac{p_S}{q_M}\frac{4}{d(i)}\left(\frac{u_{kol}i}{\pi p_S l d(i)} - 1\right) + \\ -\frac{1}{l}\frac{dl}{dt}\left[1 + \ln\left(\frac{4}{\pi K_g}\cdot\frac{g\,l}{d^2(i)}\right)\right] + \\ +\frac{2}{d(i)}\frac{dd(i)}{di}\frac{di}{dt}\left[1 - \ln\left(\frac{4}{\pi K_g}\cdot\frac{g\,l}{d^2(i)}\right)\right]$$

It follows that the value of the damping factor function is variable

(11) $$\theta_S(i) = \frac{q_M}{4}\frac{d(i)}{p_S}$$

and so is the value of dissipated power

(12) $$P_{dys}(i) = \pi p_S l d(i)$$

The static voltage-current characteristics is given by

(13) $$u_{kol}i = \pi p_S l d(i)$$

With new symbols, Eq. (7) can be represented in a simpler form as

(14) $$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_W d(i)}\left(\frac{u_{kol}i}{p_W l d(i)} - 1\right) + \\ -\frac{1}{l}\frac{dl}{dt}\left[1 + \ln\left(K_W\cdot\frac{g\,l}{d^2(i)}\right)\right] + \\ +\frac{2}{d(i)}\frac{dd(i)}{di}\frac{di}{dt}\left[1 - \ln\left(K_W\cdot\frac{g\,l}{d^2(i)}\right)\right]$$

where the constant parameters are: $\theta_W = q_M/(4p_S)$, $p_W = \pi p_S$, $K_W = 4/(\pi K_g)$.

The mean arc cross-section area is a function of such variables as the current i, the pressure p, gas type, the temperature T and the velocity v of the gas motion. In the case of welding arcs, the mean temperature depends on the kind of technology applied. It can be typically expressed as $T = 800\ U_i$, where U_i – effective ionisation potential of the gas mixture, V. As was established in [10], the arc radius is primarily a function of the current $r_{kol} = f\left(i^{2/3}\right)$. This has been confirmed experimentally [11] and the dependence $r_{kol} \cong w_r|i|^n$, cm, was established, where: n = 0.6-0.7, w_r – approximation factor. The mean current density in the arc is 10^1-10^3 A/cm^2, which is significantly lower [12] than the density near the cathode (10^3-10^8 A/cm^2) and near the anode (10^4-10^5 A/cm^2). It can be roughly assumed that the arc diameter can vary freely together with the discharge current

(15) $$d = w|i|^{2/3}$$

where $w = 2w_r$. Eq. (11) together with the condition (15) has the property that the time constant is convergent $\lim_{|i|\to 0}\theta_S(i) \to 0s$ [8], which is inconsistent with experimental data and with other mathematical models of the arc [13, 14]. Even a complete momentary decay of current does not cause plasma and electric conductivity resulting from gas ionisation to disappear immediately. At the same time, the effect of magnetic pinch ceases to hold and plasma may tend to expand. The curve representing the experimentally obtained time constant is a non-monotonic function [13], which is taken into account in hybrid arc models [14]. The modified formula for the arc diameter is

(16) $$d(i) = d_0\exp(-w_1|i|) + w_2|i|^{2/3}$$

When the current passes through zero, the time constant can take the maximum value and the arc diameter can be non-zero $d(0) = d_0$. This approximation, however, leads to a non-continuous derivative of the function, which can bring about complications in the numerical integration of the arc model.

In arc modelling, it is convenient to use normalised forms of smooth functions. Because of that, we suggest the following approximation should be used

$$d(i)=d_{01}\exp\left(-\left|\frac{i}{I_{01}}\right|^{m}\right)+d_{02}\exp\left(-\left|\frac{i}{I_{02}}\right|^{-n}\right) \tag{17}$$

The value d_{01} corresponds to the arc diameter when the current is passing through zero and the value d_{02} corresponds to the asymptote of the arc diameter function for the extreme value of the current. The power exponents m and n correspond to the steepness of falling and rising. The prescribed currents $I_{01} > 0$ and $I_{02} > 0$ determine the position of inflection points (I_{01}, d_{01}/e) and (I_{02}, d_{02}/e) of the component functions, where e – the Napier's number.

It has to be kept in mind that the time constant of high-current arcs and that of thermally insulated arcs are very low, despite large arc diameters. This is due to a strongly nonlinear character of the dependence of gas viscosity and thermal conductivity on temperature [15].

Publications [2-5] offer modifications of the Mayr and Cassie models, taking into account the additional function of the discharge ignition. A similar effect can be obtained by including in Eq. (14) an additional term including forced voltage. This causes decrease in conductance

$$\begin{aligned}\frac{1}{g}\frac{dg}{dt}&=\frac{1}{\theta_W d(i)}\left(\frac{gu_{kol}^2}{p_W l d(i)}-1\right)+\\&-\frac{\zeta(t_l)}{\theta_W d(i)}\left(\frac{gu_z^2}{p_W l d(i)}-1\right)+\\&-\frac{1}{l}\frac{dl}{dt}\left[1+\ln\left(K_W\cdot\frac{g\,l}{d^2(i)}\right)\right]+\\&+\frac{2}{d(i)}\frac{dd(i)}{di}\frac{di}{dt}\left[1-\ln\left(K_W\cdot\frac{g\,l}{d^2(i)}\right)\right]\end{aligned} \tag{18}$$

where

$$\zeta(t_l)=\begin{cases}1, & \text{if}\quad t_z<t_l\le t_1\\ 0, & \text{if}\quad t_1<t_l<t_2\end{cases} \tag{19}$$

where: t_l – local time of a single discharge cycle; t_z – moment of ignition, t_z=0s; t_1 – duration of the ignition pulse; t_2 – conditioning time of cycle duration ($t_2=t_l\leftrightarrow g(t_l)=g_{\min}$); $g_{\min}$ – prescribed minimal value of conductance just before the arc quench; u_z – voltage of the auxiliary ignition source.

Changes of the arc length in time can be described by a formula analogous to [5]

$$l(t_l)=l_0+k\alpha v_g t_l \tag{20}$$

where: l_0 – gap between the electrodes at the location when the arc is struck; α - divergence angle between the electrodes, k – correction factor. The speed of the arc motion can be also altered by means of an external magnetic field.

Computer simulations of processes in a circuit with Voronin model of a GA plasma generator

Simulation experiments were performed on an AC powered plasma generator. The rate of change in the arc length in a GA plasma generator tends to be significantly lower than the rate of change of the diameter and conductance, determined by the frequency of the electrical source. Assuming that $\left|\frac{1}{S}\frac{dS}{dt}\right|>>\left|\frac{1}{l}\frac{dl}{dt}\right|\approx 0$, it is possible to obtain the simplified equation

$$\begin{aligned}\frac{1}{g}\frac{dg}{dt}&=\frac{1}{\theta_W d(i)}\left(\frac{gu_{kol}^2}{p_W l d(i)}-1\right)+\\&-\frac{\zeta(t_l)}{\theta_W d(i)}\left(\frac{gu_z^2}{p_W l d(i)}-1\right)+\\&+\frac{2}{d(i)}\frac{dd(i)}{di}\frac{di}{dt}\left[1-\ln\left(K_W\cdot\frac{g\,l}{d^2(i)}\right)\right]\end{aligned} \tag{21}$$

This assumption makes it possible to eliminate problems related to step changes in the arc length occurring between the arc quenching and striking it again. In accordance with the approximations assumed, it is possible to establish the derivatives of the arc diameter. Approximation (16) corresponds to formula

$$\frac{dd(i)}{di}=\left[-w_1 d_0\exp\left(-w_1|i|\right)+\frac{2}{3}w_2|i|^{-1/3}\right]\mathrm{sgn}(i) \tag{22}$$

and approximation (17) to

$$\begin{aligned}\frac{dd(i)}{di}=&\left[-m\frac{d_{01}}{I_{01}}\left|\frac{i}{I_{01}}\right|^{m-1}\exp\left(-\left|\frac{i}{I_{01}}\right|^{m}\right)+\right.\\&\left.+n\frac{d_{02}}{I_{02}}\left|\frac{i}{I_{02}}\right|^{-n-1}\exp\left(-\left|\frac{i}{I_{02}}\right|^{-n}\right)\right]\mathrm{sgn}(i)\end{aligned} \tag{23}$$

The simulations of processes in a GA plasma generator were carried out by means of the software MATLAB-Simulink. First, a macromodel of a simple power supply circuit was created with a real alternating sinusoidal voltage source, connected in series with a macromodel of the electric arc with a cyclically changing length.

Two variants of the modified Voronin model were taken into account (21): one variant with the arc diameter approximated by function (16) with the parameters: d_0 = $2\cdot10^{-3}$ m; w_1=0,1; w_2 = $6\cdot10^{-5}$ m$\cdot$A$^{-2/3}$, and the other with the arc diameter approximated by function (17) with the parameters: I_{01}= 10 A; I_{02}= 20 A; d_{01}= $3\cdot10^{-3}$ m; d_{02} = $3\cdot10^{-3}$ m; m = 1.6; n = 0.5. The parameters of the supply circuit were: U_m = 1500 V, R_w=75 Ω, L = 30 mH, f = 50 Hz, U_z = 2500 V. The parameters of the model (21) and of the function defining periodical changes in the arc length (20), resulting from the construction and operation principles of the plasma generator are: l_0 = 15 mm; α = 25°; v_g = 10 m/s, k = $1\cdot10^{-3}$, t_z = $2\cdot10^{-3}$ s, $g_{\min}$ = $1.9\cdot10^{-3}$ S. Figures 1 and 2 present the simulation results of the electric processes occurring in the plasma generator with various methods of approximating the changes in the arc diameter. Besides, in the first case a generator of unipolar ignition pulses was used, and in the other it was a bipolar pulse generator. Comparing the results with those obtained in the simulations and experiments described in [2-5], we can state that the model meets the requirements with respect to the accuracy of approximating the voltage and current in GA plasma generators of any power.

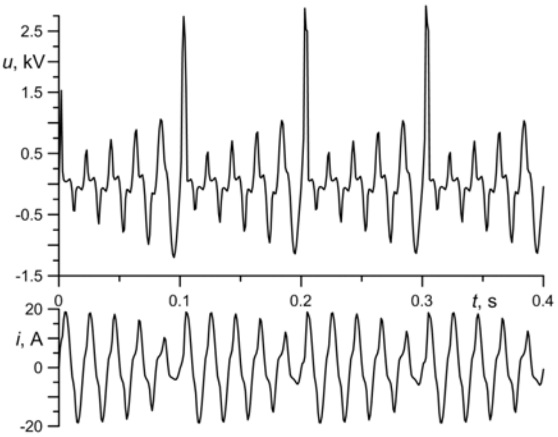

Fig. 1. Current and voltage waveforms in a circuit with Voronin arc model - column diameter approximated by function (16) (θ_W = 0.87 s/m; p_W = 6.3·10^7 W/m^2; K_W = 3·10^{-3} m/S).

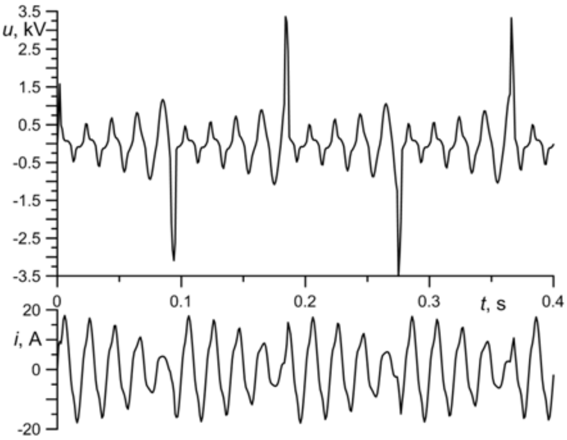

Fig. 2. Current and voltage waveforms in a circuit with Voronin arc model - column diameter approximated by function (17) (θ_W = 0.77 s/m; p_W = 5.4·10^7 W/m^2; K_W = 3.1·10^{-3} m/S).

Conclusions

1. Since the number of simplifying assumption is small, it is possible to modify the Voronin model of the electric arc so that it suits variable operating conditions in GA plasma generators.
2. The fact that the arc column expansion accompanying the passing of current through zero is taken into account ensures a better discharge stability, facilitates simulation and yields more realistic waveforms.
3. The mathematical model of the arc is to a large extent universal, since it is capable of representing processes both in low current and in high current gliding arcs in plasma generators of various powers.
4. In the more mathematically elaborate variant of the model, the arc diameter depends not only on the current but also on the temperature and velocity of the gas. These quantities are variable in the chamber of a GA plasma generator.

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