The paper presents the fuzzy model which enables to calculate temperature changes in the flat resistive wall of the furnace chamber. The rules the model is based on, are associated with the thermal state of object, more specifically with the instantaneous changes of its temperature. By using the concepts of "initial time constant", "initial gain", "saturation time constant", "saturation gain" good approximation of temperature changes in a flat wall of resistance chamber furnace has been achieved.

Keywords: fuzzy modeling, electroheat plants

Introduction

The article presents the possibility of using fuzzy logic and fuzzy sets to create a model describing heat exchange in a plane wall of a chamber electric furnace. There are several well known methods for formal description of the properties of such objects but they are either too complicated for implementation in control systems or they aren't accurate enough. Electroheat plants are important in various technologies and widely used in industrial practice. Therefore there is a need to develop more advanced modeling methods of such objects

The main goal of this paper is to show the fuzzy model and fuzzy sets to create a model describing heat exchange in a plane wall of a chamber electric furnace. There are several well known methods for formal description of the properties of such objects but they are either too complicated for implementation in control systems or they aren't accurate enough. Electroheat plants are important in various technologies and widely used in industrial practice. Therefore there is a need to develop more advanced modeling methods of such objects.

Modeling using physical laws

As insulation walls, are main constructional elements of electric resistance furnaces, the properties of the walls determine the behavior of the entire plant. The heat flow through the furnace wall can be described by partial differential equations of parabolic type with proper boundary conditions [1, 5]. Neglecting many constructional details, it can be regarded as the one-dimensional spatial model as follows:

\[
d\dot{\vartheta}(t, x) = \frac{\lambda}{\rho c} \frac{\partial^2 \vartheta(t, x)}{\partial x^2}
\]

where: \(\vartheta\) is temperature, \(\lambda\) is thermal conductivity, \(\rho\) is density, \(c\) is specific heat of the insulation material.

First order inertia model of thermal plants

Temperature changes in a resistance chamber furnace and more specifically in its plane wall can be also modeled using following first order inertia model with time delay expressed as transfer function:

\[
G(s) = \frac{K}{1 + sN} e^{-sL}
\]

where: \(K\) is gain, \(N\) is time constant, \(L\) is time delay.

In many cases the main source of time delay \(L\) in (2) is a temperature sensor [2]. Sometimes it can be assumed that temperature inside the furnace is measured by sensor with negligibly small inertia. This gives the possibility of taking \(L = 0\) and then formula (2) reduces to the form:

\[
G(s) = \frac{K}{1 + sN}
\]

Models (2) and (3) are often used in practice because they are simple and easy to implemented. However in many technologies the accuracy of such models comparing to experimental results becomes unsatisfactory.

Identification of temporary dynamic parameters of the furnace

Instantaneous values of dynamic parameters of the furnace can be obtained through the approximation of its answer by the equation:

\[
\vartheta_k^m = w_k^m \theta
\]

\[
\theta = [K(1 - e^{-\frac{t}{\lambda}}), e^{-\frac{t}{\lambda}}]^T
\]

\[
w_k = [P_k, \theta_{k-1}]
\]
where: \( \vartheta_k \) is temperature in time step \( k \), \( \theta \) is vector of model parameters, \( w \) is model input vector, \( \Delta \) is time interval.

Having set pairs of values \{ \vartheta_i, P_i \}, \( i = r, \ldots, r + n \) it is possible to determine the vector of model parameters \( \theta \). This vector will provide the best representation of the real signals using model (4) in the \( r \)-th time interval comprising \( n \) samples. Static gain \( K_r \) and time constant \( N_r \) of the object obtained for an appropriately small value of \( n \) can be regarded as instantaneous dynamic parameters of the object. The values of parameters \( K_r \) and \( N_r \) vary depending on what \( r \) and \( n \) was adopted.

For the considered class of electrothermical objects there exist both in the literature and in practice a descriptive terms: “initial time constant”, “initial gain”, “saturation time constant”, “saturation gain”. These terms are associated with variable degree of saturation of the thermal components of the object. This terms encourage to develop the model in which the parameters \( N \) and \( K \) depends on the conditions under which the object operates.

**Fuzzy modeling of dynamic properties of the furnace**

Changes of the parameters \( N \) and \( K \), depending on the thermal state of the object, are not crisp, especially if described in terms of linguistic terms given above. This is a good basis for the use of fuzzy set theory and fuzzy logic to incorporate these terms into a model. It can be done by combination of four inertia first order blocks. The behavior of an object is different in an initial state and a saturated state. It is also influenced by heating phase or cooling phase. Applied fuzzy inference machine assures smooth transition from one state to another. When temperature change (\( d\vartheta \)) is big it means that object is in “initial state”, while small changes are typical for “saturation state”. When \( d\vartheta \) is negative object is in cooling phase and when is positive object is in heating phase.

In the space of temperature changes two fuzzy sets are defined: small and big change in temperature. These sets are described by the formulas (5) and are shown in Fig. 1.

\[
\begin{align*}
\mu_{\text{small}}(d\vartheta) & = \begin{cases} 
1 & \text{for } d\vartheta \geq a \text{ and } d\vartheta \leq c \\
\frac{d\vartheta - b}{d\vartheta - a} & \text{for } d\vartheta > a \text{ and } d\vartheta < b \\
0 & \text{for } d\vartheta < a \text{ or } d\vartheta \geq b
\end{cases} \\
\mu_{\text{big}}(d\vartheta) & = \begin{cases} 
0 & \text{for } d\vartheta \geq b \text{ and } d\vartheta \leq c \\
\frac{d\vartheta - a}{d\vartheta - c} & \text{for } d\vartheta > c \text{ and } d\vartheta < b \\
1 & \text{for } d\vartheta < c \text{ or } d\vartheta \geq d
\end{cases}
\end{align*}
\]

where: \( \mu_{\text{small}}(d\vartheta) \) is membership function of the fuzzy set “small change in temperature”, \( \mu_{\text{big}}(d\vartheta) \) is membership function of the fuzzy set “big change in temperature”, \( d\vartheta \) is temperature difference.

It is then possible to formalize the qualitative description of thermal plant using a fuzzy model in the form of Takagi-Sugeno-Kanga system [4]. Fuzzy model diagram is shown in Fig. 2

**Optimization of model parameters**

Estimation of the parameters of this model using the least sum of squares requires solving the following optimization problems:

\[
IF d\vartheta \geq 0 \\
\vartheta_k = w_k[\mu_{\text{big}}(d\vartheta_k) \cdot \theta_{UP} + \mu_{\text{small}}(d\vartheta_k) \cdot \theta_{UP}] \\
ELSE \\
\vartheta_k = w_k[\mu_{\text{big}}(d\vartheta_k) \cdot \theta_{DOWN} + \mu_{\text{small}}(d\vartheta_k) \cdot \theta_{DOWN}]
\]

![Fig. 1. Fuzzy sets: small and big change in temperature](image)

![Fig. 2. Fuzzy model diagram](image)
For heating phase:
\[
\{c, d, N_{iUp}, N_{sUp}, K_{iUp}, K_{sUp}\} = \arg \min_n \sum_{k=1}^n (\vartheta_k - w_k [\mu_{big}(d\vartheta_k) \cdot \theta_{iUp} + \mu_{small}(d\vartheta_k) \cdot \theta_{sUp}])^2
\]

For cooling phase:
\[
\{a, b, N_{iDown}, N_{sDown}, K_{iDown}, K_{sDown}\} = \arg \min_n \sum_{k=1}^n (\vartheta_k - w_k [\mu_{big}(d\vartheta_k) \cdot \theta_{iDown} + \mu_{small}(d\vartheta_k) \cdot \theta_{sDown}])^2
\]

Since results of solution of (8) and (7) using Matlab lsqnonlin function were not satisfactory genetic optimization methods were adopted.

**Results**

In order to assess the adequacy of the proposed fuzzy model of thermal objects its results have been compared to real furnace temperature as well as to a single first order inertia transfer function. Measurements were performed on the industrial furnace of rating power of 10kW with insulation fiber.

Chosen examples of the step input responses of analyzed models are shown in Fig. 3.

**Fig. 3.** Comparison of temperature in real furnace, with inertia first order model and fuzzy model

The set of best values of the parameters:
\[
a = -0.18863, \quad b = -0.02024, \quad c = 0.064065, \quad d = 0.30448
\]
\[
N_{iUp} = 340.30, \quad N_{sUp} = 1036.78
\]
\[
K_{iUp} = 17.07, \quad K_{sUp} = 33.05
\]
\[
N_{iDown} = 1359.51, \quad N_{sDown} = 7503.54
\]
\[
K_{iDown} = 18.64, \quad K_{sDown} = 41.27
\]

The parameters of inertia first order model:
\[
N = 3221.85, \quad K = 57.75
\]

It is noticeable that the graphs of furnace temperature and those obtained from the fuzzy model (6) almost overlap whereas first order transfer function model gives unsatisfactory results. It proves that proposed fuzzy modeling can be a very promising approach to describe the properties of thermal devices with distributed parameters.

**Conclusions**

Fuzzy approach to modeling of dynamic properties of thermal systems has been proposed. In particular, by this approach distributed parameters of such a system and their influence on its dynamics can be taken into account. The proposed method allows the qualitative description of thermal plant to be formalized using Takagi-Sugeno-Kang fuzzy structure. The antecedents of IF-THEN rules describe different thermal states of insulation walls of the plant while their consequences realize first order inertia dynamics including both "initial" and "final" state.

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