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# Using of continuous wavelet transform for de-noising signals accompanying electrical treeing in epoxy resins

***Streszczenie**. W artykule zaprezentowanie zastosowanie metod redukcji szumu w sygnałach emisji akustycznej towarzyszących zjawisku drzewienia dielektryków stałych, w szczególności żywic epoksydowych, opartych na falkowej analizie sygnałów. Opisany został algorytm usuwania szumów z sygnału m. in. algorytm miękkiego oraz twardego progowania stworzonych przez Donoho i Johnsona. Wszystkie obliczenia wykonano w Matlabie z wykorzystaniem dodatku Wavelet Toolbox.* ***Zastosowanie metod redukcji szumu w sygnałach emisji akustycznej towarzyszących zjawisku drzewienia dielektryków stałych, w szczególności żywic epoksydowych***

***Abstract**. The following paper presents application of methods of noise reduction in acoustic emission signals, accompanying phenomenon electrical treeing of solid dielectric such as epoxy resin, based on time-frequency signal analysis. For signal estimation was applied method of soft and hard thresholding described by Donoho and Johnson. All calculations ware obtained with use of Matlab software, especially Wavelet Toolbox.*

**Keywords:** signal filtering, wavelet transform, CWT, electrical treeing, epoxy resins
**Słowa kluczowe**: filtrowanie sygnałów, transformata falkowa, drzewienie

## Introduction

Major factor in solid dielectrics, used as insulation for high-voltage devices, are partial discharges (PD). They are present on the surfaces of dielectrics or in their structures, causing deterioration of electrical insulation properties. With PD occurring in solid dielectrics is often linked to the electrical treeing process involving the formation of a conductive or semi-conductive dielectric channels, taking the shape of a tree. Tubular discharge causes the further development of the tree, leading eventually to low-resistance short circuit.

Current research on PD, in particular on electrical treeing phenomena relate to either the same process in modern insulating materials [1] or relating to the use new and improvement of well-known research methods [2]. One of the non-invasive methods of examination PD is AE method consists in analyzing the acoustic wave propagating in a discharge surroundings.

The authors of the article in their research involved the study of using an AE method in solid materials such as epoxy resins used for insulation of i.e. high voltage cables.

The paper presents an application of wavelet transform DWT for filtration recorded acoustic signals.

## Measurement stand and analysis

To study process of electrical treeing was used acoustic emission method (AE), involving the measurement, acquisition and analysis of acoustic signal accompanying the process. The measurement stand (Figure 1) and method of measurements was described in detail in [3] and [4].

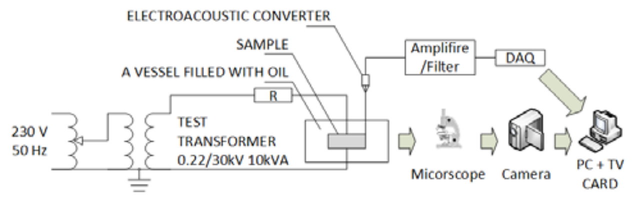


Fig.1. Measurement stand for AE signals accompanying electrical treeing [based on 1]

Characteristic features of studied phenomenon are small, comparable to noise level, amplitudes of vibrations. Therefore, prior to submission to the recorded signals analysis it is required to reduce the noise. The complexity of the process of formation of the acoustic wave, non-deterministic nature of its parameters and the presence of strong interference mean that it becomes necessary to use modern signal processing methods. The article presents the filtration method with the use of discrete wavelet analysis.

## Multiresolution signal decomposition

The idea of wavelet analysis is representation of signal in time-frequency (equivalent to time-scale) is described in equation:

$$c(a,t) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(\tau)\gamma^{*}\left(\frac{\tau - t}{a}\right) d\tau \,, \tag{1}$$

where: $a$ – scale, $x$ – signal in time domain, $t$ – time, $\gamma$ – wavelet function, $c$ - wavelet coefficient for scale-time domain.

In many cases for complete representation of underlying signal sufficient is wavelet transform calculated in discrete, dyadic domain of scale and time, such that:

$$t = n2^{-m} \quad \text{and} \quad a = 2^{-m} \,, \tag{2}$$

where: $m$ – discrete order of signal representation, $n$ - time shifting. The transformation described in such a discretized time-frequency domain is called Discrete Wavelet Transform (DWT) and is defind by:

$$x(t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} d_{m,n} g_{m,n}(t) \,, \tag{3}$$

where: $d_{m,n}$ – wavelet coefficient for $n$-$th$ time shifting and $m$-$th$ dilatation of wavelet function $g_{m,n}(t)$

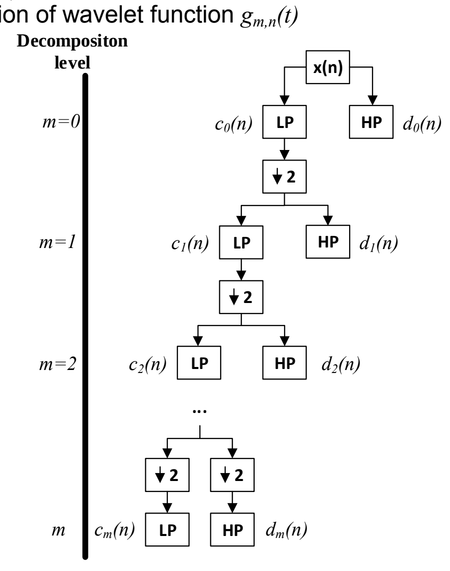


Fig. 2. Discrete Wavelet Transform algorithm

In this case signal on every discrete *m-th* level of decomposition, after $2^m$ times downsampling, can be separated to lowpass band and its complement to decomposition form lower level with use of pair of lowpass and highpass filter (Quadrature Mirror Filters QMF). Fig.2 shows algorithm of such a representation.

Described algorithm allows to introduce transform body at *k-th* level (for $k < m_0$) as a projection of the analyzed signal at the base of time shifted functions $\Phi(t)$ (hereinafter referred to as scaling functions) and their orthogonal complement - base functions $\psi(t)$[8]:

(4) $$x(t) = \sum_n c_{m_0,n}\phi(t)_{m_0,n} + \sum_{m=m_0}^{k-1}\sum_n d_{m,n}\psi(t)_{m,n},$$

where: $m$ – order of decomposition for base function, $m_0$ - order of decomposition for base function and:

(5.a) $$c_{m,n} = \int_{-\infty}^{\infty} x(t)^{m/2}\phi^*(2^m t - n)dt,$$

and:

(5.b) $$d_{m,n} = \int_{-\infty}^{\infty} x(t)^{m/2}\psi^*(2^m t - n)dt,$$

Frequency spectrum of scaling functions $\Phi(t)$ is low-pass, so coefficients $c_{m,n}$ contains low-pass part of signal. The function at the *k-th* level, acts as a low pass filter, the coefficients $h_0(k)$ is a linear combination of the scaling function from level $k + 1$:

(6) $$\phi(t) = \sum_k h_0(k)\sqrt{2}\phi(2t-k),$$

Frequency spectrum of wavelet function $\psi(t)$ is band-pass filter (witch completes scaling function spectrum). It can be shown, that wavelet function at level k can interpreted as high-pass filter with coefficients $h_1(k)$, which are linear combination of wavelet functions at lower than k levels:

(7) $$\psi(t) = \sum_k h_1(k)\sqrt{2}\phi(2t-k),$$

Such a multi-frequency signal representation allows to separate from signal generally (low frequency signal), and a detail (approximations signal for *k-th* order). Figure 3 shows part of recorded signal (a) and its FFT (b). Figure 4 shows five level decomposition of signal form Figure 3.

**DWT signal denoising**

For further consideration let be recorded signal as equation:

(8) $$x(t) = v(t) + e(t),$$

where: $x(t)$ – recorded signal, $v(t)$ – unknown real value of measured AE signal, $e(t)$ – difference between recorded signal and real value (measurements error). Figure 3 shows samples of measured signal and its Fast Fourier Transform. It clearly shown that recorded signal is very noised (SNR=5,55 – see summery). Frequency of AE signal is between 15 and 40 kHz, rest part of signal is white noise.

The wavelet approximation of measured signal from equation (1) can be expressed as:

(9) $$\langle x, g_k\rangle = \langle v, g_k\rangle + \langle e, g_k\rangle,$$

where: $\langle x,g_k\rangle$ - wavelet transform of measured signal, $\langle v,g_k\rangle$ - wavelet transform of AE signal, $\langle e,g_k\rangle$ - wavelet transform of error (noise) signal, $g_k$ – orthogonal base of functions at *k-th* order.

In order to de-noise signal we can define estimator of AE signal in wavelet domain:

(10) $$\hat{X} = \sum_{k=0}^{N-1}\langle x, g_k\rangle\theta[k],$$

where – $\theta[k]$ – adaptively selected, for each implementation of the signal $v[n]$, coefficient of value from set {0,1}.

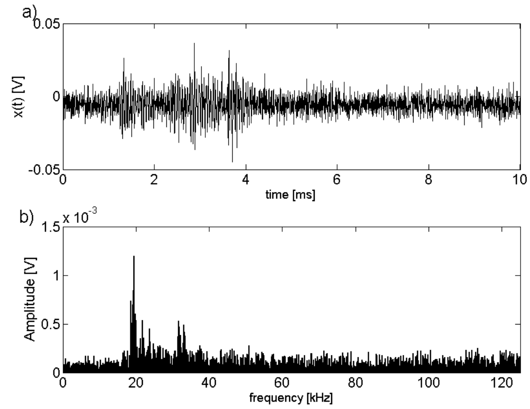


Fig.3. Recorded signal: a) time domain b) frequency domain (FFT)

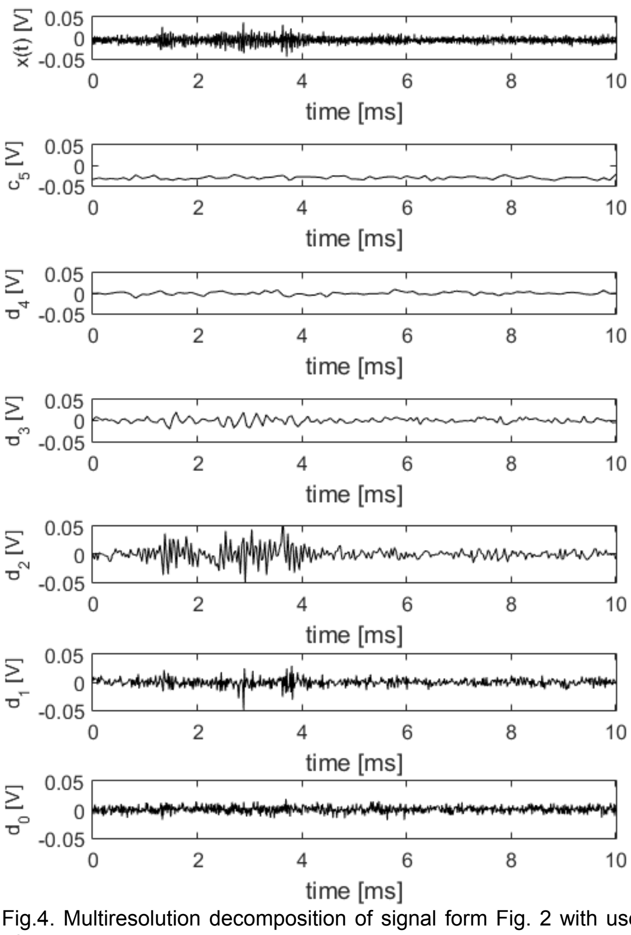


Fig.4. Multiresolution decomposition of signal form Fig. 2 with use of Daubechies wavelet (denoted as 'db10' in Matlab).

Such a definition of estimator eliminates from recorded signal selected *k-th* order decomposition (for $\theta[k]$ equal to zero). It now remains to specify the criterion according to which are assigned $\theta[k]$ values for the *k-th* order of decomposition, thus criteria for the *k-th* decomposition. For this purpose a mean square error estimation of signal $v[n]$ described the formula:

(11) $$\varepsilon = \mathbf{E}\left\{\left|v - \hat{X}\right|^2\right\} = \sum_{k=1}^{N}\mathbf{E}\left\{\left|\langle v, g_k\rangle - \langle x, g_k\rangle\theta[k]\right|^2\right\},$$

Additionally insert to the formula (8) noise variance equal to:

$$\sigma^2 = \mathbf{E}\left\{ \left| \left\langle sz, g_k \right\rangle \right|^2 \right\}, \tag{12}$$

and insert relationship (9). Then the final dependence for mean square error is:

$$\varepsilon = \sum_{k=1}^{N} \left| \left\langle v, g_k \right\rangle \right|^2 \left(1-\theta[k]\right)^2 + \left(\sigma\theta[k]\right)^2 , \tag{13}$$

To minimize mean square error, $\theta[k]$ derivative of (11) must be equal to zero for each order $k$:

$$\frac{\partial \mathbf{E}}{\partial \theta[k]} = \frac{\partial \left( \left| \left\langle v, g_k \right\rangle \right|^2 \left(1-\theta[k]\right)^2 + \left(\sigma\theta[k]\right)^2 \right)}{\partial \theta[k]} = 0 , \tag{14}$$

After simplify equation (12) give formula for $\theta[k]$ at $k$-$th$ order:

$$\theta[k] = \frac{\left| \left\langle v, g_k \right\rangle \right|^2}{\left| \left\langle v, g_k \right\rangle \right|^2 + \sigma^2} , \tag{15}$$

hence:

$$\theta[k] = \begin{cases} 1 \quad \text{when} \quad \left| \left\langle v, g_k \right\rangle \right|^2 > \sigma^2 \\ 0 \quad \text{when} \quad \left| \left\langle v, g_k \right\rangle \right|^2 \le \sigma^2 \end{cases} , \tag{16}$$

**Wavelet signal approximation with method of soft and hard thresholding**

Presented earlier estimation method makes during implementation difficulties, mainly from ignorance of functions $v[n]$, resulting from the variability of the acoustic wave produced during electrical treeing. In addition, the algorithm removes, through the function of $\theta[k]$, the selected signal decomposition. More accurate noise reduction can be achieved applying filtration method described by D. L. Donoho and I. M. Johnstone in [3, 4]. These authors report a method of estimating the signal from the noisy samples of the use of certain relationships:

$$\hat{X} = \sum_{k=0}^{N-1} \left\langle x, g_k \right\rangle \theta\left( \left\langle x, g_k \right\rangle \right), \tag{17}$$

The solution is to correct each of the wavelet coefficients by means of a non-linear wavelet shrinkage function whose value depends on the wavelet coefficients values.

Donoho and Johnston in [6], in addition to the criterion of minimum mean square error introduced the smoothness of the functions using the projection functions for Besova functional space. The authors proposed two variants of wavelet shrinkage functions. First variant called *soft thresholding* function:

$$\theta\left( \left\langle x, g_k \right\rangle \right) = \begin{cases} \left\langle x, g_k \right\rangle - \lambda & for \quad \left\langle x, g_k \right\rangle \ge \lambda \\ \left\langle x, g_k \right\rangle + \lambda & for \quad \left\langle x, g_k \right\rangle < -\lambda \\ 0 & for \quad |x| < \lambda \end{cases} , \tag{18}$$

and second *hard thresholding* function:

$$\theta\left( \left\langle x, g_k \right\rangle \right) = \begin{cases} \left\langle x, g_k \right\rangle & for \quad \left| \left\langle x, g_k \right\rangle \right| > \lambda \\ 0 & for \quad \left| \left\langle x, g_k \right\rangle \right| \le \lambda \end{cases} , \tag{19}$$

where: $\lambda$ – of threshold level.

Figures 5a and 5b show an example of a plot for threshold function of soft and hard thresholding function, depending on a value of wavelet coefficient.

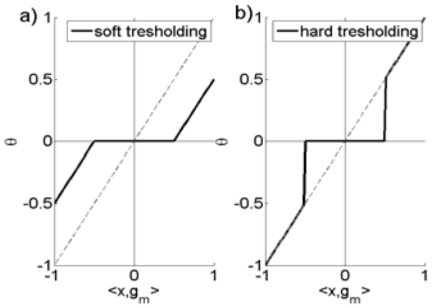


Fig.5. Example of wavelet shrinking function $\theta$ for: a) soft thresholding b) hard thresholding.

The primary determinant of the quality described above methods for noise reduction is the choice of an appropriate threshold level for each signal. The authors of soft and hard thresholding methods proposed in [6] an algorithm called universal trigger. The trigger level is determined by using the formula

$$\lambda = \sigma \sqrt{2\log(N)/N} , \tag{20}$$

where: $\lambda$ – threshold level, $\sigma$ – noise level, $N$ – length of input signal (number of samples).

Thus, clearly showing that the quality of the algorithm depends mainly on the length of the analyzed signal samples. For the analysis of a small sample quantities method may become insufficient.

In [10] Luisier et al proposed shrinkage function (*SURE-shrink function*) defined as:

$$\theta\left( \left\langle x, g_k \right\rangle \right) = \sum_{k=1}^{K} a_k \varphi_k \left( \left\langle x, g_k \right\rangle \right), \tag{21}$$

where: $K$ – number of terms, $a_k$ – parameters and $\varphi_k$ defined as:

$$\varphi_k(y) = y \exp\left( -(k-1) y^2 / 12\sigma^2 \right), \tag{22}$$

It is shown that value of $K \geq 2$ results of shrinking function is quite similar [9].In practical application clear signal wavelet coefficients are unknown. In described algorithm $a_k$ parameters are determine with use of Stein's Unbiased Risk Estimator (SURE) as mean squared error (MSE) estimator.

$$\hat{\varepsilon} = \frac{1}{N} \sum_{n=1}^{N} \theta^2\left( \left\langle x, g_k \right\rangle \right) - 2\left\langle x, g_k \right\rangle \theta\left( \left\langle x, g_k \right\rangle \right) + \\ + 2\sigma^2 \theta'\left( \left\langle x, g_k \right\rangle \right) , \tag{23}$$

Figure 6 shows part of recorded noised signal and estimations of clear signal with used of described in article algorithms. For validation of presented methods SNR was calculated as:

$$SNR = 10\log\left( \frac{\sigma_x^2 - \sigma_n^2}{\sigma_n^2} \right), \tag{25}$$

where: $\sigma_x$ – signal variance, $\sigma_n$ –noise signal variance.

To get SNR value from recorded signal, noise variance must be estimated. In [10] a robust estimate of $\sigma_n$ is given by:

$$\hat{\sigma}_n = \frac{MAD}{0.6745} \sqrt{\frac{2p}{2p+1}} , \tag{26}$$

where MAD is median absolute deviation of finest wavelet coefficients.

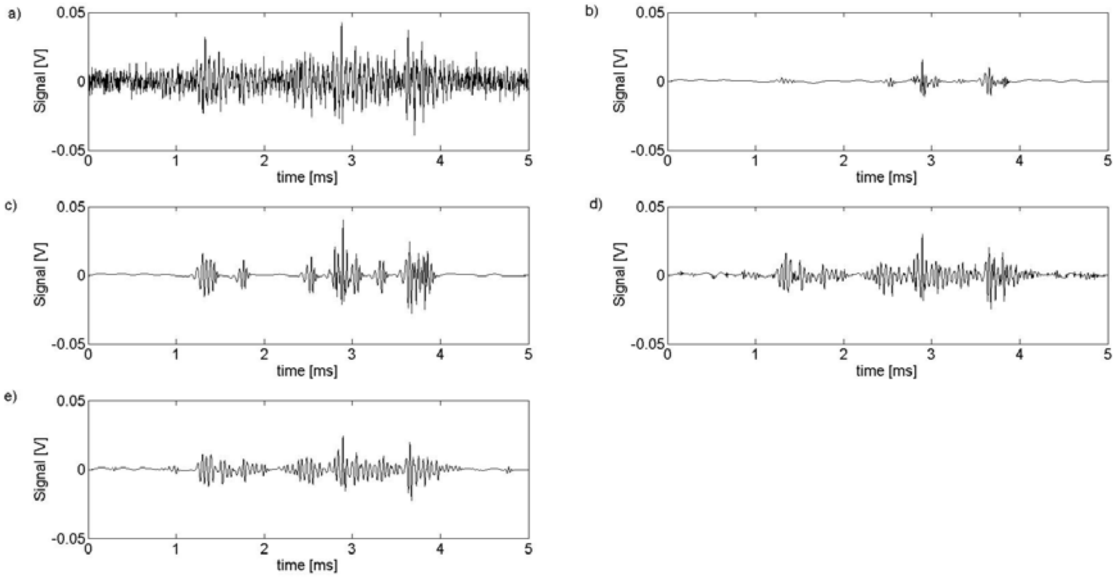


Fig.6. Example of signal estimation for described algorithms: a) noised signal, b) soft thresholding with universal threshold, c) hard thresholding with universal threshold, d) SURE-shrinkage function, e) heuristic variant of SURE-shrinkage function.

Table 1 shows SNR value estimated with for signals from figure 6.

Table 1 Comparison of estimated SNR value for wavelet shrinkage methods.

| Signal | SNR [dB] |
|---|---|
| Noised | 5,5519 |
| *Soft Thresholding* | 6,0317 |
| *Hard Thresholding* (Universal Treshold) | 10,927 |
| *SURE-shrink* | 9,7977 |
| *SURE-shrink* - heuristic variant | 9,1974 |

**Summary and comments**

The article presents application for discrete wavelet transform in study on PD in epoxy resins. It is clearly shown that wavelet shrinkage methods give good result for AE signal noise reduction. In studied example best algorithm gives almost two times higher SNR value.

Described in the paper standard method of hard thresholding with universal threshold gives best results, better than *SURE-shrink* functions. It can be explained that in this application, when sampling frequencies are much higher than frequencies of AE signals in studied phenomena, high resolution of signal gives possibility to get high samples number $N$ for decomposition. In presented example $N$ = 1250 samples.

In conclusion, it should be noted that the solution consisting in the analysis of signals with low SNR requires analysis and simulation. These studies are designed to determine the best methods to minimize the impact of disruption on the result for the issues under consideration. In the present case, the AE signals associated with the electrical treeing process are a type of relaxation with amplitudes comparable to the noise, the best proved to be one of the basic methods for removing noise.

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