

Application of artificial bee colony algorithm to auto-tuning of linear-quadratic regulator for PMSM position control

Abstract. In this paper auto-tuning process of linear-quadratic regulator (LQR) for position control of permanent magnet synchronous motor (PMSM) is presented. The novelty of the proposed solution lies in use of artificial bee colony (ABC) optimization algorithm to calculate gain coefficients of controller. In order to maintain selected state and control variables of drive in a permissible level, the ABC algorithm has been extended for solving constrained optimization problem. Proper operation of auto-tuning procedure was investigated by numerical simulations.

Streszczenie. W artykule przedstawiono proces samostrojzenia regulatora liniowo-kwadratowego przeznaczonego do regulacji położenia silnika synchronicznego o magnesach trwałych. Oryginalność proponowanego rozwiązania polega na zastosowaniu algorytmu optymalizacyjnego sztucznej kolonii pszczoł do wyznaczenia współczynników wzmocnień regulatora. W celu utrzymania wartości wybranych zmiennych stanu i sygnałów sterujących w dopuszczalnym zakresie, algorytm optymalizacyjny rozszerzono o możliwość rozwiązywania problemów optymalizacyjnych z ograniczeniami. Poprawne działanie algorytmu automatycznego strojenia zostało zbadane w testach symulacyjnych (**Zastosowanie algorytmu sztucznej kolonii pszczoł do samostrojzenia regulatora liniowo-kwadratowego sterującego położeniem PMSM**).

Keywords: linear-quadratic regulator, artificial bee colony algorithm, auto-tuning, PMSM.

Słowa kluczowe: regulator liniowo-kwadratowy, algorytm sztucznej kolonii pszczoł, samostrojzenie, PMSM.

Introduction

Because of its excellent dynamic behavior and compact structure, permanent magnet synchronous motors (PMSM) are widely used in motion control applications such as industrial robots and machine tools [1]. A new field of application for PMSM is to use it in automotive: as an auxiliary drive in electric power steering (EPS) or in heating, ventilating, air conditioning (HVAC) applications [2, 3].

Position control of servo-drive with PMSM is usually realized by using cascade control structure with PID type controllers [4]. In order to ensure better properties of the drive (i.e. robustness, disturbance compensation), advanced control techniques such as: fuzzy logic and neural networks can be used in cascade control structure [5, 6].

Other approach to position control of servo-drive with PMSM is to utilize state feedback controller. In the structure mentioned above, one controller for selected state variables is designed. In spite of a few advantages of state feedback control approach (i.e. non-linearity tolerance [7] and superior disturbance compensation [8]), the main drawbacks are: limitation of state and control variables and determination of controller coefficients. Good knowledge of PMSM mathematical model causes, model predictive approach to constraints introduction (MPAC) can be used to cope with the constraints [8]. Coefficients of state feedback controller can be determined by using pole placement technique or linear-quadratic optimization method. In the first approach positions of closed loop poles have to be determined, while in the second method values of the weighting matrices should be selected. Currently the most popular method to obtain positions of the poles or values of weighting matrices is trial-and-error procedure. On the other hand computer-aided optimization methods such as: genetic algorithm (GA) [9] or particle swarm optimization (PSO) [10] can be applied. One of relatively new optimization method is the artificial bee colony algorithm (ABC) proposed by Karaboga [11] and applied to solving multimodal numeric problems [12]. A very good performance and low computational effort of the ABC in comparison to PSA and GA reported in [12] causes that mentioned algorithm seems to be a promising approach for determination of state feedback controller coefficients.

Our goal is to adapt the ABC optimization algorithm for auto-tuning process of state feedback position controller. Firstly, performance index for minimization will be selected. Next, the ABC algorithm will be modified for solving constrained optimization problems, in order to maintain selected control and state variables of drive in permissible level. Finally, calculations of linear-quadratic regulator and step response of simplified model of drive will be incorporated with optimization procedure.

Model of the plant

Synthesis process of linear-quadratic regulator (LQR) requires state-space representation of the plant. In the proposed approach mechanical variables of the PMSM (i.e. angular speed and position) are controlled with the help of state feedback controller while internal model control (IMC) with PI controllers is used in a current control loop [13]. The simplified model of the mechanical part of PMSM is as follows

$$(1) \quad \frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} + \mathbf{B}u,$$

with:

$$(2) \quad \mathbf{x} = \begin{bmatrix} \omega_m \\ \theta \end{bmatrix}, \mathbf{A} = \begin{bmatrix} -\frac{B_m}{J_m} & 0 \\ 1 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \frac{K_t}{J_m} \\ 0 \end{bmatrix}, u = i_q,$$

where: ω_m – angular speed, θ – angular position, J_m – moment of inertia, B_m – viscous friction, K_t – torque constant, i_q – q -axis current. During modeling process it was assumed that: (i) an external load torque can be neglected and (ii) the dynamics of the current control loop can be omitted. The second assumption is valid for sufficiently small rising time of current step response in comparison to speed step response of the motor.

In order to control angular position of the PMSM without steady-state error (in a case of step variations of the reference angular position and load torque), an internal model of reference input should be introduced [14]. An augmented state equation takes the following form

$$(3) \quad \frac{d\mathbf{x}_i}{dt} = \mathbf{A}_i\mathbf{x}_i + \mathbf{B}_i u + \mathbf{F}r,$$

where:

$$(4) \quad \mathbf{x}_i = \begin{bmatrix} \omega_m \\ \theta \\ e_\theta \end{bmatrix}, \mathbf{A}_i = \begin{bmatrix} -\frac{B_m}{J_m} & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \mathbf{B}_i = \begin{bmatrix} \frac{K_t}{J_m} \\ 0 \\ 0 \end{bmatrix},$$

$$\mathbf{F}^T = [0 \quad 0 \quad -1], r = \theta_{ref}$$

The new state variable e_θ introduced in the augmented state equation (3) corresponds to the integral of the position error as follows

$$(5) \quad e_\theta = \int_0^t [\theta(\tau) - \theta_{ref}(\tau)] d\tau$$

Linear-quadratic regulator

Synthesis procedure of position controller can be divided into two main tasks: (i) determination of controller structure, and (ii) calculation of controller coefficients. The control law for a system described by augmented state equation (3) can be calculated from the following formula

$$(6) \quad u = -\mathbf{K}_i \mathbf{x}_i = -\mathbf{K}_x \mathbf{x} - K_e e_\theta,$$

where: \mathbf{K}_i , \mathbf{K}_x are gain matrices of state feedback controller. An additional gain coefficient K_e was introduced into control law (6) to assure zero steady-state position error. Schematic diagram of the considered control system with state feedback position controller is shown in Fig. 1.

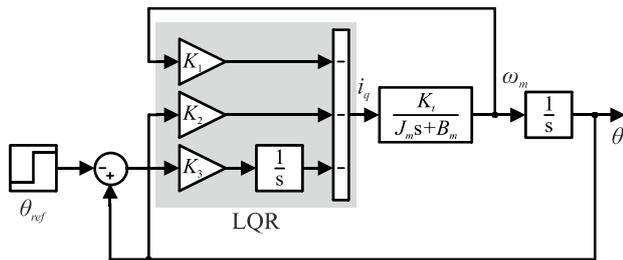


Fig. 1. Block diagram of simplified control system

Gain coefficients of state feedback controller can be determined with the help of pole placement technique [14] or linear-quadratic optimization [7]. In the presented approach the optimization method is employed. In such a case weighting matrices \mathbf{Q} and R that minimizes the performance index

$$(7) \quad J = \int_0^t (\mathbf{x}_i^T \mathbf{Q} \mathbf{x}_i + u^T R u) dt,$$

have to be determined. For a system depicted above (3) the dimension of weighting matrices are as follows

$$(8) \quad \mathbf{Q} = \text{diag}([q_1 \quad q_2 \quad q_3]), \quad R = r_1$$

The most popular tuning approach for determination gain coefficients is based on the trial-and-error procedure. In this paper, coefficients of LQR will be determined by applying computer aided optimization algorithm.

Artificial bee colony algorithm

The artificial bee colony (ABC) optimization algorithm was described by Karaboga in 2005 [11]. It is based on the foraging behavior of honey bees that can be divided into three groups: employed bees, onlookers and scouts. Typically, the first half of the colony consists of the

onlookers and the second one of employed bees. Employed bees are responsible for visiting food sources, onlooker bees make the decision to choose a food source, while scouts are responsible for random search of a new food sources. The position of a food source is a potential solution of the optimization problem. The fitness of the associated solution is described by the nectar amount of a food source.

An employed bee generates a modification of the position on the basis of the local information about fitness value of the potential source (new solution). The bee forgets the old position and memorizes the new one if the new fitness value of potential solution is higher. At the end of search process realized by all employed bees, an information about fitness and position of the food sources is shared with onlooker bees. Knowledge about fitness of all food sources is used by onlookers during selection procedure. It is made on the basis of the probability value associated with that food source. When a food source is abandoned by the bees, a scout bee randomly determines a new one. The employed bees from abandoned sources become scouts.

The main parameter associated with ABC algorithm is the number of colony size – NP . It is equal to the sum of employed and onlooker bees. Initially, the number of food sources is equal to the half of NP . Each potential solution is represented as a D -dimensional vector, where D is the number of optimization parameters [11]. Initially, the lower (lb) and the upper bounds (ub) of D are also defined.

An important control parameter of the ABC algorithm, is the predetermined number of cycles called *limit* [11]. If a better solution associated with the food source cannot be found before iteration counter reaches *limit*, the food source is abandoned. Typically, *limit* value is a multiple of a food number [11]. More detailed information and performance evaluation of ABC optimization algorithm during solving multimodal numeric problems can be found in several publications [11, 12, 15].

An application of ABC for calculation of LQR coefficients

In order to apply ABC algorithm to determine coefficients of linear-quadratic regulator, auto-tuning procedure have been incorporated with optimization process. Firstly, performance index for minimization was selected. In this approach an integral of the time multiplied by absolute position error (ITAE) performance index is minimized. Next, weighting matrices (8) were chosen as parameters modified during optimization procedure.

For proper operation of electrical servo-drive, phase currents and angular speed of the motor should be maintained in a permissible level. Because of this, auto-tuning of LQR should be treated as a constrained optimization problem. As it was shown in [15], the ABC algorithm can be successfully adopted for solving constrained optimization problems. The main modifications of ABC algorithm are stated below.

Contrary to unconstrained version of ABC algorithm, where only one randomly chosen parameter in employed bees phase is modified, here the modification of parameter depends on the relationship between a uniformly distributed random real number ($0 \leq rm \leq 1$) and the modification rate (MR) control parameter. In addition the requirement of modification of at least one parameter has to be met [15].

Selection process of modified ABC algorithm is realized with the help of constrained handling method. In the proposed approach Deb's rules are employed. This method uses a tournament selection parameter, where two solutions are compared by using the following criteria [15, 16]:

- for two feasible solutions, the one having better objective function value is chosen,

- any feasible solution ($violation \leq 0$) is preferred to any infeasible solution ($violation > 0$),
- for two infeasible solutions, the one having smaller constraint violation is chosen.

Since two signals are limited in our case, $violation$ parameter is calculated as a maximum value of violations obtained for normalized q -axis current and angular speed. Method of calculation $violation$ parameter proposed above guarantees that limits of signals are not exceeded.

At the end of searching process realized by employed bees, information about fitness and position of all food sources is shared with the onlooker bees. It is used to calculate the probability values as follows [15]:

$$(9) \quad p_i = \begin{cases} 0.5 + \left(\frac{fitness_i}{\sum_{j=1}^{NP} fitness_j} \right) \times 0.5 & \text{for feasible solution} \\ 1 - \left(\frac{violation_i}{\sum_{j=1}^{NP} violation_j} \right) \times 0.5 & \text{for infeasible solution} \end{cases}$$

with: $violation_i$ – the penalty value of i -th solution, $fitness_i$ – the fitness value of i solution (it is proportional to the quality of food source):

$$(10) \quad fitness_i = \begin{cases} 1/(1+f_i) & \text{if } f_i \geq 0 \\ 1+abs(f_i) & \text{if } f_i < 0 \end{cases}$$

where: f_i – the cost value (ITAE) of i -th solution.

The last difference between constrained and unconstrained ABC algorithms consists in the scouts production. In this case scouts are produced at predetermined period of cycles named scout production period (SPP). As it was stated in [15], scout production process is carried out if there is an abandoned food source exceeding $limit$ for SPP cycle.

Block diagram of constrained ABC algorithm used for auto-tuning of LQR is shown in Fig. 3. As it can be seen in Fig.3, in each phase of ABC algorithm *the main calculations* block exist. Its content is presented in Fig. 2.

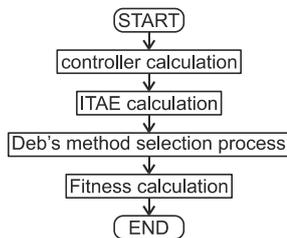


Fig.2. Content of the main calculation block

In order to obtain gain coefficients of state feedback controller, the Matlab lqr function is used. Values of penalty matrices (8) are provided by optimization algorithm. Next, state feedback model of the simplified control system (Fig. 1) is applied to determine values of state variables. The step response of the model is obtained with the help of the Matlab $lsim$ function. Values of reference and output position are used to calculate ITAE performance index while q -axis current and angular speed are examined in a Deb's method selection process. Finally, fitness value is calculated on the basis of (10).

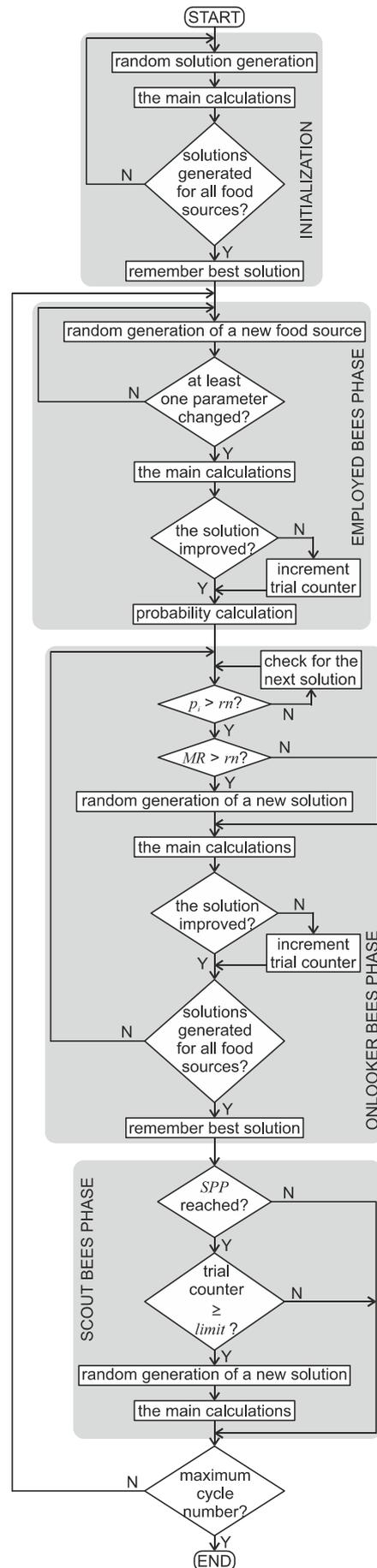


Fig.3. Block diagram of constrained ABC

Numerical experiments

Efficiency of the proposed ABC optimization algorithm was investigated on two models of PMSM drive. The main parameters of drives are listed in Table 1.

Table 1. The basic parameters of drives

Parameter	Symbol	Unit	Drive-1	Drive-2
PMSM power	P_N	kW	0.6	2.7
PMSM rated current	I_N	A	3	5.8
PMSM resistance	R_s	Ω	0.85	1.05
PMSM inductance	L_s	mH	4	12.7
PMSM inertia	J_m	kgm^2	1×10^{-4}	6.2×10^{-4}
PMSM viscous friction	B_m	Nms/rad	1.1×10^{-3}	1.4×10^{-3}
PMSM no of pole pairs	p	–	3	3
PMSM torque constant	K_t	Nm/A	0.35	1.64
DC-link voltage	U_{dc}	V	325	560
PWM frequency	f_s	kHz	16	10
Sampling period	T_s	μs	62.5	100

A few control parameters of ABC optimization algorithm have to be set at the beginning of auto-tuning procedure. For two models of the drive almost the same sets of control parameters were used. The only difference is a lower boundary value lb_4 that corresponds to R weighting matrix minimum value. Value of lb_4 chosen for Drive-1 is 1×10^{-3} while for Drive-2 is equal to 1. During simulation tests of Drive-2 it was observed that value of lb_4 (an in consequence value of R) lower than 1 causes rapid changes of PMSM q -axis current. For illustrative reason, different values of PMSM limits were selected. Maximum value of Drive-1 angular velocity was set to $\omega_N = 300$ rad/s while of Drive-2 $\omega_N = 180$ rad/s. Maximum values of q -axis current are equal to its rated values I_N respectively (see Table 1). Control parameters of ABC optimization algorithm that are common for two drives are summarized in Table 2.

Table 2. Control parameters of ABC optimization algorithm

Parameter	Symbol	Value
The number of optimized parameters	D	4
The number of colony size	NP	20
The number of food sources	FN	$NP/2$
Maximum number of cycles	MCN	60
Control parameter	$limit$	$4 \times FN$
Scout production period	SPP	$4 \times FN$
Modification rate control parameter	MR	0.8
Lower bounds of parameters	$lb_1 \div lb_3$	1×10^{-3}
Upper bounds of parameters	$ub_1 \div ub_4$	1×10^4

Similarly, to the study presented in [11], the percentage of employed bees was 50% of NP and it is equal to the percentage of onlooker bees. The number of scout bees was chosen as one.

Evolution of ITAE performance index observed for Drive-1 during auto-tuning procedure is presented in Fig. 4. Note, that after 13 iterations, ITAE changes slightly. The final value of performance index is equal to 0.0172.

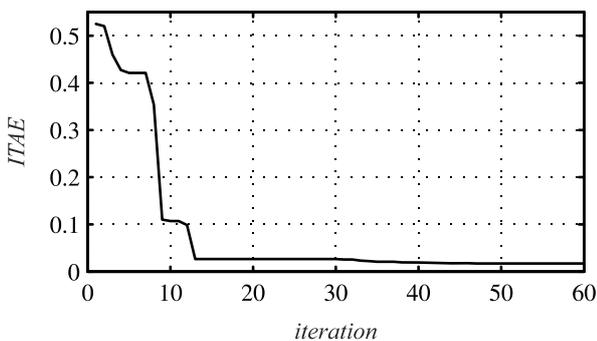


Fig.4. Evolution of ITAE for Drive-1.

Efficiency of ABC based auto-tuning process was examined in Matlab 2010a installed on PC with Core 2 Duo T9300 @ 2,5 GHz with 4GB memory. The total time required to complete 60 iterations is equal to 46 s.

Evolution of Drive-1 angular position during auto-tuning process is shown in Fig. 5. Since initial solution is generated randomly, very long settling time of the first step response is observed. The final values of linear-quadratic regulator coefficients are as follows:

$$(11) \quad \mathbf{K}_i = [0.0321 \quad 1.8698 \quad 40.6284]$$

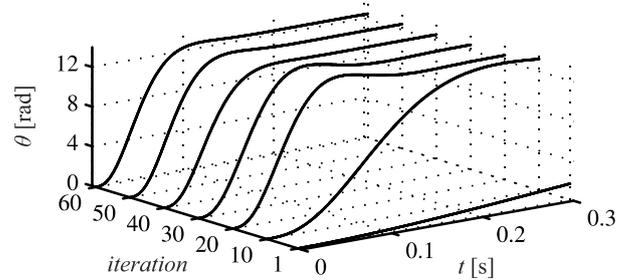


Fig.5. Evolution of Drive-1 angular position

In order to evaluate performance of the proposed optimization algorithm, simulation of the simplified control system shown in Fig. 1. was accomplished in Matlab/Simulink environment. The dynamical behavior of Drive-1 with auto-tuned linear-quadratic regulator is presented in Fig. 6.

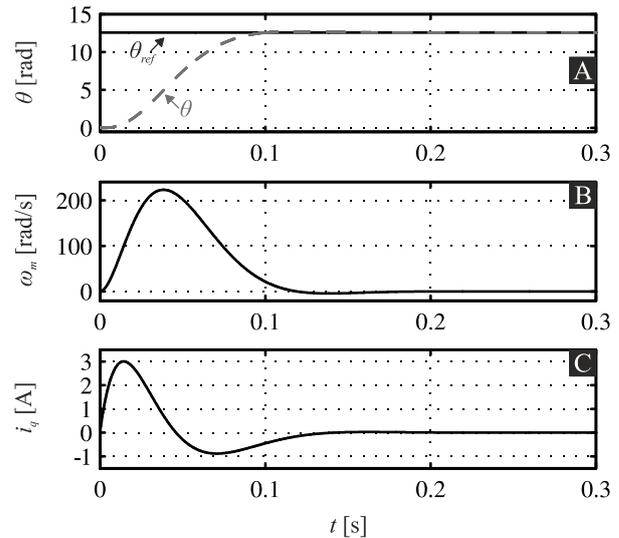


Fig.6. Behaviour of simplified model of Drive-1.

From Fig. 6.A it can be seen, that angular position is controlled properly, without steady-state error and overshoot. Proper operation of the constrained ABC algorithm can be observed in Fig. 6.C – maximum value of q -axis current is equal its rated value. In such a case boundary value of current determinates dynamic behavior of the PMSM.

Evolution of ITAE performance index observed for Drive-2 during auto-tuning procedure is presented in Fig. 7. In this case, significant changes of ITAE are observed in the first half of optimization process. The final value of performance index is equal to 0.0247. The total time required to complete 60 iterations is equal to 18 s. More than 2 times shorter value obtained for Drive-2 is caused by higher step size used in $lsim$ function during auto-tuning

process. Since mechanical time constant of Drive-1 is smaller, lower value of step size parameter should be used.

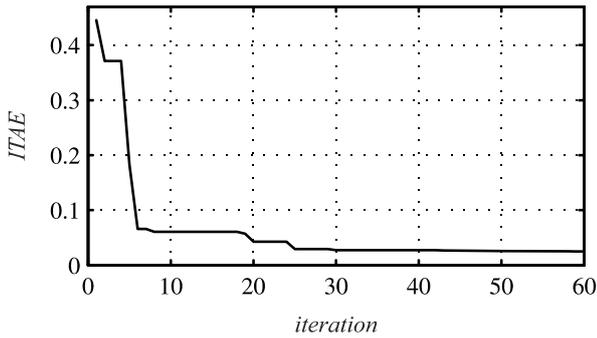


Fig.7. Evolution of ITAE for Drive-2.

Evolution of Drive-2 angular position during auto-tuning procedure is shown in Fig. 8. Similarly to auto-tuning of Drive-1, very long settling time of the first step response is observed due to randomly generated initial solution. The final values of linear-quadratic regulator coefficients for Drive-2 are as follows:

$$(12) \quad \mathbf{K}_i = [0.1038 \quad 4.9133 \quad 96.111]$$

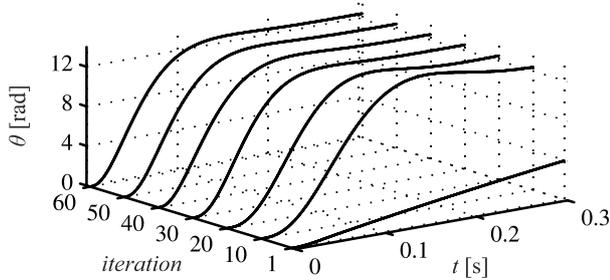


Fig.8. Evolution of Drive-2 angular position

Simulation results of simplified control system (Fig. 1.) are presented in Fig. 9. An angular position of Drive-2 shown in Fig. 9.A is controlled properly. The steady-state error is not present. Contrary to the situation observed for Drive-1, in this case angular speed of PMSM (Fig. 9.B) reaches its maximum value. It proves that the constrained ABC algorithm with Deb's method selection process works properly.

In order to evaluate designed linear-quadratic regulators in a more precise way, detailed model of a drive was implemented in Matlab/Simulink/Plecs environment (Fig. 10).

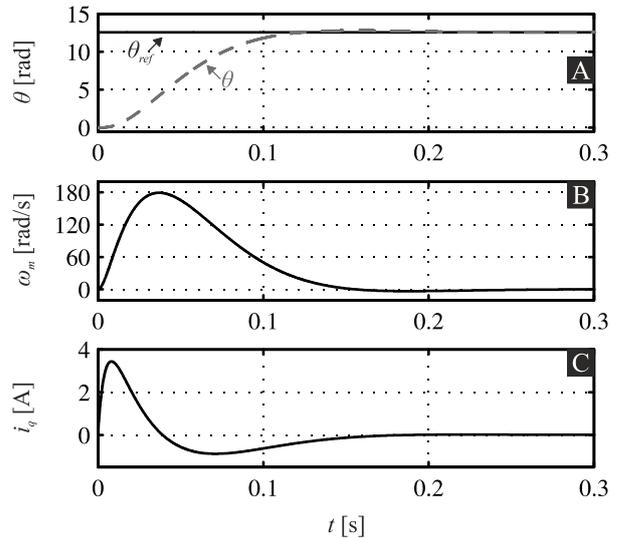


Fig.9. Behaviour of simplified model of Drive-2.

Model of the 2-level voltage source inverter (VSI) and PMSM was developed using PLECS blockset. Carrier-based sinusoidal modulation (CB-SPWM) with zero sequence signal (ZSS) is applied in the control system [13]. An internal model control (IMC) theory is utilized to design PI controllers in current control loop [13]. The rise time of Drive-1 current control loop was set to $t_r = 500 \mu\text{s}$. For Drive-2 the rise time is equal $t_r = 2 \text{ ms}$ respectively. Since model of PMSM is non-linear [14], feedback linearization method has been implemented in decoupling block. Detailed information about linearization method used can be found in [14]. In order to ensure proper generation of control signals, discrete forms of controllers (i.e. LQR and IMC) were implemented in triggered subsystems. Triggered synchronization block was used to ensure measurements in a midpoint of the PWM pulse length. The PWM frequency as well as the sampling period chosen for Drive-1 and for Drive-2 were summarized in Table 1.

As a reference signal $\theta_{ref} = 4\pi \text{ rad}$ was used. For a comparative purposes, value of reference angular position is the same as in auto-tuning process. Since the proposed control structure operates without constraints of state and control variables, for larger value of reference signal, current and angular velocity could exceed its rated values. In order to cope with the constraints, additional modifications of control algorithm should be performed (e.g. proposed in [8] model predictive approach to constraints introduction (MPAC) could be applied), but those are beyond the scope of this paper.

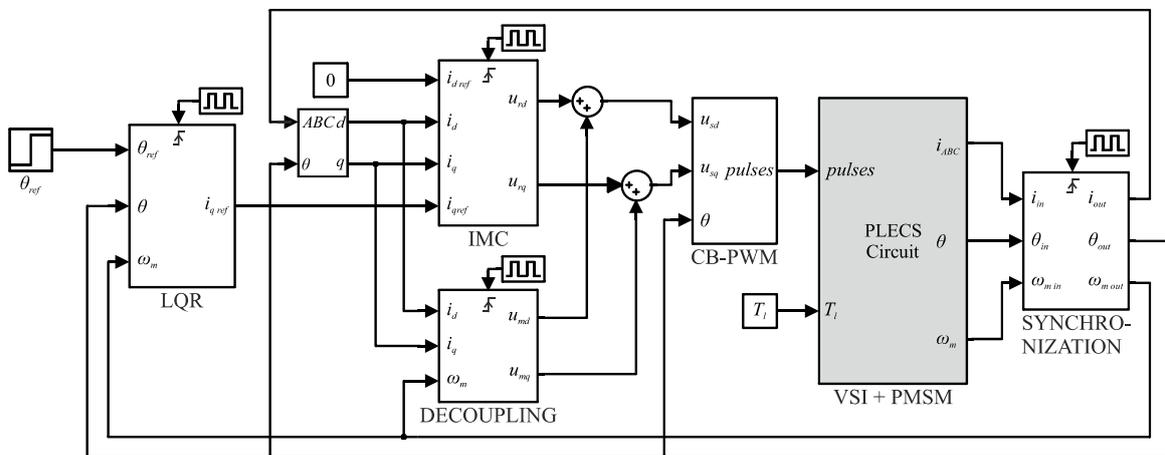


Fig.10. Schematic diagram of PMSM fed by VSI

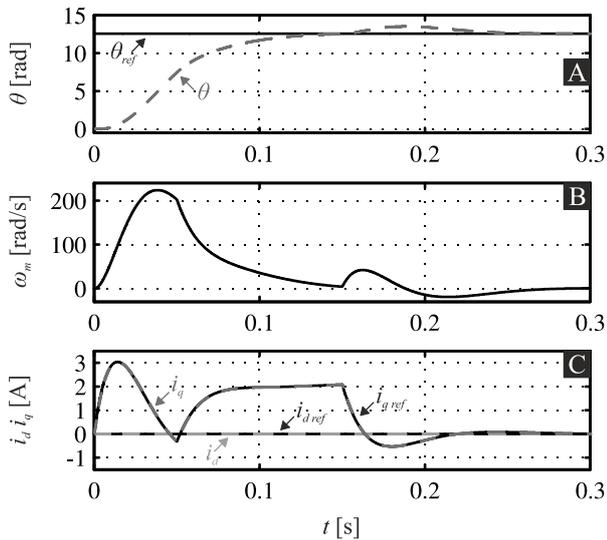


Fig.11. Behavior of detailed model of Drive-1

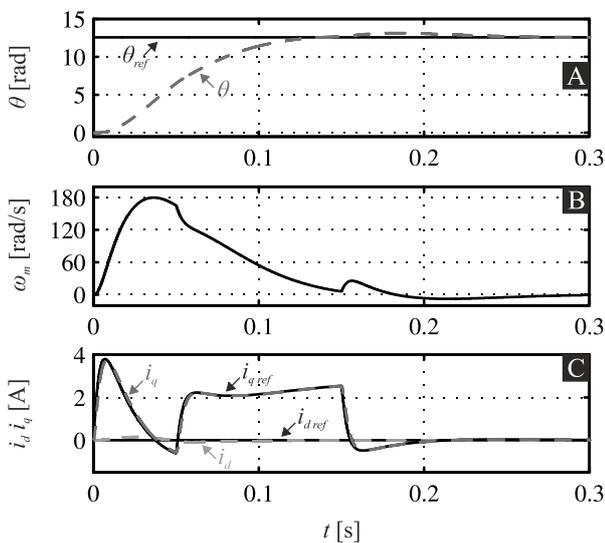


Fig.12. Behavior of detailed model of Drive-2

Dynamical behavior of drive was examined with presence of external disturbance.

Simulation results of Drive-1 are shown in Fig. 11. It should be noted, that angular position of the drive is controlled without steady-state error. Transient position error caused by step change of load torque ($T_l = 0.75$ Nm for $t \in <50; 150>$ ms) is eliminated properly. Shown in Fig. 11.C q -axis current doesn't exceed its boundary value. From Fig. 11.C it can also be seen that, control strategy with zero d -axis current is successfully employed.

Simulation results of Drive-2 are presented in Fig. 12. Proper control of angular position, in a case of reference position and external disturbance ($T_l = 4.4$ Nm for $t \in <50; 150>$ ms), is observed. Similar to results shown in Fig. 9.B, angular speed of PMSM doesn't exceed its boundary value.

Conclusions

It was found that ABC optimization algorithm, used before to solving multimodal numeric problems, can be successfully applied to auto-tuning procedure of linear-quadratic position regulator of PMSM. By applying Deb's method selection process with modified calculation of violation parameter, coefficients of LQR are selected with respect to boundary values of control and state variables of

PMSM (i.e. q -axis current and angular speed). During synthesis procedure of state-feedback position controller, ITAE performance index was minimized. Auto-tuning process was successfully examined for two different models of PMSM. Thanks to simplified model of the drive used in optimization procedure, computation effort of the proposed solution is relatively short. Similar results obtained for simplified and detailed models of the drive indicate that: (i) gain coefficients of linear-quadratic controllers are selected properly by ABC optimization algorithm and (ii) simplified model of the drive can be successfully used for auto-tuning process. Experimental verification is planned.

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