Design Estimations and Channel Capacity Calculations for Spatial, Polarized MIMO Antenna System for Mobile Applications

Abstract. Mobile MIMO (Multiple Input Multiple Output) communications provides an improved transmission capacity and error performance over traditional digital transmission systems. In this paper the spatial, polarization technique is used in order to improve the channel capacity of the proposed system. The potential for integrating MIMO systems comprising both 2x2 and 3x3 elements into a mobile handset environment will be considered. The MIMO channels will be subject to Rayleigh fading, and the results are compared against linear (or planar) arrays. In addition, different azimuthal spectra will be considered in evaluating the actual system performance.

Keywords: MIMO, Channel Capacity, Polarization, Rayleigh Fading Channel.

Introduction

Multiple Input Multiple Output communication systems are able to provide an increased data rate, with consequently improved error performance as compared to systems without multiple antennas [1-4]. For antenna systems a particular area of interest is the practical utilisation of multi-path propagation [5, 7]. During the propagation of signals from the transmitter side, these transmitted signals reflected from intermediate objects which cause these signals to travel along separate paths with different time of arrival at the receiver end. These effects combined with digital beam-forming enhance the potential for greater system capability and bandwidth. In order to realize this potential it is necessary to understand how the use of spatial correlation based on polarization states impacts on the achievable volume of the radiating structures.

This paper presents the modeling and design concepts of 2x2 and 3x3 MIMO antenna systems for mobile handsets. The spatial, polarization technique has been used to improve the channel capacity of the system. The channel capacity is estimated and discussed over Raleigh fading channel and compared to planar antenna array MIMO channel. In order to calculate the channel capacity several power spectrum distribution functions has been used [8-15].

Channel Capacity Model

A communication system which is comprised of N number of transmitter and receiver antennas, the channel capacity can be calculated by using the well-known formula [16].

\[ C = \sum_{i,j} W_i^H G_{ij} W_j \]

Which is an expectation value incorporating, the variance of the noise power (\( \sigma^2 \)), the mean of the total transmitted power (\( P_t \)), and the channel transfer matrix, H. The matrix size is \( n_r \times n_t \), \( I \) is the identity, and \( H \) is the conjugate transpose operation. If the spatial matrices characterising the receiver (\( W_i \)) and transmitter (\( W_j \)) are also known, then \( H H^H \) may be obtained simply as follows:

\[ H H^H = W_i G_{ij} W_j^H \]

The characteristic channel properties are encoded in the system matrix \( G_{ij} \), so for a channel with Rayleigh fading these matrix elements are drawn from a complex Gaussian distribution [17-19]. A further simplification can be introduced into the modeling, where the spatial transmitter matrix for maximum channel capacity is given as the identity, this is justifiable on physical grounds. For the maximum channel capacity the spatial matrix of the transmitter can be reduced to identity matrix due to the space availability at the transmitter side. Now equation (2) can be simplified to the following:

\[ HH^H = W_i G_{ij} W_j \]

The receiver spatial matrix \( W_i \) can be defined by the Following equation:

\[ W_{ij} = \sqrt{\frac{\int d\Omega (E_{ai} \cdot E_i)}{\sigma_{ij}}} \]

Where

\[ \sigma_{ij} = \int d\Omega (E_{ai} \cdot E_i)^2 \]

The spatial integrals used in the above equations are understood to be parameterised as \( \int d\Omega = \sin \theta d\theta d\phi \). The \( E_{ai} \) are the electric fields of the radiating elements, whilst the \( E_i \) are the electric fields incidents on the receiver side.

Fig.1. The basic antenna geometry for 3x3 system

The polarization geometry assumes a set of three collocated dipoles antennas present over the azimuthal axis whereas they are centred at the origin. Fig.1 and Fig.2 shows the geometric representation of the 3x3 and 2x2 proposed antenna systems. This analysis is restricted to a
maximum of three radiating elements; at this stage the mutual couplings are ignored. The elements are arranged according to the elevation angle which has been considered as polarization angle in this case. To reduce complexity, short dipoles are employed in which their radiating field can be easily computed.

Fig. 2. The basic antenna geometry for 2×2 system

However, for arbitrary dipole orientation \( (\theta_d, \phi_d) \), the following unit vector, which defines the local axis of the dipole, can be expressed as follows:

\[
\vec{d} = \sin \theta_d \cos \phi \hat{e}_x + \sin \theta_d \sin \phi \hat{e}_y + \cos \theta_d \hat{e}_z
\]

where \( \{\hat{e}\} \) form an orthonormal right handed triad. The radiated dipole field can be given as follows:

\[
E_\theta = E_\theta(\theta, \phi) \hat{e}_\theta + E_\phi(\theta, \phi) \hat{e}_\phi
\]

The components \( E_\theta \) and \( E_\phi \) are defined as the (spatially averaged) scalar products,

\[
E_\theta(\theta, \phi) = \langle \vec{d} \cdot \vec{u}_\theta \rangle
\]

\[
E_\phi(\theta, \phi) = \langle \vec{d} \cdot \vec{u}_\phi \rangle
\]

and

\[
\vec{u}_\theta = \cos \theta \cos \phi \hat{e}_x + \cos \theta \sin \phi \hat{e}_y + \sin \theta \hat{e}_z
\]

\[
\vec{u}_\phi = -\sin \phi \hat{e}_x + \cos \phi \hat{e}_y
\]

Power Spectrum Distribution Functions

For the calculation of the channel capacity of MIMO system, several power distribution functions have been used [20-23]. Gaussian, Laplacian and Secant squared, uniform distributions have been used over the azimuthal angle. In addition raised cosine and the nth order raised cosine distributions are considered along the zenith angle. These distributions are combined with one another to form the incident plane wave illuminating the receiver elements. The following analysis considers all possible combinations between these distributions in such a way as to predict the maximum degradations and/or improvements of the proposed MIMO system performance. In general, there are twelve combinations can be considered to compute the channel capacity of the MIMO system.

Gaussian distribution

The power spectrum over the azimuthal direction using a Gaussian distribution can be defined as follows:

\[
P_\phi(\phi) = \sqrt{\Lambda} \exp \left\{ -\frac{(\phi - m_\phi)^2}{2\sigma_\phi^2} \right\} - \pi \leq \phi \leq \pi
\]

where \( \Lambda \) is the normalization factor, this can be given by:

\[
A = 2\sigma_\phi^2 \pi \left\{ -\text{erf} \left( \frac{\pi + m_\phi}{\sqrt{2}\sigma_\phi} \right) + \text{erf} \left( \frac{-\pi + m_\phi}{\sqrt{2}\sigma_\phi} \right) \right\}^2
\]

\( m_\phi \) and \( \sigma_\phi \) are the mean and variance of the Gaussian function respectively. Several plots of this function for different values of \( m_\phi \) and \( \sigma_\phi \) are illustrated in Fig. 3.

Fig. 3. Gaussian distribution functions for different values of \( m_\phi \) and \( \sigma_\phi \)

Laplacian distribution

A well known practical channel capacity approximation uses the Laplace power distribution along azimuthal direction [24-26]. Laplacian distribution can be simply expressed as follows:

\[
P_\phi(\phi) = \sqrt{\Lambda} \exp \left\{ -\frac{|\phi - m_\phi|}{\sigma_\phi} \right\} - \pi \leq \phi \leq \pi
\]

The behaviour of this distribution for different values of \( m_\phi \) and \( \sigma_\phi \) are illustrated in Fig. 4. Note that these distributions show narrower, and sharp variations, around the mean of the incident angle compared to that found in the Gaussian distributions.

Secant square distribution

Raised (nth order) cosine distributions applied along an angular direction are also used in some applications when calculating the channel capacity. This distribution function can be expressed as follows:

\[
P_\phi(\phi) = \frac{1}{\pi} \sec(\phi - |\pi - \phi_0|), -\pi \leq \phi \leq \pi
\]

\[
\left\{ \begin{array}{ll}
\sec(\phi_0 - |\pi - \phi_0|), & |\phi - m_\phi| \leq \phi_0 \\
1, & |\phi - m_\phi| > \phi_0
\end{array} \right.
\]
where \( 2\phi_0 \) is the width of the angle when the distribution is not constant, and \( m_\phi \) is the mean angle in which the incident fields scattered on the antenna. Several distribution functions of this kind of PAS are shown in Fig.5.

For suburban channels the variation over elevation angle is similar to the urban channel model. The azimuth is taken at 5° intervals from 5° to 20°, and the corresponding Laplacian spectra are computed and compared.

The \( E_p \) and \( E_m \) of the incident fields were assumed to be independent over the angular range \([\theta, \phi]\), and their variations are uniform for the channel models under consideration. It was also assumed that the phase variations are uniform over \([0, 2\pi]\).

Channel capacities for an urban channel model are simulated for both the 2x2 and 3x3 MIMO systems, the results are shown in Fig 7 and Fig.8. The antennas for the 2x2 system are located at \( \phi = 0^\circ \) and \( 180^\circ \), whereas for the 3x3 system \( \phi = (0^\circ, 120^\circ, 240^\circ) \). The data used to generate these figures employs a closed form solution for the elements of \( W \). The capacity was evaluated over a Rayleigh fading model, the average was taken over 1000 complex samples. The transfer function was normalized at each point to provide a good prediction of the maximum variation of the spatial matrices for these MIMO antennas. It should be noted that the maximum capacities for the 2x2 and 3x3 cases occur at 55° and 63° respectively. These angles should be selected for the required orthogonalization of the spatial field distributions for the antenna geometry (in Fig.1).

The same sequence of results is presented for the suburban channel model in Fig.9 and Fig.10 for various values of \( \sigma_\phi \). Here the maximum capacity limits for the 2x2 MIMO case are reached for all the presented values of \( \sigma_\phi \); in the 3x3 case there is a slight, but detectable reduction, as \( \sigma_\phi \) increases. The elevation angle is varied uniformly over 30 degrees at the horizontal plane, whereas azimuth angle varied as Laplacian spectrum of different values of \( \sigma_\phi \) (5, 10, 15, 20 degrees for the geometry presented in Fig.1) in which the azimuth direction randomly selected between 0 and \( 2\pi \).

Channel capacities for the 2x2 and 3x3 cases using the Secant square distribution along azimuth angle are shown in Fig.11 and Fig.12. The antennas for the 2x2 case are located at \( \phi = 0^\circ \) and \( 180^\circ \), whereas for the 3x3 case \( \phi = (0^\circ, 120^\circ, 240^\circ) \). For the 2x2 and 3x3 systems the maximum channel capacities have been observed at 55° and 62° respectively. Fig.14 and Fig.15 show the channel capacities of 2x2 and 3x3 systems as described in figures 1 and 2 respectively. The channel capacity has been calculated by assuming the Raised cosine distributions along the zenith angle whereas Gaussian distribution has been used along the azimuth. In both cases the calculated channel capacity is seen to approach the maximum limit.
It should be noted that the channel capacity results shown in Fig. 13 and Fig. 14 are approximately similar to the capacities calculated using Laplacian and Secant squared distributions. The maximum capacities for both 2×2 and 3×3 MIMO systems have been observed at the polarization angles of 64°.

The channel capacities for different transmitted powers are shown in Fig. 15, here the elevation angle varies uniformly over 180° from the centre (with zero crossings), but with the same azimuthal variations used for Fig. 8 and Fig. 10. The channel capacities can be seen to increase with increasing SNR (this is in fact approximately linear); with the maxima located at an elevation angle of 63°.
Conclusion

This research paper has thoroughly discussed the channel capacities for 2×2 and 3×3 Multiple Input Multiple Output systems with respect to spatial polarization technique. Furthermore, verity of channel capacity results have been computed for MIMO systems presented in Figure 1 and Figure 2, these results have been simulated over Raleigh fading channel with different azimuth spectra. The simulated results have been compared and verified using the dominant Eigen values related to the channel matrix. It has been concluded from the presented results that the maximum channel capacity can be reached for the tightly collocated multiple antennas by careful selection of the intrinsic orthogonalities of spatial field distribution. The estimated results for the proposed small volume geometry are comparable with planar array which has much larger antenna volume.

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REFERENCES


