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Assessment of Stability Margin of Linear Parametric Amplifiers by the Frequency Symbolic Method

Abstract*. This paper considers the problem of assessment of stability margin of linear periodically-time-variable circuits, in particular parametric amplifiers, which was investigated using the frequency symbolic method. The assessment of circuit stability is carried out by the real parts of the denominator roots of a normal parametric transfer function, which is defined by the frequency symbolic method in the form of the approximation of Fourier polynomials. The calculation is performed in an MATLAB environment.*

Streszczenie. *Praca przedstawia problem oceny marginesu stabilności obwodów liniowych, zmiennych w czasie. Rozważania dotyczą w szczególności wzmacniacza parametrycznego, analizowanego przy użyciu metody symbolicznej w dziedzinie częstotliwości. Stabilność jest określana na podstawie części rzeczywistej biegunów transmitancji obwodu parametrycznego, zdefiniowanej przy użyciu aproksymacji wielomianami Fouriera. Obliczenia numeryczne zostały wykonane przy zastosowaniu programu Matlab. (**Ocena stabilności pracy wzmacniacza parametrycznego przy użyciu metody symbolicznej w dziedzinie częstotliwości**).*

Keywords: frequency symbolic method; linear periodically time-variable circuits; asymptotic stability; linear parametric amplifiers.
Słowa kluczowe: metoda symboliczna analizy, obwody o parametrach okresowo-zmiennych, stabilność asymptotyczna, wzmacniacz parametryczny.

Introduction

The problem of assessment of stability is an urgent problem for linear periodically-time-variable (LPTV) circuits. Nowadays, there are methods for assessing the stability of these circuits, which are based on the construction of Lyapunov functions. The absence of a clear method for the selection of Lyapunov functions that was used on cumbersome calculations and on the inaccuracy of the results [1, 2].

The paper considers issues of assessment of asymptotic stability and stability margin of LPTV circuits, when you change the parameters of its elements by the frequency symbolic method (FS-method) [3,4] and by the system functions MAOPCs (Multivariate Analysis and Optimization of the Parametric Circuits), which is developed on its basis [5].

In the FS-method, the approximation $\hat{W}(s,\xi)$ of a normal transfer function $W(s,\xi)$ as well as its dual transfer function $W(s,t)$, the LPTV circuit is defined as a Fourier polynomial [3,4]:

$$\hat{W}(s,\xi)=W_{\pm 0}(s)+\sum_{i=1}^{k}\begin{bmatrix} W_{-i}(s)\cdot\exp(-j\cdot i\cdot\Omega\cdot\xi)+ \\ +W_{+i}(s)\cdot\exp(+j\cdot i\cdot\Omega\cdot\xi)\end{bmatrix}, \tag{1}$$

where $W_{\pm 0}(s)$, $W_{-i}(s)$, $W_{+i}(s)$ - are time-independent rational fractional functions of a complex variable *s*; ξ is the moment of applying a delta function to the circuit input; k is the number of harmonic components in the polynomial; $\Omega = 2\cdot\pi/T$, T is the period of parameter change of a parametric element of the circuit under the influence of the periodic pumping signal. The number of harmonic components k determines the accuracy of approximation of the transfer function and should be selected with a sufficient value. Complex variable S and the required circuit parameters are given in symbolic form and are determined numerically at the last stages of the calculations.

Assessment of asymptotic stability in the system functions MAOPCs is performed by calculating the real parts of roots of a denominator $\Delta(s)$ of normal transfer function $\hat{W}(s,\xi)$. If the real parts of all such roots are negative, then the circuit is asymptotically stable, otherwise it is unstable [3]. If the circuit is stable, then the value of the negative real part of the root, which is the closest to the axis $j\omega$ of the complex plane, characterizes such an important parameter of circuit as stability margin. The farther to the left from the axis $j\omega$ this root is located, the greater stability margin is. Chapters 2 and 3 of the paper shows the application of FS- method for assessment of stability margin of single-circuit and double-circuit parametric amplifiers.

Assessment of stability margin of single-circuit parametric amplifier

The paper is representing a single-circuit parametric amplifier (Fig. 1) with two periodic time-variable elements *c(t)* and *L(t)*. The analysis of the single-circuit parametric amplifier is fulfilled by the system functions MAOPCs [5]; as a result, we have obtained the denominators $\Delta(s)$ for required polynomials k (1 and 6) of harmonic components and for parameters s, m_c, m_L given in symbolic form. Further, these polynomials can be repeatedly calculated for different values of these symbols.

Let's pay attention to the following problems.

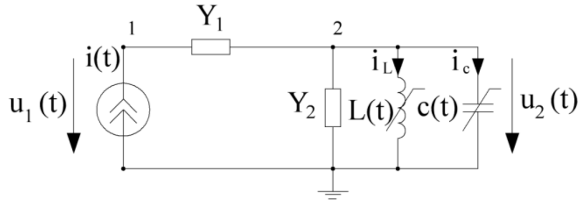

Fig. 1. Single-circuit parametric amplifier with two parametric elements, where t is time; $i(t)=I_m\cdot\cos(\omega_c\cdot t+\varphi)$; $c(t)=c_0\cdot(1+m_c\cdot\cos(\Omega\cdot t+\varphi_c))$; $c_0=10\cdot 10^{-12}F$; $L(t)=L_0\cdot(1+m_L\cdot\cos(\Omega\cdot t+\varphi_L))$; $Y_1=0.25S$; $L_0=0.2533\cdot 10^{-6}H$; $\Omega=4\cdot\pi\cdot 10^{8}\,rad/s$; $Y_2=0.0004S$

A. The dependence of the number of roots in the denominator *Δ(s)* on the number of harmonic components in approximation of the transfer functions

According to the FS-method, the coefficients of the approximation expressions (1) are determined from system of linear algebraic equations (SLAE) in which they are unknown. In this interrelation, in the presence of the symbolical variable s each unknown is calculated in the form of a rational fractional function of s; the degree of the denominator is equal to $n(2k+1)$, where n is the degree of differential equation, which is describing the circuit. Since the denominator $\Delta(s)$ is the same for all the approximating coefficients, it will be definitely the denominator of a normal transfer function (1), the roots of which are carrying the information about the stability of the circuit [3]. Thus, the polynomial $\Delta(s)$ has the degree r, which corresponds to the number k of the harmonic components through such a dependence: $r=n(2k+1)$. The number of roots of the polynomial $\Delta(s)$ is also equal to r. Fig. 2 shows the roots of the polynomial $\Delta(s)$ for the circuit, which are shown in Fig. 1 when $k=1$ and $k=6$ (conjugate roots are not shown in the figure). Thus, for each value of k we have the set A of the roots, which correspond to $(k-1)$ harmonic components and, additionally, the set B of "new" roots. Since k-th harmonic component refines the value of $\Delta(s)$, the roots that belong to the set A should be considered as accurate. Obviously, the roots of the denominator $\Delta(s)$, that are obtained at $k=1$ in the calculations for $k=6$ will be most accurate. The roots of the set B are the most inaccurate, but they may be refined taking into account the $\Delta(s)$ for $(k+2)$-th, $(k+3)$-th, etc. harmonic components. Therefore, the roots of $\Delta(s)$ that are obtained at the maximum value of k (in our case, $k=6$) and are concurrently present in the set of roots of the polynomial $\Delta(s)$, should be selected for further consideration. In Fig.2,

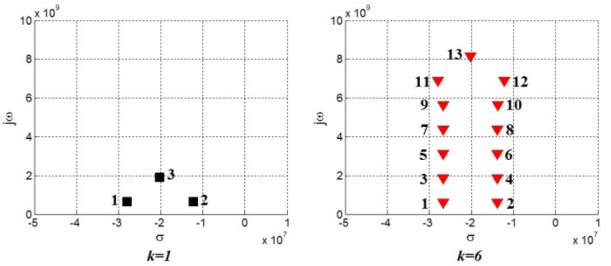

Fig. 2. The roots of the polynomial $\Delta(s)$ of the amplifier, when $m_L=0.1$, $m_c=0.15$, $(\varphi_c-\varphi_L)=0$

For $k=1$ and $k=6$, these roots are denoted by numbers «1» and «2», respectively. Thus, we have obtained two different real parts -1.378935e + 7 and -2.666838e + 7, corresponding to roots 1 and 2, of which the stability margin determines the value -1.378935e + 7, because it is closer to the axis $j\omega$.

B. Construction of stability maps

In [1] two different operating modes of the amplifier were qualitatively defined: $(\varphi_c-\varphi_L)=0$ and $(\varphi_c-\varphi_L)=180^o$.
The operating mode at $(\varphi_c-\varphi_L)=0$ is named synchronous; and it was shown, that in this mode the energy

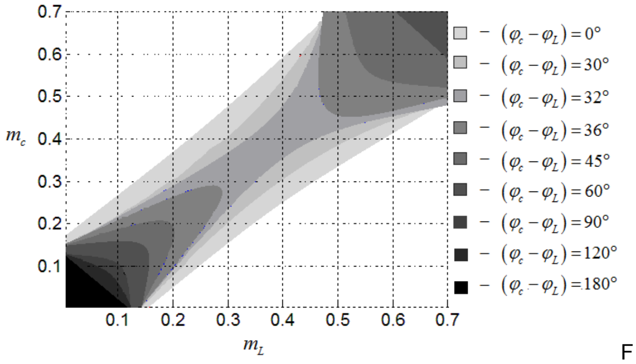

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Fig. 3. The stability areas of the single-circuit parametric amplifier at $k=6$ and different values of phases $(\varphi_c-\varphi_L)$

delivered to the contour owing to change $c(t)$, is compensated by the energy delivered to the contour owing to change $L(t)$. The working stability of such amplifier is the biggest one. The operating mode at $(\varphi_c-\varphi_L)=180^o$ is named as asynchronous; and it was shown that in this mode the energy delivered to the contour owing to change $c(t)$ and the energy delivered to the contour owing to change $L(t)$, so the working stability of the amplifier is the smallest one. The results of simulation of the specified modes by the system MAOPCs are shown in Fig. 3. Here are presented nine values $(\varphi_c-\varphi_L)=0^o,30^o,32^o,36^o,45^o,60^o,90^o,120^o,180^o$. For each value has been calculated the map of stability of the amplifier in the coordinates m_c and m_L when

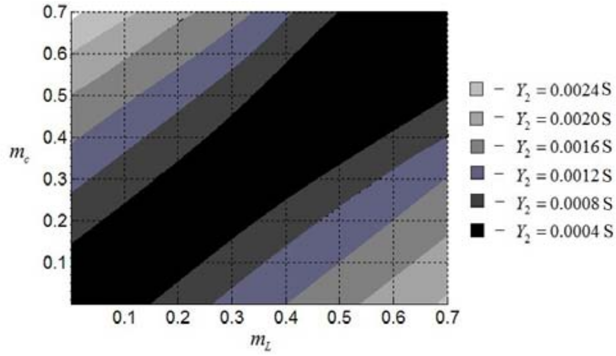


Fig. 4. The stability areas of the amplifier shown in Fig.1 for $k=6; \varphi_c-\varphi_L=0°$ and for different Y_2 values

they are changing their values from 0 to 0.7. The stability areas are marked by different shades of gray. The instability area is left without shade. For example, for $(\varphi_c-\varphi_L)=0$ the stability area is the biggest (shown in fig. 3 by the shade ☐) and it occupies the largest area on the map. For $(\varphi_c-\varphi_L)=30°$ the stability area is reduced (in Fig. 3 it is shown by the shade ☐) and it becomes more lucid in the center of the map. For $(\varphi_c-\varphi_L)=32°$ this area becomes even more lucid, while for $(\varphi_c-\varphi_L)=36°$, it is broken into two parts which shrink for $(\varphi_c-\varphi_L)=45°,60°,90°,120°$. For $(\varphi_c-\varphi_L)=180°$ the stability area is located only in the left lower corner of the map. This fact demonstrates that in such case the amplifier has the largest instability area.

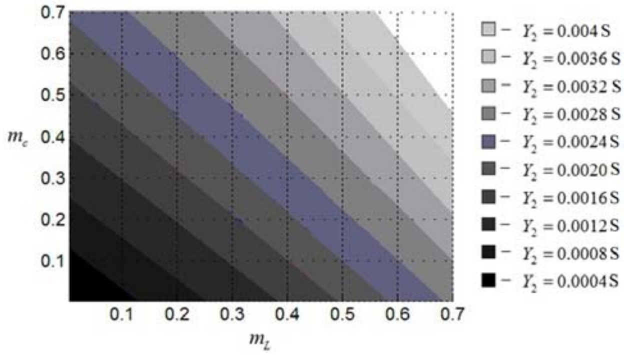


Fig. 5. The stability areas of the amplifier shown in Fig.1 for $k=6, \varphi_c-\varphi_L=180°$ and for different Y_2 values

These results are fully consistent with the conclusions obtained in [1] for synchronous and asynchronous operating modes of the amplifier, and, in addition, show the set of the maps for the intermediate values.

In Fig. 4 and Fig.5 are shown the areas of stability/instability, at a fixed phase difference $(\varphi_c-\varphi_L)=0°$ and $(\varphi_c-\varphi_L)=180°$, respectively, while changing

$m_c = 0:0.7$, $m_L = 0:0.7$ and the conductivity Y_2 increases from the value 0.0004 S (that is shown in Fig. 1) to the value Y_2=0.004 S.

In Fig. 4 and Fig.5 the stability areas increase with the growth of Y_2. This fact is understandable, because the growth of conductivity means a reduction of quality factor (*Q* factor) of the amplifier circuit.

C. Assessment of the stability margin

Assessment of the stability margin by the system

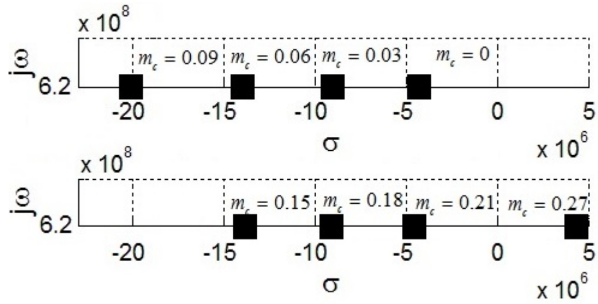


Fig. 6. Root №2 of polynomial $\Delta(s)$ of the amplifier at k=6, $\varphi_c - \varphi_L = 0$, m_L=0 and different m_c values

functions MAOPCs can be performed by changing arbitrary parameters of the circuit that affect its stability. For example, we have built such assessment for m_L=0.1, when changing m_c within the range 0-0.27.

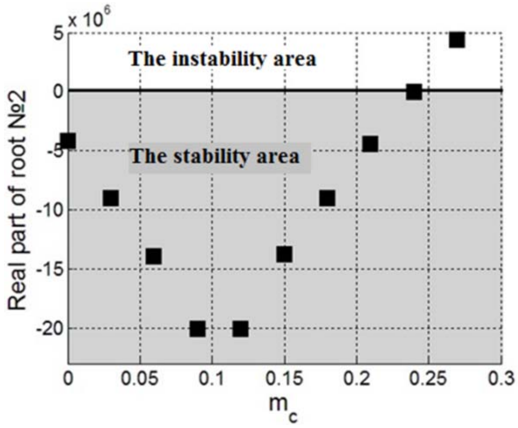


Fig. 7. The map of the stability margin of the single-circuit parametric amplifier at k=6, $\varphi_c - \varphi_L = 0°$ and m_c values varying in the range 0-0.27

It can be noted that while m_c increases from 0 to 0.09, the root №2 (that determines the stability margin) shifts to the left, increasing this margin. Further, when m_c increases from 0.09 to 0.21, this root shifts to the right, reducing stability margin, and when m_c=0.27 the circuit is already unstable. The transition from the stable condition to the unstable one occurs, when m_c=0.25, which is fully consistent with the results of the verification, using the Micro-Cap software. The dependence of the stability margin on m_c is shown in Fig. 7, and its physical meaning can be understood from the map shown in Fig. 3: for $\varphi_c - \varphi_L = 0°$ and m_L=0.1 we observe that, in the course of varying from m_c=0 to m_c=0.27 (along the vertical axe), we are moving away from the instability area to the bottom part of the map, then we are approaching the instability area at the top part of the map and finally we are entering it.

Assessment of stability margin of double-circuit parametric amplifier

Double-circuit parametric amplifier with parametric time-varying capacity *c(t)* is shown in Fig. 8.

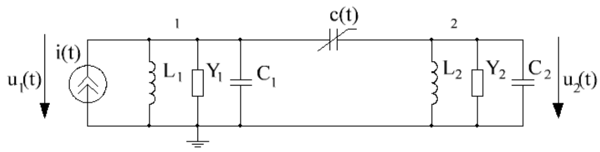


Fig. 8. Double-circuit parametric amplifier

$$i(t) = I_m \cdot \cos(\omega_c \cdot t + \varphi), c(t) = c_0 \cdot (1 + m \cdot \cos(\Omega \cdot t)),$$
$$c_0 = 1pF, \varphi = \pi/4, \Omega = 2 \cdot \pi \cdot 298.573 \cdot 10^6 \, rad/s, I_m = 0.1mA,$$
$$\omega_c = 2 \cdot \pi \cdot 10^8 \, rad/s, Y_1 = Y_2 = 0.0001S, \varphi = 0, C_1 = C_2 = 68pF,$$
$$L_1 = 36.70795nH, L_2 = 9.312609nH,$$

In Fig.9 is shown the location of the roots of the polynomial $\Delta(s)$, taking account in the normal transfer function of the

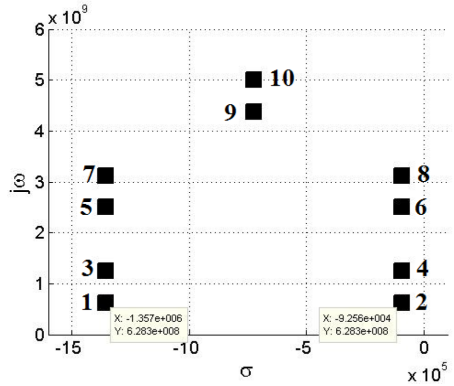


Fig. 9. The roots of polynomial $\Delta(s)$ of the amplifier circuit from Fig.4, when the number of harmonic components is in the normal transfer function k=2 and m=0.2

Circuit of the two harmonic components k=2, that provides, in this case, the requirements of accuracy. The degree of the polynomial $\Delta(s)$, considering the 4th order of differential equation (n=4), that describes the circuit, will be $r=n(2k+1)=20$ [4]. There are not shown the conjugate roots in Fig. 9. Since the real parts of all roots from figure 9 is negative, then the circuit is stable for selected values of the parameters. In figure 10 is shown the trajectory of the roots of the polynomial $\Delta(s)$ from figure 9, by changing the modulation depth m of parametric capacity *c(t)* within limits m=0.2:0.01:0.25. Roots 2,4,6,8, that are having the same real part and moving in the axis to the side $j\omega$ (marked by white squares) and they are between values m=0.2292 and m=0.2293, simultaneously are crossing it. It is fully consistent with the results, that obtained by using the program Micro-Cap.

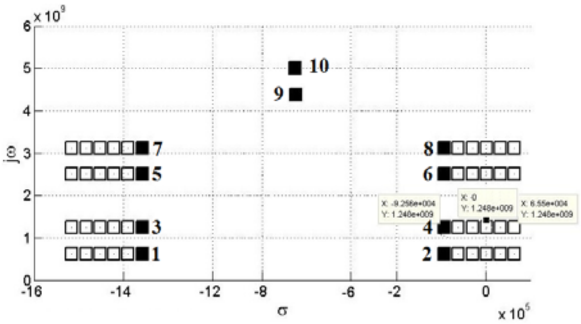
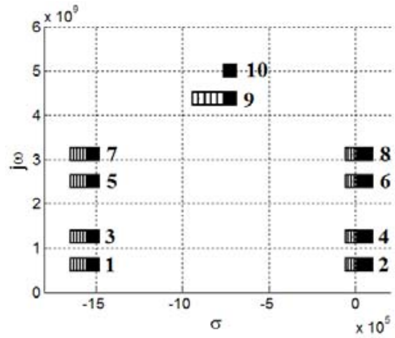
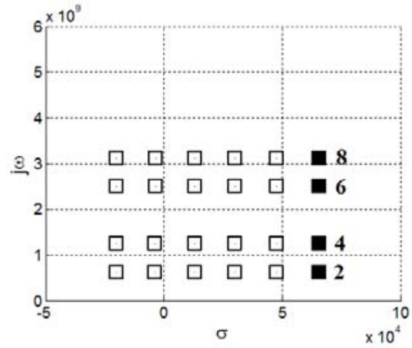


Fig. 10. The roots of polynomial $\Delta(s)$ amplifier circuit from Fig.8 when changing m=0.2:0.01:0.25. Roots for m=0.2 are marked by black color



a)



b) trajectories of the roots in a larger scale

Fig. 11. The roots of polynomial $\Delta(s)$ amplifier circuit from Fig. 8 at m=0.25 and changing $Y_1 = 1 \cdot 10^{-4} : 0.05 \cdot 10^{-4} : 1.25 \cdot 10^{-4}$. Roots for $Y_1 = 1 \cdot 10^{-4}$ are marked by black color

Thus, the circuit at m=0.2292 is still stable, while m=0.2293 - is not stable.

In Fig. 11 is shown the movement of the roots in the reverse direction. For this initial value of the roots we fixed for m=0.25 (the endpoint of the trajectories of the roots from figure 10) and began to increase Y_1. In this case, the quality factor of the signal contour is reduced, and between values $Y_1 = 1.15 \cdot 10^{-4}$ and $Y_1 = 1.2 \cdot 10^{-4}$ the amplifier becomes stable again.

In Fig. 12 is shown the stability map of double-circuit amplifier for m=0.25 and by changing conductivity Y_1 and Y_2 (quality factor) of the signal and idler contours, respectively. By the symbols ● and ✕ are marked stable and unstable conditions of circuit, respectively. In Fig. 12, by rectangle is selected an area, that corresponds to the (upward) value of the roots 2,4,6,8 from Fig.11.

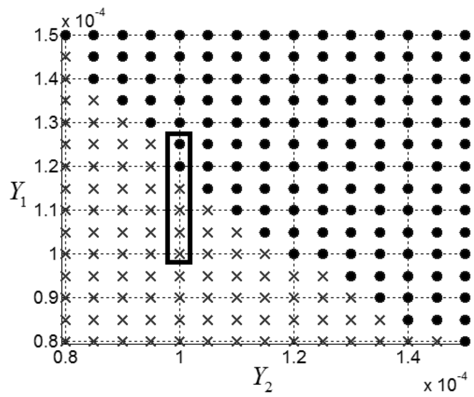


Fig. 12. Stability map of amplifier from fig. 8 at $m = 0.25$ and changing $Y_1 = 8 \cdot 10^{-5} : 0.5 \cdot 10^{-5} : 15 \cdot 10^{-5}$ $Y_2 = 8 \cdot 10^{-5} : 0.5 \cdot 10^{-5} : 15 \cdot 10^{-5} : 15 \cdot 10^{-5}$ Is marked by the black rectangle selected fragment that corresponds to a trajectories of the movement of roots from fig.11b

The results are fully consistent with the obtained conclusions, using the program Micro-Cap.

Conclusions

1. The FS-method and based on this method system functions MAOPCs are a perspective tool for assessment of asymptotic stability and stability margin of linear periodically time-variable circuits, in particular parametric amplifiers.
2. The necessary refinement of roots and, therefore, the refinement of stability margin can be done by increasing in the number of harmonic components in a normal transfer function.
3. For the first time, stability maps for single-circuit and double-circuit parametric amplifiers have been obtained by software application.
4. Full coincidence of results between programs MAOPCs and Micro-Cap proves the adequacy of the transfer functions, generated by the FS-method, and high accuracy of assessment of stability through the roots of a polynomial $\Delta(s)$.
5. FS- method allows effectively assess the stability and generate the trajectories of roots or of stability maps for changing arbitrary parameters of circuit, that is comfortable in control of stability in statistical character problems and optimization of parametric devices.

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