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Comparative assessment of digital filters for microprocessor-based relay protection

Abstract. *This article presents the implementation of digital filters used in microprocessor-based (digital) relay protection current measuring elements. It has been shown that in order to reliable estimate the digital filter performance its input signals waveforms must be close to the actual secondary current waveform of the current transformer (CT) to which the digital protection with the estimated digital filter is connected. Ways of digital filtering improvement based on the window functions usage are proposed.*

Streszczenie. *W pracy przedstawiono przykład wdrożenia filtrów cyfrowych stosowanych w elementach pomiarowych zabezpieczenia przekaźnika prądowego na bazie mikroprocesora cyfrowego. Wykazano, że w celu wiarygodnego oszacowania wydajności filtrów cyfrowych ich przebiegi sygnałów wejściowych powinny być zbliżone do rzeczywistego przebiegu prądu wtórnego transformatora prądowego do którego jest podłączono zabezpieczenie cyfrowe z pomiarowym filtrem cyfrowym. Zaproponowano sposoby poprawy filtracji cyfrowej w oparciu o wykorzystanie funkcji okna. (**Ocena porównawcza filtrów cyfrowych stosowanych w mikroprocesorowym zabezpieczeniu przekaźnika**).*

Keywords: digital filter, discrete Fourier transform, least error squares technique, orthogonal component, Hamming window, current transformer, saturation, digital relay protection, MATLAB, Simulink.
Słowa kluczowe: filtr cyfrowy, transformata Fouriera, metoda najmniejszych kwadratów, składowa prostopadła, okno Hamminga, transformator prądu, nasycenie, zabezpieczenie cyfrowe przekaźnika, MATLAB, Simulink.

Introduction

Microprocessor-based (digital) relay protection devices continuous improvement allows to implement more sophisticated algorithms for processing signals under control [1]. Nevertheless, the signal magnitude estimation by digital filters is not significantly improved. It occurs due to the fact that the digital filter coefficients are calculated with the reference to the simplest input signals the waveform of which are substantially differs from the real emergency signals.

The purpose of this work is to study the different digital filters performance and to select the most appropriate their implementation. In order to get that done the digital filters coefficients calculation algorithms were developed and the simulation model for obtaining the complex test signals similar in waveform to the current transformer (CT) secondary current was implemented in MATLAB-Simulink environment.

Main part

The digital relay protection input signal $y(nT)$ contains useful signal $u(nT)$ and noise $e(nT)$:

(1) $$y(nT) = u(nT) + e(nT)$$

The main function of the digital filter consists in useful signal $u(nT)$ determination and in as much as possible noise $e(nT)$ suppression. In general, noise signal consist of: higher order harmonics, exponentially decaying DC component, and decaying and non-decaying harmonic components off-nominal frequency. That is why the precise accounting of the all noise signal components is essentially impossible, therefore the useful signal determination by digital filters will always come with the error which is proportional to the difference between the input signal $y(nT)$ and the useful signal $u(nT)$.

To solve such problem the Least Error Squares technique is widely spread. It is based on minimizing the mean-square error between the actual and assumed waveforms in accordance to the next expression:

(2) $$S = \sum_{n=1}^{N} (y(nT) - u(nT))^2 \to min$$

Depending on the error signal components a different digital filter implementations are exists: digital filter based on the Least Error Squares technique (LES) [2], based on the Discrete Fourier Transform (DFT) [3] and also based on the Orthogonal Components Former (OCF) [4], etc. Let us consider in more details the structure of the mentioned above digital filters.

Least error squares technique

Input signal identification based on LES was first proposed in [2]. The signal is modeled as a combination of the fundamental frequency component (useful signal), the exponentially decaying DC component with the time constant τ and the third harmonic component (error signal):

(3) $$y(nT) = U_a e^{-\frac{nT}{\tau}} + U_{m1} sin(\frac{2\pi n}{N} + \varphi_1) + U_{m3} sin(\frac{6\pi n}{N} + \varphi_3)$$

Equation (3) in simplistic terms represents a current signal waveform during a short circuit in the power system. Such components combination is explained because of even harmonics are not presented in the fault current and higher order harmonics started from fifth are significantly attenuated by an analog low-pass filter (LPF).

Exponentially decaying DC component with the time constant τ is decomposed by using Taylor series and only 3 or 4 terms of this series is used. Harmonic components of the signal are transformed with the help of trigonometric identities (angle addition formulae for the sine): $sin(\alpha + \beta) = sin\alpha \cdot cos\beta + cos\alpha \cdot sin\beta$.

As a result, for any sample number n=1…N, the equation (3) appears as follows:

(4) $$y(nT) = U_a (1 - \frac{nT}{\tau} + \frac{n^2T^2}{2\tau^2}) + U^c_{m1} sin(\frac{2\pi n}{N}) + U^s_{m1} cos(\frac{2\pi n}{N}) + U^c_{m3} sin(\frac{6\pi n}{N}) + U^s_{m3} cos(\frac{6\pi n}{N})$$

where $U^s_m = U_m sin\varphi$, $U^c_m = U_m cos\varphi$ – magnitude of the sine and cosine orthogonal components of the signal (3).

In the equation (4) there are next unknowns: $X_1 = U_a$,

(5) $$X_2 = -\frac{U_a}{\tau},\ X_3 = \frac{U_a}{2\tau^2},\ X_4 = U^c_{m1},\ X_5 = U^s_{m1},\ X_6 = U^c_{m1},\ X_7 = U^s_{m3}.$$

Taking into account the accepted symbolic notation, the equation (4) can be written as follows:

(6) $$y(nT)=X_1+X_2nT+X_3n^2T^2++X_4\,sin(\frac{2\pi n}{N})+X_5\,cos(\frac{2\pi n}{N})+$$
$$+X_6\,sin(\frac{6\pi n}{N})+X_7\,cos(\frac{6\pi n}{N})$$
or in matrix form **Y=AX**.

In this expression: **A** – coefficients matrix (main matrix); **X** – column vector of unknowns; **Y** – column vector of free terms.

Squared linear equation system can be resolved by multiplying column vector of free terms **Y** by inverse matrix $\mathbf{A}^{-1}$:

(7) $$\mathbf{X}=\mathbf{A}^{-1}\mathbf{Y}.$$

Practically, the number of samples N per fundamental frequency period is often greater than the number of unknowns. Such equation systems are called overdetermined, therein the coefficients matrix **A** – is rectangular matrix with the number of rows greater than the number of columns. To solve such equation system the pseudoinverse matrix is used. This matrix can be obtained with the help of any math software package, e.g. in MATLAB for this purpose the *p=pinv(a)* function can be used [5].

As a result, 2 rows of this matrix are the digital filter coefficients a_{cn} and a_{sn} for the sine and cosine fundamental frequency orthogonal components determination:

(8) $$u_{cn}=\sum_{n=1}^{N}a_{cn}y(nT)$$

(9) $$u_{sn}=\sum_{n=1}^{N}a_{sn}y(nT)$$

The magnitude of the fundamental frequency component can be estimated as follows:

(10) $$U_{m1\,n}=\sqrt{u_{sn}^2+u_{cn}^2}$$

Discrete Fourier transform

The inability of accurate accounting of the all emergency components in the fault signal gives the opportunity to propose as low as practicable model of the signal, which contains only the fundamental frequency component:

(11) $$y(nT)=U_{m1}^{c}\,sin(\frac{2\pi n}{N})+U_{m1}^{s}\,cos(\frac{2\pi n}{N})$$

In this case, calculated by analogy with (3) digital filter coefficients are fully coincide with the fundamental frequency orthogonal components coefficients of the DFT.

In the digital filter design practice for relay protection purposes the Cosine filter (CF) is widely used [3]. This filter can be obtained from the cosine fundamental frequency orthogonal component of the DFT and its coefficients can be obtained as follows:

(12) $$A_c(nT)=(\frac{2}{N})cos(\frac{2\pi kn}{N})$$

Inheriting all the advantages of the DFT, the CF is also practically immune to the exponentially decaying DC component of the signal.

Orthogonal components former

In the signal mathematical models discussed above the number of equation exceeds the number of unknowns. However, if to capture on the finite observation time interval with the duration about one fundamental frequency period (data window) an even number of signal samples L it is become possible to form the square matrix with L rows and L columns (LxL) and to calculate the digital filter coefficients in accordance with (7). In this case the mathematical model of the signal is formed considering that the exponentially decaying DC component is represented by the first two terms of the Taylor series, so that the each row of the coefficients matrix **A** has $M=L$-2 elements for the fundamental frequency component and higher-order harmonics [4]. Since the each harmonic component of the signal is described by the two orthogonal components then for the even L the maximum harmonic order in the signal has the number M/2:

(13)
$$y(nT)=\sum_{n=0}^{L-1}[U_a-U_a\frac{nT}{\tau}+\sum_{k=1}^{M/2}(U_{m1}^{c}\,sin(\frac{\pi nk}{N})+U_{m1}^{s}\,cos(\frac{\pi nk}{N}))]$$

The key feature of the OCF coefficients calculation is that the number of samples N per fundamental frequency component period can be either an integer when $N{\cdot}T$=0.02 s or a real number in case of $N{\cdot}T\neq$0.02 s. Such an opportunity of choosing the sampling period T enables to change the digital filter coefficients in order to obtain the most optimal frequency and transient response [6]. Furthermore, it provides a way to the on-line recalculation of the digital filter coefficients, e.g during power system transients along with the frequency fluctuation.

If the exponentially decaying DC component is neglected, then the OCF coefficients will be the same as the sine and cosine fundamental frequency orthogonal components coefficients of the DFT filter.

Table 1. Digital filters coefficients.

Sample number	CF	OCF	LES
1	0.0833	0.0000	-0.1407
2	0.0805	0.1638	-0.0690
3	0.0722	-0.0112	-0.0129
4	0.0589	0.1423	0.0146
5	0.0417	-0.0417	0.0200
6	0.0216	0.1049	0.0229
7	0.0000	-0.0833	0.0420
8	-0.0216	0.0618	0.0823
9	-0.0417	-0.1250	0.1301
10	-0.0589	0.0244	0.1603
11	-0.0722	-0.1555	0.1511
12	-0.0805	0.0028	0.0969
13	-0.0833	-0.1667	0.0127
14	-0.0805	0.0028	-0.0727
15	-0.0722	-0.1555	-0.1315
16	-0.0589	0.0244	-0.1505
17	-0.0417	-0.1250	-0.1363
18	-0.0216	0.0618	-0.1085
19	0.0000	-0.0833	-0.0859
20	0.0216	0.1049	-0.0733
21	0.0417	-0.0417	-0.0571
22	0.0589	0.1423	-0.0123
23	0.0722	-0.0112	0.0830
24	0.0805	0.1638	0.2346

Test inputs

All the considered above digital filters were represented by the pair of orthogonal components. From the each pair was chosen the one filter with the better performance. The coefficients of the chosen filters for N=24 are provided in the table 1.

Knowing the first orthogonal component of the signal it is possible to calculate the second orthogonal component. In order to get that done 2 samples can be used: current u_n and previous u_{n-1} obtained through the sampling period T [4]. Then the signal magnitude can be estimated as follows:

(14) $$U_{mn}=\frac{\sqrt{u_n^2+u_{n-1}^2-2u_nu_{n-1}\,cos(\frac{2\pi}{N})}}{sin(\frac{2\pi}{N})}$$

Minimum time delay of the one sampling period T during signal magnitude estimation as well as the impossibility of denominator vanishing are the main advantages of the equation (14).

Therefore, the equation (14) will be further applied to the all digital filters.

Signal of the fundamental frequency component superimposed with the exponentially decaying DC component is a widespread test input for the digital filters. All the considered digital filters have no significant errors in magnitude estimation of such a signal.

In reality the information about power system operation is transferred to the digital relay protection devices via current transformers (CT). That is why the digital filters performance evaluation is necessary to carry out with the help of the test signal whose waveform must be close to the CT secondary current waveform during power system faults. Such a signal can be obtained with the help of mathematical modeling.

Using the Simulink-SimPowerSystems [5] dynamic simulation environment the power system and the current measuring element models were developed (Fig.1).

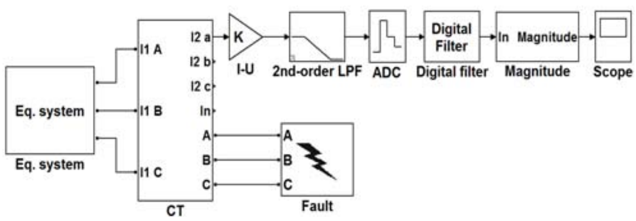


Fig.1. Model structure.

The model consists of the next elements:

- wye-connected CTs model (*CT*) in accordance to [7];
- three-phase equivalent supply system model (*Eq. system*) with the next parameters: rated voltage U_n, three-phase short-circuit current I_{sc} and the power system time constant τ;
- short circuitor block (*Fault*) which is used to set the time of the fault occurrence and its type;
- input transducer ($I \rightarrow U$) which provides at its output the voltage proportional to the input current;
- anti-aliasing second-order analogue low-pass filter (*2nd-order LPF*);
- ADC with the quantization time of T=0.02/N (*ADC*) which is equal to the sampling period;
- digital filter block (*Digital filter*) which is used for the fundamental frequency orthogonal component determination;
- fundamental frequency component magnitude estimation block (*Magnitude*) in accordance to equation (14).

The settings of the model elements

To obtain the digital filter input signal, the CT model with the next parameters was used: CTs ratio – 300/5 A and rated accuracy limit factor ALF =20. It should be noted that the CTs actual resistive secondary burden is equal to the rated resistive secondary burden. All shown below waveforms are referred to the CT located in phase A. In all cases a three-phase fault was considered.

The steady-state three-phase short circuit current with a power system time constant τ=0.05 s is assumed to I_{sc}=5000 A, that is a priori less than the maximum permissible current, which is defined by the ALF value.

The presence of the decaying DC component in the short-circuit current, even in case of its permissible value, drives the CTs into saturation and as a consequence the secondary current waveform becomes distorted. The harmonic content at the input (primary current) and at the output (secondary current) of the CT is shown in Fig.2.

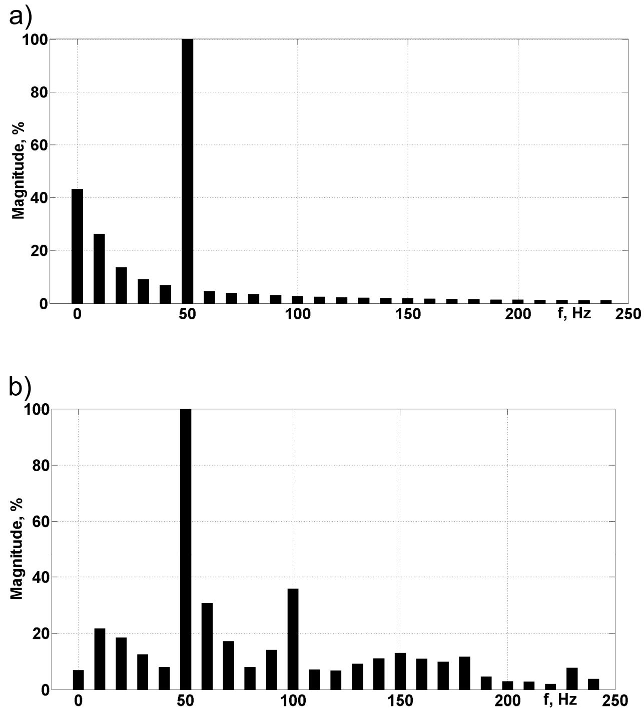


Fig.2. Harmonic content of the input (*a*) and output signal (*b*).

The key feature of the distorted CT secondary current is the presence of interharmonics, which are not included in any considered model of the signal.

Thus, the CT provides the undistorted secondary current to the digital relay protection devices only in case of primary fault current waveform is near to the sinusoidal and its value doesn't exceed the maximum permissible level, defined by ALF, and also the CT secondary burden must be equal or less then the rated. That's why the digital filter must insure the proper functioning not so much in case of the exponentially decaying DC component presence as in the case of the distorted CT secondary current.

Fig.3 illustrates the digital filters response to the signal which is proportional to the distorted CT secondary current (curve 3). LES-based digital filter (curve 2) shows the worst performance and cannot be recommended for use in the digital relay protection devices.

CF and OCF have the most reliable signal magnitude estimation and show almost equal results (curve 1).

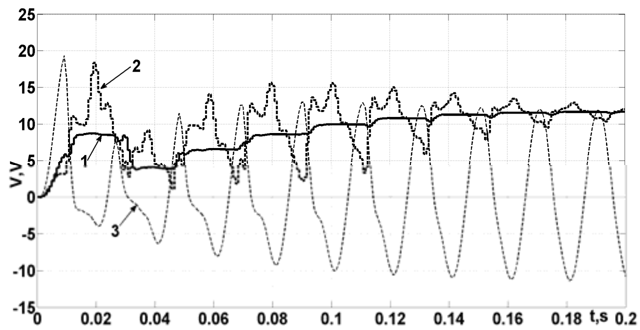
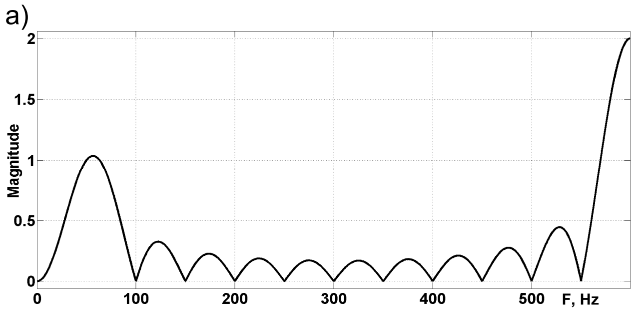


Fig.3. Signal magnitude estimation by digital filters.

The usage of the window functions helps significantly improve the digital filtering of the distorted signals. Each digital filter has the frequency response similar to the frequency response of the OCF, which is shown in Fig.4a. Frequency response has the main lobe in which center the frequency of interest (50 Hz) is located and sidelobes which has zeros at the harmonics of fundamental frequency.

Whereas the interharmonics which are located between the suppressed frequencies will be significantly amplified. The signal processing with the help of window functions allows dramatically reduce the amplitude of the sidelobes (Fig.4b).

a)



b)

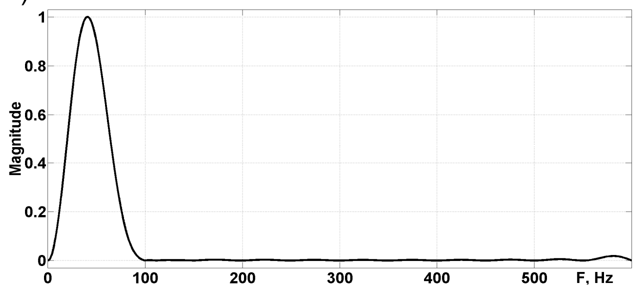


Fig.4. Frequency response: OCF (*a*), OCF with Hamming window (*b*)

For this purposes a cosine windows are widely used, e.g. Hamming window [8]:

$$w(n) = 0{,}54 - 0{,}46 \cdot cos(\frac{2\pi n}{N-1}) \quad (16)$$

This window function is an extra digital filter, which is connected in series with the main digital filter of the presented current measurement element (Fig.1). That's why the signal under control is undergoing the double processing – by window function and the main digital filter. Mathematically, such a process is called discrete linear convolution $s(n)$ of two discrete signal vectors: digital filter coefficients $a(n)$, n=0…N-1 and Hamming window coefficients $w(n)$, n=0…M-1, wherein, in general case, $N \neq M$:

$$s(n) = \sum_{m=0}^{n} a(n) \cdot w(n-m), \; n = 0...(N+M-2) \quad (17)$$

Fig.5 allows to estimate the effect of window function usage when the analogue LPF-2 is excluded from the presented model (Fig.1).

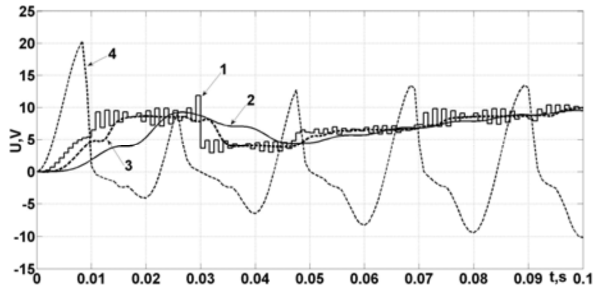


Fig.5. 1-OCF output signal, 2-HF output signal, 3-output signal of the HF with the shortened Hamming window, 4-fault signal which is proportional to the CT secondary current.

Output signal waveform of the hybrid digital filter (HF), which is formed of the OCF and Hamming window, is smoother (curve 2), but the settling time is greater than that of OCF (curve 1).In addition, if the number of the window function coefficients is less than the number of samples per cycle N, then the hybrid filter performance is increasing.

In Fig.5 curve 3 corresponds to this case in which the OCF has 24 coefficients and the Hamming window – 8.

In general, the current measurement element performance is affected by the next 3 filters: LPF-2, digital filter and Hamming window. Moreover if the reliable output signal settling time of the digital filter in case of periodic signal input is fixed and approximately equal to one fundamental frequency period, the settling time of the LPF-2 and Hamming function depends on desired filtering performance. Therefore by varying the LPF-2 cut-off frequency and the Hamming window size it is possible to achieve the current measurement channel optimal performance.

Conclusion

1. The algorithms for calculation the coefficients of the digital filters used in microprocessor-based (digital) relays are developed.
2. It was found that the Cosine and Orthogonal Components Former digital filters have the best performance in case of input waveform is close to the real distorted CT secondary current.
3. Ways of digital filtering improvement based on the window functions usage are considered.

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