Algorithm of periods determination of various priority tasks implementation in the measurement and control node

Abstract. In this paper a new algorithm for periods determination of tasks implementation in the uniprocessor measuring and control system (MCS) node with elastic scheduling model is presented. Assigned solution means modification of frequency of all tasks. The level of modification depends on the scope of permissible periods changes, the tasks execution times and the tasks priority.

Streszczenie. W artykule przedstawiono nowy algorytm doboru okresów realizacji zadań w jednoprocesorowym węźle systemu pomiarowo – sterującym z elastycznym modelem szeregowania zadań. Wyznaczono rozwiązanie oznacza modyfikację częstotliwości wykonywania wszystkich zadań jednak w różnym stopniu w zależności od zakresu dopuszczalnych zmian okresu wykonywania, czasu wykonywania zadania oraz jego priorytetu. (Algorytm doboru okresów realizacji zadań o różnym priorytecie w węźle pomiarowo sterującym).

Keywords: scheduling tasks, elastic model of task scheduling, modification of task periods.

Słowa kluczowe: szeregowanie zadań, elastyczny model szeregowania zadań, modyfikacja okresów wykonywania zadań.

Introduction

In a distributed management control system (MCS) at each node tasks are carried out on different periods of exercise. In the so-called static scheduling task model (SSTM) [1], for a single-node MCS, each task is described by the following parameters: $C$ – maximum execution time for the task, $T_{nom}$ – the period of occurrence of the task and $D$ – indicating the relative deadline time limit constraint within which the processor must finish the task. Parameter useful at the design stage is the utilization coefficient of resources $U$, which may not exceed a predetermined value $U_{su}$:

\begin{align}
(1) \quad & U = \sum_{i=1}^{n} C_i / T_{nom_i}, \\
(2) \quad & U \leq U_{su},
\end{align}

where: $n$ – number of tasks in MCS node.

$U_{su}$ – value of the utilization coefficient established during the design process. For example, in the method Earliest Deadline First Scheduling (EDF) $U_{su}$ is equal to one (full utilization of resources junction) [1] and in the method of Rate Monotonic Scheduling (RM) is calculated using a formula [1]:

\begin{equation}
(3) \quad U_{su} = n \cdot \left(2^{1/n} - 1\right).
\end{equation}

Due to the limitations of the SSTM an elastic scheduling task model (ESTM) was introduced. When the MCS node has to implement additional critical tasks (tasks like "hard" [1] or "safety-critical" [1]) then execution frequency of other tasks must be reduced. It is necessary preserve stability of the MCS node. In the ESTM, the period length of individual tasks may be modified by the node in the range of $T_{min}$ to $T_{max}$. Additionally selected values of periods $T_{sel}$ for all tasks have to fulfill the condition (2).

An exemplary timing diagram of two tasks $t_1$ and $t_2$ implementation is shown in Figure 1a). The periods of these tasks must be changed to enable implementation of the additional tasks $t_1$, as shown in Figure 1b). There the gray color is used to indicate nominal periods of tasks $t_1$ and $t_2$. The black color is used for show periods $T_{sel_1}$ and $T_{sel_2}$ of these tasks after modification necessary to enable execution of task $t_1$. This is the way the ESTM makes possible to carry out additional tasks properly utilizing the resources of the MCS node.

There are different ways to solve the scheduling problem. For example, in [2] the concept of macro and microcycle was introduced. Microcycle is the time interval in which it may be reported to execute one or more tasks. The macrocycle is determined after finding a repeating pattern of microcycles. An evolutionary algorithm is used to solve the problem. The solution obtained this way may be acceptable, but not optimal.

Determine the timing of tasks in [3, 4, 5] has been performed by SMSZ and ranges $T_{min}$ and $T_{max}$ of possible modifications the period $T$ for each task were used. Added flexibility coefficients $e$ impose proportionate possible modification of the period $T$ of each task. The idea of a flexible scheduling model, as presented in [3, 4, 6] can be compared to the phenomenon of springs tension, which are connected to each other. In this solution, the periods of all tasks are iteratively increased in proportion to the value of $T_{sel}$ satisfying the condition (2).

In papers [7,8] the other solution is presented, the periods of tasks selection is performed by means of heuristic algorithms. In addition, the validity weighting factors $(w_i)$ of tasks were introduced. This approach allows to take tasks priorities into account, i.e. periods of tasks of lower importance are modified firstly.
This article describes a novel algorithm for selecting periods in ESTM. The proposed algorithm takes into account: values of \( vwf \) for each task, maximum execution time for the task \( C \) and the scope of possible periods changes in the range of from \( T_{\text{min}} \) to \( T_{\text{max}} \). The algorithm results always meet the established value of the utilization coefficient \( U_{\text{nom}} \) with the specified precision \( \delta \). The design of the algorithm is focused on the smallest calculations load of a MCS single-node.

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Algorithm for tasks periods selecting in the MCS node

In the proposed algorithm determining the periods of perform of various tasks \( T_{sel} \) involves increasing the initial (nominal) periods values. Hence for each period \( T_i \), multiple \( k_{sel} \) of period increments is calculated.

\[
(4) \quad T_{seli} = T_{nomi} + k_{seli} \cdot A \Delta T_i, \\
\text{where the multiple } k_{seli} \text{ does not have to be an integer, } \Delta T_i \text{ is an increment of } T_i.
\]

Increments of \( T_i \) values are determined individually for each task with the following parameters: the participation of the particular task in resource use node expressed as quotient of \( C_i/T_i \) and the scope of possible changes periods in the range of from \( T_{nomi} \) to \( T_{max} \), and the validity weighting factor of the task \( vwf_i \). The value of the \( vwf \) depends on the tasks priority and is the smallest for the tasks with the highest priority.

\[
(5) \quad \Delta T_i = (T_{maxi} - T_{nomi}) \cdot \frac{C_i}{T_{nomi}} \cdot vwf_i.
\]

It can be expected that in the target solution periods of certain tasks, particularly those with low priority, can achieve the maximum value (i.e. saturation):

\[
(6) \quad T_{seli} = T_{maxi} = T_{nomi} + ks_i \cdot \Delta T_i, \\
\text{where } ks_i = \text{ multiple of } i\text{-th task saturation when } T_{nomi} = T_{maxi}. 
\]

It is determined for every task and it can be calculated using the formula:

\[
(7) \quad ks_i = \frac{T_{maxi} - T_{nomi}}{\Delta T_i} = \frac{T_{nomi}}{C_i \cdot vwf_i}.
\]

The algorithm for selecting periods of tasks \( T_{seli} \) is carried out in two stages. The first step is to select two successive (ordered cumulative) multiples of saturation, between which the sought multiple \( k_{seli} \) value is located. The second stage involves the determination \( k_{seli} \) value by bisection method. A flow diagram is shown in Figure 2.

The first operation, after entering data is to check the condition of reachability of the predetermined ratio of resource use \( U_{nom} \).

If there is a solution, then for each task the increment of period \( \Delta T_i \), (5) and the saturation multiple \( ks_i \), (7) are determined.

Tasks are ordered according to increasing values of a saturation multiple. This is important, because with increasing multiple \( k \), value of the utilization coefficient of resources \( U \) decreases monotonically. Then, for saturation multiple \( k \) taking value of each ascend \( ks_i \), the values of resource use \( U \) are calculated by the formula:

\[
(8) \quad U = \sum_{i: k_{seli} \leq k} \frac{C_i}{T_i} + \sum_{i: k_{seli} > k} \frac{C_i}{T_{maxi}}, \\
\text{where } T_i \text{ is calculated using the similar to (4) formula, i.e.:}
\]

\[
(9) \quad T_i = T_{nomi} + k \cdot \Delta T_i, \\
\]

\[
(10) \quad 0 < U_{usu} - U < \delta.
\]

The last used multiple \( k \) value is the sought \( k_{sel} \) solution which is used to calculate all \( T_{sel} \) periods for all tasks.

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Fig. 2. Flowchart of algorithm of periods determination of various priority tasks implementation in the MCS node.

These calculations are repeated until the next obtained value \( U \) is greater than the established value \( U_{nom} \). The wanted multiple \( k_{sel} \) value is located between the current \( ks_i \), and the previous \( ks_{i-1} \) value. This completes the first stage of the calculation. This calculation phase on the graphs in Fig. 3, is indicated by the vertical dashed line.

The second stage of algorithm is the iterative calculations narrowing \( k \) value range containing the search \( k_{sel} \) value by the bisection method. The value of \( U \) is determined by means of the formula (8) taking into consideration the value of multiple \( k \) in the middle of the current \( k \) value range. The half of range without \( k_{sel} \) solution is discarded. The calculations are continued until the difference values of the just calculated utilization factor \( U \) and the predetermined value \( U_{nom} \) is greater than zero and less than the selected precision \( \delta \).
Periods of tasks that multiple $k_s_i$ saturation is less than $k_{sel}$ are equal to maximum possible period value $T_{max}$.

$$T_{sel_i} = \begin{cases} T_{nom_i} + k_{sel} \cdot A T_i, & \text{when } k_{sel} < k_s_i \\ \frac{1}{T_{max_i}}, & \text{when } k_{sel} \geq k_s_i \end{cases}$$

The results of simulation studies

Algorithm simulation tests were conducted for a sample single-node MCS. It was assumed implementation of 81 tasks with parameters which are combinations of possible settings values given in Table 1. The course of the algorithm implementation is shown in Fig. 3.

Table 1. Data for the simulation studies

<table>
<thead>
<tr>
<th>$T_{nom}$</th>
<th>$T_{max}$</th>
<th>$C$</th>
<th>$vwf$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>5</td>
<td>1000</td>
<td>2</td>
<td>0.1</td>
</tr>
<tr>
<td>10</td>
<td>5000</td>
<td>5</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The graph in Fig. 3a) shows successive values of the multiplicity factor $k_s$, for which subsequent values of coefficient $U$ were determined. The graph in Fig. 3b) shows progress of $U$ value determination. The eighth determined value of the utilization coefficient is greater then the predetermined value $U_8 > U_{thr}$, while another, the ninth value is less $U_9 < U_{thr}$. On the graph it is indicated by the vertical dashed line. These values correspond to the multiple values $k_{s8}=50$ and $k_{s9}=100$. Then the calculation process goes on to the next stage of the algorithm where in next iterations the bisection method is used to determine values of $k$ more closer to the target value $k_{sel}$. When the next obtained $U$-value is close to the $U_{thr}$ with established precision $\delta$ the $k_{sel}$ is reached.

Simulation studies were made for the proposed iterative algorithm and selected heuristic algorithms to their comparison. Computational complexity determined by the number of determinations of the fitness function was compared. It has been applied for various input data sets which are described in the work [7] and [8]. The various timing parameters was prepared for 10 measurement scenarios. The studies, included a different number of tasks implemented in a MCS node, are summarized in table 2.

Table 2. The number of tasks in different measurement scenarios

<table>
<thead>
<tr>
<th>Measuring scenario</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5-6</th>
<th>7-9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>The number of tasks</td>
<td>3</td>
<td>81</td>
<td>3888</td>
<td>81</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

In each of the scenarios were used different input parameters: $T_{nom}$, $T_{max}$, $C$ and $vwf$. Some of the data used in each scenario was derived from the literature (scenario 2 is taken from [7], the scenarios from 5 to 9 are taken from [4]).

These simulation studies were designed to investigate how the computational complexity depends on the pre-established values of the accuracy $\delta$ (Formula 10). During the simulation studies, for each scenario (Table 1), $\delta$ value was taken as a number from 0.2 to 0.00002 with tenfold reduction step. The results of simulation are shown in Fig. 4 and 5.
Reducing the δ value (Figure 4) increases computation cost. The reason for this is necessity of more computations to obtain a predetermined δ value by means of the bisection method. For the sixth scenario, with different δ values, the same computational complexity value is obtained. The assumptions of this scenario cause that the formula 2 meets only one combination of timing parameters. The first solution is together the final. On the basis of obtained results, it seems that, selection of a specific value δ means finding a compromise between computational complexity and quality (accuracy) of the resulting $U_\delta$-value.

In the new algorithm, always, if it is possible, a predetermined δ value is achieved. In Fig. 5 relationship of required and obtained δ values for each scenario is shown. There are not provided results for 6 scenario because obtained δ values are always equal 0.

In next simulation studies the new algorithm is compared with selected heuristic algorithms used earlier in the ESTM. For comparative purposes the number of basic operations has been accepted. The fitness function was adopted as basic operation in the heuristic algorithms. And as a basic operation in the in the proposed algorithm was accepted the number of calculations $U_\delta$ necessary to achieve the accuracy δ=0.002. In the case of heuristic algorithms simulation studies were carried out for different sizes of the initial populations. Populations were 2 to the powers of: 2, 5 and 10. Figure 6 shows the number of basic operations required for various algorithms and for different measuring scenarios.

On the basis of the adopted criteria (number of basic operations), the new proposed algorithm is the least computationally expensive solution. The study also showed that, for the tested heuristic algorithms, increase the size of the initial population causes increase the number of basic operations.

**Summary**

The new original algorithm for selecting periods of the tasks with different priority in the MCS node is presented. The algorithm is simple in structure and always finds a solution (if exists). The algorithm determines periods of the tasks in the ESTM, taking into account on the scope of permissible periods changes, the tasks execution times and the tasks priority, to achieve the $U_\delta$ rate of resources with established precision δ. In relation to the tested heuristics the proposed algorithm has much smaller computational cost.

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**REFERENCES**


