

About the Biot-Savart-Laplace law and its use for calculations in high-voltage AC installations

Abstract. Several forms of writing the Biot-Savart-Laplace law and formulas obtained using modern methods for calculating magnetic fields are given. It is mentioned that there are inaccuracies in the interpretation of the Biot-Savart-Laplace law. It is shown that in the electric power industry a simple formula based on the use of the Biot-Savart-Laplace law and the experimentally obtained correction coefficients can be used to calculate the magnetic fields produced by alternating currents in the buses and in power lines. The nomogram and the table for their determination are presented.

Streszczenie. Niekiedy można spotkać niedokładności w interpretacji prawa Biot-Savart-Laplace. Stosowane są poprawki do obliczania pola magnetycznego wytwarzanego przez prąd przemienny. Zaprezentowano program ułatwiający określenie tych poprawek. **O prawie Biota-Savarta_Laplace w zastosowaniu do obliczania wysokonapięciowych instalacji prądu przemiennego**

Keywords: law, magnetic field, correction coefficient.

Słowa kluczowe: Prawo Biota-Savarta, obliczenie pola magnetycznego

Introduction

In 2020, it will be 200 years since the great Ampere discovered the effect of direct current (DC) on a magnetic needle, and Laplace, on the basis of studies by Biot and Savart, presented the induction \vec{B} of magnetic field as a function (1) of the distance R between the point under consideration and a section $d\vec{l}$ of a conductor with a DC I (known as the Biot-Savart-Laplace law [1-6])

$$(1) \quad \vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{[d\vec{l} \times \vec{R}_0]}{R^2},$$

where: μ_0 - the magnetic permeability of air, \vec{R}_0 - a unit vector drawn from $d\vec{l}$ to the point at which the magnetic induction is determined (Fig. 1).

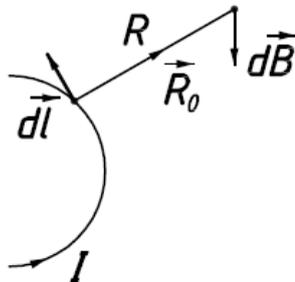


Fig. 1. Explanatory scheme for the Biot-Savart-Laplace law

Formula (1) is the first form of law recording and allows integration over the entire length of a closed loop with a current.

If in (1) the current I is entered as a constant value into the vector product and replaced $I d\vec{l}$ by $\vec{j} dV$ (where dV is the volume element of the conductor with the current density \vec{j}), then

$$(2) \quad \vec{B} = \frac{\mu_0}{4\pi} \int_V \frac{[\vec{j} \times \vec{R}_0]}{R^2} dV$$

Formula (2) - the second form of the law; Integration is performed over the volume occupied by the current.

During these two hundred years, the law was recorded very differently with all sorts of refinements, depending on the depth of the examination and the required accuracy of the calculations. However, most recently it has been shown

that some interpretations of the Biot-Savart-Laplace law (sometimes used even in textbooks) should be refined, for example, as was done in [7] in determining the magnetic potential vector \vec{A} in the two-dimensional case for a wire with alternating current (AC), located above the metal surface.

At present, most modern systems of modeling magnetic fields are based on the finite element method [8, 9], an alternative to which, under certain conditions, can be the method proposed in [10]. In addition, due to the improvement of parallel computing technologies, the model of a magnetic field based on the theory of similarity and the Monte Carlo method has been developed [11]. The method of determining magnetic fields by solving the Dirichlet and Sturm-Liouville problems [12], as well as some others, are sometimes used. The basic formulas for determining the induction B of magnetic field, obtained by the methods listed above are given in Table 1.

Table 1. Methods for determining the induction of a magnetic field

Method	Formula
The similarity theory and the Monte Carlo method	$(3) \quad \vec{B} = -\frac{\mu_0}{m_H dS} \sum_{i=1}^{dN} (M_i v_{yi} z_i) \vec{i} +$ $+ \frac{\mu_0}{m_H dS'} \sum_{i=1}^{dN} (M_i v_{xi} z_i) \vec{j}$ <p>where: m_H - the scale of the strength of the magnetic field, dS' - the elementary region of the model, dN - the number of particles with a mass M_i that emits each time slot at a speed v_i, $z_i = \pm 1$ - a coefficient that takes into account the direction of the current that emitted the particle.</p>
Solution of the Dirichlet and Sturm-Liouville problems	$(4) \quad B = -\frac{2}{\alpha \sqrt{\lambda}} \times$ $\times \sum_{k=1}^N U_k (\cos \sqrt{\lambda} \varphi_k - \cos \sqrt{\lambda} \varphi_{k-1})$ <p>where: α - the normalizing coefficient, λ - the real parameter, U_k - the scalar magnetic potential, φ_k - the coordinates of the calculated point.</p>

In this variety of fairly complex methods it is difficult to navigate not only the bachelor, but also the master. Meanwhile, in the design and operation of devices in

various fields of technology, elementary formulas derived from the use of the Biot-Savart-Laplace law can be useful. For example, in the electric power industry, in order to calculate the induction of magnetic fields created by the AC flowing in the buses of complete switchgear (CSG) and complete current conductors (CCC), as well as in power transmission lines (PTL), we propose the following formula

$$(5) \quad B = k_{corr} B_{calc} = k_{corr} \frac{\mu_0 I}{4\pi R} (\cos \alpha_1 - \cos \alpha_2)$$

where: R - the distance between the calculated point m and the conductor (Fig. 2).

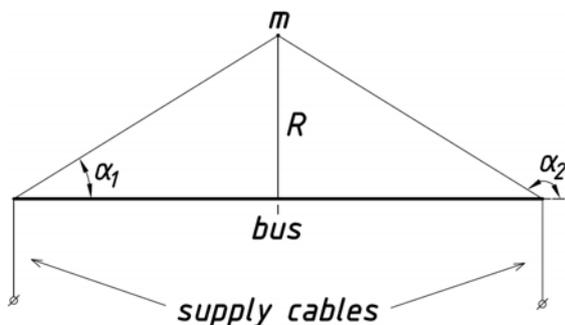


Fig. 2. Scheme for calculating the induction of a magnetic field

B_{calc} was obtained in [3], starting from (1), for thin rectilinear conductors of circular cross section of finite length when DC flow in them. The coefficient k_{corr} takes into account that the current in the electrical installations under consideration is AC, the CSG buses are not circular, the CCC buses are not thin, and the PTL have sags that do not allow them to be considered as rectilinear conductors.

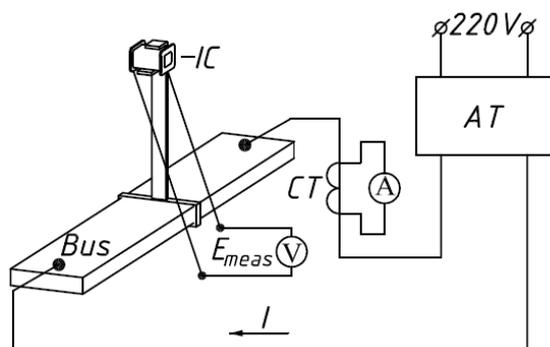


Fig. 3. Scheme of the experimental installation

Determination of correction coefficients

The value k_{corr} is determined experimentally for industrial-manufactured and widely used flat buses of CSG with a cross-section $(60 \times 6) \text{ mm}^2$ and a length L of 0.3 to 1.2 m, as well as for tubular buses of CCC with a length $L = 2$ m (diameters 0.12 m, 0.14 m, 0.18 m, 0.28 m, 0.424 m, 0.65 m). The current I in the buses (Fig.3) was regulated by the autotransformer TDGC2-20K and measured by the current transformer TTE-125-4000/5A and the multimeter Fluke 87V. The induction coil (IC) with the number of turns $\omega = 2250$ and the cross-sectional area of the winding $s = 554 \text{ mm}^2$, which was located at the center of the buses length at distances of $R = (20 \div 300) \text{ mm}$ from them (Fig. 2) by means of the construction developed by us using 3D printing, was used as the sensor of magnetic field. The measurement of the magnitude of the electromotive force (EMF) $E_{meas.1}$ induced by the magnetic field of the

buses was measured on the terminals of the IC by the multimeter Fluke 87V (all devices are verified by metrological services).

EMF $E_{meas.1}$, measured on the terminals of the IC for the flow of AC in the buses I from 80 to 640 A, on the basis of the well-known formula from [13], we can represent as:

$$(6) \quad E_{meas.1} = 2\pi f \omega s B_{\Sigma} = k_1 B_{\Sigma} = k_1 (B_{calc} + B_{int} + B_{sh})$$

where: f - the frequency of the current ($f = 50 \text{ Hz}$), B_{Σ} - the total induction of the magnetic field, which induces EMF $E_{meas.1}$ on the terminals of the IC.

B_{Σ} is decomposed into the following components: B_{calc} is the calculated component of the induction, determined from (5) without a coefficient k_{corr} for given values I , R , $\cos \alpha_1$ and $\cos \alpha_2$ for the bus under consideration, B_{int} is the component created by interference from all nearby electrical installations, supplying cables and the Earth, B_{sh} is the component, determined by the influence of the shape and size of the bus.

To determine B_{sh} , experiments were carried out, the only difference from which was the use, instead of flat and tubular buses, of thin rectilinear conductors (cables with the diameter of $d = 4,5 \text{ mm}$) of the same lengths. The EMF values $E_{meas.2}$ measured on the terminals of the IC for experiments with thin conductors, in analogy with (6), are as follows:

$$(7) \quad E_{meas.2} = k_1 (B_{calc} + B_{int})$$

From the joint consideration of (6) and (7), taking into account that $E_{meas.1}$ and $E_{meas.2}$ were measured at the same currents I in the buses and in thin conductors, we have

$$(8) \quad B_{sh} = \frac{E_{meas.1} - E_{meas.2}}{k_1}$$

Then the correction coefficient k_{corr} , taking into account the influence of the shape and size of the bus, in which the AC flows, is defined as follows:

$$(9) \quad k_{corr.1} = \frac{B_{calc} + B_{sh}}{B_{calc}} = 1 + \frac{B_{sh}}{B_{calc}}$$

The nomogram (Fig.4), which allows determining the coefficient $k_{corr.1}$ for the CSG and CCC buses, was constructed from the results of experiments with the help of power-law approximation (tested by the Fisher criterion). Note that the value $k_{corr.1}$ does not depend on the current I in the bus and is determined only by its shape, length L (for flat buses) or diameter d (for tubular buses) and the distance R between the calculated point and the bus. To determine the coefficient $k_{corr.1}$ of the nomogram on its horizontal scale, select the point corresponding to the required value R , and, restoring a perpendicular from it, find the point of intersection with the curve corresponding to the considered length L (in the case of a flat bus) or the diameter d (in the case of a tubular bus). Then, from the intersection point, drop the perpendicular to the vertical scale, along which the value $k_{corr.1}$ is determined.

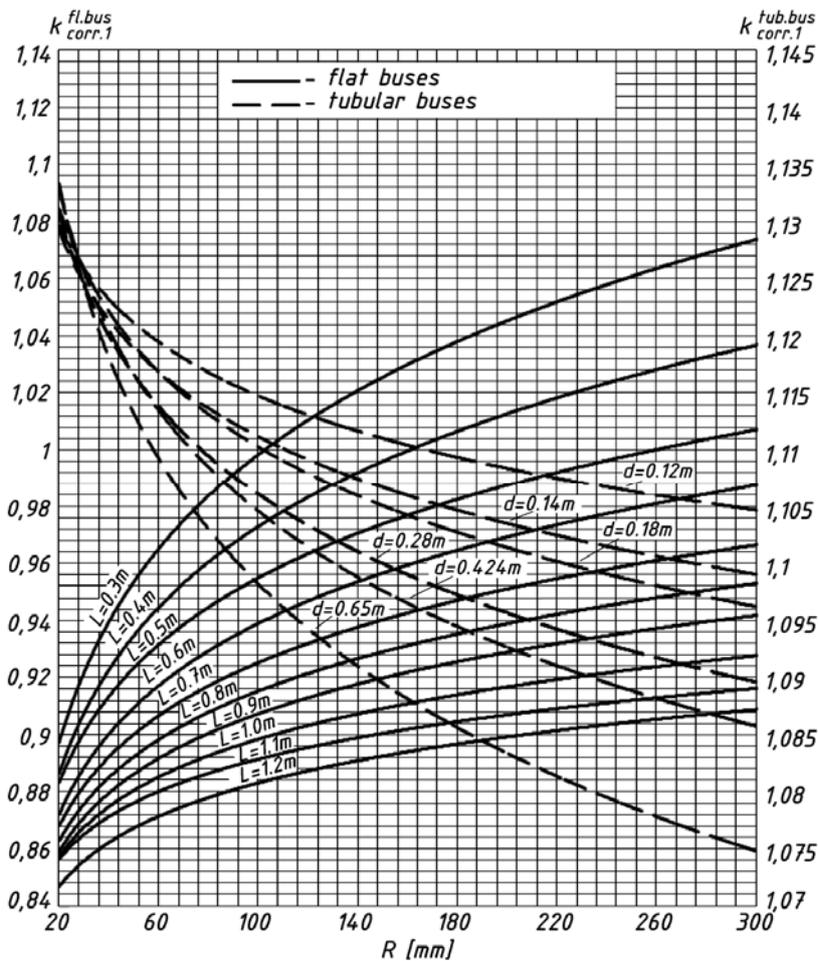


Fig. 4. Nomogram for determining the coefficient $k_{corr.1}$

Table 2. Parameters of the PTL models

PTL voltage [kV]	Support height, H_{sup} [mm]	Span length, L_{sp} [mm]	Sag, f_s [mm]	Reduction scale
10	560	3000	25	1÷25
35	576	3030	28	1÷33
110	373	2985	64	1÷67
220	416	3488	132	1÷86
500	388	3604	191	1÷111

We conducted studies in which the well-known [14] method of complete simulation and similarity was used to determine the coefficient $k_{corr.2}$ for the PTL. Under laboratory conditions, taking into account the accepted reduction scales (Table 2), the models of the PTL (Fig. 5) of different voltage classes with the parameters regulated in [15, 16] were collected.

As the sensor of magnetic field the same IC was used as in the experiments with CSG and CCC buses, which was located in the middle of the span length of each model of the PTL at the calculated points $m_1 \div m_7$. The range of AC I in the PTL was 80÷640 A. Current control and measurement of its values, as well as the EMF values $E_{meas.3}$ induced by the magnetic field of the PTL, was carried out using the scheme shown in Fig. 3. EMF $E_{meas.3}$ is represented as follows:

$$(10) \quad E_{meas.3} = k_1 (B_{calc} + B_{add})$$

where: B_{calc} is the calculated component of the induction, determined by (5) without the coefficient k_{corr} using the

scheme (Fig. 2), B_{add} is an additional induction component determined by the influence of the sag of the PTL and interference created by the earth's magnetic field and other PTLs.

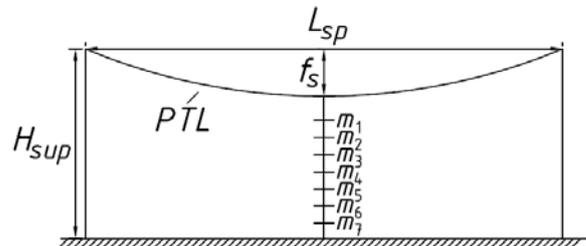


Fig. 5. Scheme of the PTL model

From (5) and (10), bearing in mind $k_{corr.2} = k_{corr}$, we obtain a coefficient that takes into account the influence of the sag of the PTL in which the AC flows and the interference created by the earth's magnetic field:

$$(11) \quad k_{corr.2} = \frac{B_{calc} + B_{add}}{B_{calc}}$$

Based on the results of experiments with PTL models using power-law approximation (tested by the Fisher criterion), formula was obtained:

$$(12) \quad k_{corr.2} = ar^{-b}$$

where: r - the distance between the lowest point of the PTL in the span and the given sensor point.

Table 3 shows the values of the coefficients a and b , which make it possible to determine $k_{corr.2}$ for the PTLs of all voltage classes. So, for example, it is established that when the sensor of magnetic field is placed under the PTL in the middle of the span length at a distance of $h_E = 1$ m from the Earth's surface, the value $k_{corr.2}$ is: for PTL-10 kV - 1.11; 35 kV - 1.06; 110 kV - 1.15; 220 kV - 1.34; 500 kV - 1.75.

Table 3. The values of the coefficients a and b in (14)

PTL voltage [kV]	a	b
10	1.46	0.11
35	1.57	0.14
110	2.95	0.31
220	5.32	0.43
500	12.62	0.64

Conclusions:

1. Inaccuracies in the interpretation of the Biot-Savart-Laplace law, admitted in some textbooks, must be identified and eliminated, for example, as was done in [7]. The refinements given should, if it is possible, be confirmed by the results of the experiments.

2. In the electric power industry, to calculate the magnetic fields created by alternating currents flowing in the buses of complete switchgears and complete current conductors, as well as in power transmission lines, one can use formula (5) that does not require special education based on the use of the Biot-Savart-Laplace law and correction coefficients k_{corr} obtained in this paper.

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