High performances of Polynomial and Nonlinear Backstepping Control Strategies of an Induction Motor fed by Matrix Converter

Abstract. The main objective of this paper is to present the performance analysis of the oriented flux control of an induction motor associated with a matrix converter. A polynomial technique of RST type is used for speed control. As for the control of internal current loops, the technique used is based on the nonlinear approach. Overall, the proposed feedback law is asymptotically stable, which is shown in the context of the Lyapunov theory. The design of the control laws by the backstepping technique has been detailed while taking account of the non-linearities in the design phase of the control system. The objective is to obtain a good transient response and a good capacity of rejection of charge disturbance. The induction motor incorporating the proposed control techniques (RST-Backstepping) has been successfully implemented in numerical simulation using Matlab/Simulink under different operating conditions where the static and dynamic responses of the system are evaluated. It can be seen that the proposed control technique provides good speed monitoring performance. For internal loops, overall stability is ensured and the proposed approach presents good robustness to the uncertainties of the system parameters.


Keywords: RST, Backstepping, Induction Motor (IM), Nonlinear Control, Matrix Converter (MC).

Słownikiowe: silnik indukcyjny, sterowanie nieliniowe, przetwornik macierzowy.

Introduction

The extraordinary progresses recorded in power semiconductor technology, digital electronics and control theory have permitted to AC motors face the high requirements in terms effectiveness of control with high dynamic performances difficult to obtain in industrial sector. Actually, induction motors are the most widely used at variable speed and torque due to their simplicity, robustness, efficiency and reliability. The significant progresses mentioned above has made it possible to implement effective controls for driving the induction motor [1-3].

Currently, high-performance electrical drives require quick and accurate responses, with rapid rejection of all disturbances and insensitivity to parameter variations. The dynamic behavior of an AC motor can be significantly improved by using the vector control theory where the machine variables are transformed into a set of orthogonal axes so that flux and torque can be controlled separately [4-8].

Moreover, the matrix converter is a power converter of great importance. It has been introduced and put into operation over the past two decades. In the literature, there are only a few references concerning the use of matrix converters in drives based on inductive motors [9-12]. Thus, the drive of the induction motor supplied by a matrix converter presents a superiority compared to the voltage source in an inverter driven by the conventional pulse width modulation (PWM - VSI) technique due to the absence of short-lived capacitors, bi-directional electrical capacity, sinusoidal input/output current, and adjustable input power factor.

Conventional regulators remain, until today, the most used in many industrial applications based on Induction motors in conjunction with the oriented flow control method for speed control. About 90% of industrial controllers are PI/PID controllers [13]. The others are constructed of control systems that are based on various modern control techniques.

Although relatively easy to adjust, the PI/PID correctors do not always provide the required dynamic performance for target tracking and disturbance rejection, particularly for systems: (i) with Pure delay/ important Dead time (ii) of order greater than two (thus possessing more than one vibratory mode), (iii) with parameters varying in time, etc. [14-16].

However, the considerable scientific progress noted in the theory of non-linear control has enabled many researchers to propose systematic approaches, dealing with non-linearities, applied to the speed and/or position control of induction machines in order to improve the robustness of the control in spite of the parametric variations such as the variation of the rotor resistance of the motors. Note that these techniques require knowledge of the parameters of the system, usually used in the case of electrical machines [17-19].

In this paper, we present a polynomial control of type, associated with a nonlinear control strategy based on the backstepping approach applied to the control of an induction motor fed by a matrix converter. The objectives of this control strategy are to combine the RST-Backstepping control diagrams in order to improve the dynamic performance of the system and guarantee a total rejection of disturbances. We take advantage of the matrix converter which minimizes the ripples of internal variables such as current and torque.

The main topic of this paper is to design a simple control law compared to the works presented in the recent literature for the three-phase induction motor allowing high static and dynamic performances. The method based on the Backstepping approach establishes successive relationships to iteratively construct a systematic and robust control law, asymptotically stable according to Lyapunov stability theory, where the variation effect of some parameters and load perturbation can be considerably reduced by adding an integral action of the tracking errors at each step of the control of the currents which makes it possible to ensure a high precision of control with respect to the uncertain parametric variations.

Speed control is provided by the RST controller where the pole placement technique is used to ensure the stability of the closed loop system. This digital corrector (RST) can offer a very good alternative for high order and delayed systems. The structure of the regulator RST which acts differently on the setup and on the output which is the main
reason for this success, of which it can easily replace the
PID regulator in the industry.

The effectiveness of the control of the proposed
algorithm is verified by several simulation tests.

Mathematical modelling system
IM drive model
The system equation of induction motor in the Park
reference frame (d-q) model can be expressed as follows
[20-22]:
\[
\frac{d\theta}{dt} = \Omega
\]
\[
\frac{d\Omega}{dt} = \frac{1}{J} \left[ \frac{3}{2} p(\psi_{ad} V_{sq} - \psi_{sq} V_{id}) - f_c \Omega - T_l \right]
\]
\[
\frac{d\psi_{ad}}{dt} = V_{id} - R_s i_{id}
\]
\[
\frac{d\psi_{sq}}{dt} = V_{sq} - R_s i_{sq}
\]
\[
\frac{di_{id}}{dt} = \frac{1}{\sigma_L} \left( \frac{1}{T_s} + \frac{1}{T_f} \right) i_{id} - p\Omega \psi_{ad} + \frac{1}{\sigma_L T_f} \phi_{id} + \frac{1}{\sigma_L} \frac{V_{id}}{R_s}
\]
\[
\frac{di_{sq}}{dt} = \frac{1}{\sigma_L} \left( \frac{1}{T_s} + \frac{1}{T_f} \right) i_{sq} + p\Omega \psi_{sq} - \frac{1}{\sigma_L T_f} \phi_{sq} + \frac{1}{\sigma_L} \frac{V_{sq}}{R_s}
\]

In this model, \( \Omega \) and \( \theta \) are the mechanical speed and
angle respectively, \( V_{\text{d-axis}}, i_{\text{d-axis}} \) and \( \psi_{\text{d-axis}} \) represent the stator
voltages, currents and flux in the \((d-q)\) frame respectively.
Moreover, \( p \) denotes the number of pole pairs, \( R_s \) is stator
phase resistance, \( L_s \) is the leakage inductance in the stator
windings, \( T_s \) and \( T_f \) represent the stator and the rotor
time constant respectively, \( \sigma_L \) is the dispersion coefficient.
\( J \) is the moment of rotor inertia, \( f_c \) is the viscose friction
coefficient and \( T_l \) is the load torque.

Matrix converter model and Scalar algorithm strategy
The matrix converter has several advantages compared
to the conventional voltage or current source inverters. It
converts energy directly from the source to the load without
any intermediate power storage element and provides
sinusoidal input with minimal higher order harmonics and no
sub harmonics. It has inherent bi-directional energy flow
capability and a better control of the input displacement
factor with minimal energy storage requirements allowing to
get rid of bulky and lifetime-limited energy-storing
Capacitors.

In order to ensure operation, the scalar method
proposed by Roy and April [23-24] in 1987 uses a typical
method among several modulation methods to achieve a ratio
of 0.87 between the output voltage and converter input
voltage so that the switch actuating signals are calculated
directly from measurements of the input voltages. The
motivation behind their development is usually given as the
perceived complexity of the method of Venturini [25-27],
the value of any instantaneous output phase voltage \( V_j \)
\((V_a, V_b, V_c)\) is expressed as follows:

\[
v_{jN} = \frac{1}{T_J} (i_{K} V_K + i_{L} V_L + i_{M} V_M)
\]

In the scalar method, the switch actuating signals are
Calculated directly from measurements of the instantaneous
input voltages followed by a comparison of the quantities as
mentioned in the following algorithm:
1. Assign the subscript \( M \) to one of the three-phase input
voltages having a different polarity to the other,
2. Assign the subscript \( L \) to the smaller voltage (in absolute
value) of the two input voltages,
3. Assign the subscript \( K \) to the third input voltage.
where:

\[
M_{lj} = \frac{(v_{jN} - v_{iM})}{1.5V_j^2}
\]
\[
M_{Kj} = \frac{(v_{jN} - V_M)}{1.5V_j^2}
\]
\[
m_{MK} = 1 - (M_{lj} + M_{Kj})
\]
The output voltage is given by:

\[
v_{jN} = m_{Kj} V_K + m_{lj} V_L + m_{MK} V_M
\]
The modulation \( m_{ij} \) for the scalar coefficients method with
the value of \( Q_{\text{max}} = \frac{\pi}{2} \) is shown in equation (3).

\[
m_{ij} = \frac{V_{jN} - V_M}{1.5V_j^2} = \frac{2V_{jN} - 2V_M + \frac{2}{3} \sin(\omega t + \beta)}{1.5V_j^2}
\]

Fig 1 shows the structure of the matrix converter feeding
the induction motor. The input and the output voltages and
current can be expressed as vectors de

\[
V = \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}, \quad I = \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}
\]

where \( M \) is the transfer matrix given by:

\[
M(t) = \begin{bmatrix} m_{AA} & m_{AB} & m_{AC} \\ m_{BA} & m_{BB} & m_{BC} \\ m_{CA} & m_{CB} & m_{CC} \end{bmatrix}
\]

\[V = M(t) . V_i \text{ and } i=\text{[M(t)]}^{-1} i_j\]

![Fig.1: Circuit scheme of 3 phase to 3-phase matrix converter.](image)

**IM Control Strategy**

**Design of nonlinear Backstepping currents control**

The control objective is to design a suitable control law
for the IM servo drive system given by equations (1) so that
the state trajectory of the stator currents \( i_{sa,sa} \) can track the
desired stator currents \( i_{sa,sa} \) trajectory despite the variation
parameters and the presence of external load disturbance.
When all IM dynamics are well known, the backstepping
design for the uncertain IM servo drive system can be
described step-by-step.

**Stator current \( i_{sa} \) loop**

In this section we will present the design of the
asynchronous motor controls inputs \( V_{sa} \) and \( V_{sa}. To design
the control input \( V_{sa} \), we introduce the following tracking error:

\[
e_q = i_{sa} - i_{sa}
\]

Let the variable \( e_q \)
\[ V_1 = \frac{1}{2} (\ddot{\xi}^2 + \ddot{\xi}^2) \]

The time derivative of \( V_1 \) is given by:

\[ dV_1 = \ddot{\xi}(K_{q2} \dot{\xi} + K_{q2} \dot{\xi}) \]

\[ = \dot{\xi} \left[ \frac{d\dot{\xi}}{dt} + K_{q2} \dot{\xi} + K_{q2} \dot{\xi} \right] + \dot{\xi} \dot{\xi} \dot{\xi} \dot{\xi} \dot{\xi} \]

\[ V_2 = \frac{1}{2} (\ddot{\xi}^2 + \ddot{\xi}^2) \]

Stator current \( i_{sd} \) loop

To design the control input \( V_{sd} \), like for \( V_{sq} \) we introduce the following tracking error

\[ e_d = i_{sd} - i_d \]

Let the variable \( \xi \),

\[ \dot{\xi} = \frac{1}{K_{q2}} \dot{\xi} \]

With \( K_{q2} \) a twining gain. If we set \( \dot{\xi} = \frac{1}{K_{q2}} \dot{\xi} \),

\[ \dot{\xi} = \frac{1}{K_{q2}} \dot{\xi} \]

according to equation (4), we write:

\[ \dot{\xi} = \frac{1}{K_{q2}} \dot{\xi} \]

Let:

\[ \Phi_1 = -\dot{\xi} K_{q2} \]

With \( K_{q2} \) is a twining gain. Then, \( dV_1 \) in equation (9) can be written as:

\[ dV_1 = -K_{q2} \dot{\xi}^2 + \dot{\xi} \dot{\xi} K_{q2} (\dot{\xi} - i_{sd}) + \dot{\xi} \dot{\xi} K_{q2} (\dot{\xi} - i_{sd}) \]

\[ = -K_{q2} \dot{\xi}^2 + (\dot{\xi} + \dot{\xi} K_{q2}) (\dot{\xi} - i_{sd}) \]

Therefore, under the constraint given by equation (9), and the conditions:

\[ \dot{\xi} > 0, \]

\[ K_{q2} > K_{q2}, \quad \Rightarrow \frac{dV_1}{dt} \leq 0 \]

and the control input \( V_{sd} \) can be found by solving the constraint (9). So, by replacing \( \Phi_1 \) from equation (10) in equation (11) we can write:

\[ dV_1 = -K_{q2} \dot{\xi}^2 + K_{q2} (\dot{\xi}^2 - \dot{\xi}^2) \]

\[ = -K_{q2} \dot{\xi}^2 + K_{q2} (\dot{\xi}^2 - \dot{\xi}^2) \]

\[ = -K_{q2} \dot{\xi}^2 + K_{q2} (\dot{\xi}^2 - \dot{\xi}^2) \]

\[ V_{sd} = \alpha L_s [K_{q2} \dot{\xi} + \frac{d\dot{\xi}}{dt} + p\Omega_i \dot{\xi} + L_s \left( \frac{1}{\tau_s} + \frac{1}{\tau_r} \right) \dot{\xi}] + p\Omega_s \frac{d\dot{\xi}}{dt} - \frac{1}{\tau_r} \dot{\xi} \]

After obtaining the \( V_{sd} \) and \( V_{sq} \) control signals, they are tuned into three phases referential by means of the inverse Park transformation and are given as a reference to the Matrix Converter or the PWM block in order to generate the converter signals pulse.

Polynomial speed control

Pole placement synthesis

For good control of speed, cascade control scheme requires that the internal loop (current) is faster than the external loop (speed). The torque adjustment is effected by action on the quadrature stator current (\( i_{dq} \)). Therefore, the output of the external loop controller is the reference for the internal loop.
Imposing a strong dynamics for the loop of the torque control, the speed control is carried out by using a polynomial corrector of RST type (Fig. 2). This consists of a multi-objective problem that easily leads to optimization of the dynamics response time and disturbance rejection. It is considered as a corrector with two degrees of freedom.

The adjustment of the RST corrector is to synthesize three polynomials $R$, $S$ and $T$ on the basis of a robust pole placement.

The currents loops are extremely fast, the transfer function from the current $i_{dq}$ to the motor speed can be given by:

$$ G(s) = \frac{\Omega}{i_{dq}} \approx \frac{3L_m\phi^*_r}{2L_rJ_s} + js + f_c $$

The association of the RST structure with the system allows to imposing a global dynamic of the second order. However, the desired transfer function is given by:

$$ G_{BF}(s) = \frac{a_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} $$

The imposition of poles in closed loop is based on the desired specifications. However, we compute the damping ratio $\zeta$ and the natural frequency $\omega_n$.

The polynomials $S$, $R$ and $T$ are given by:

$$ S(s) = s_0 + s_1s $$
$$ R(s) = r_0 + r_1s $$
$$ T(s) = t_0 $$

The coefficients of the RST controllers in speed loops are designed based on the parameters of the motor and the driver system. However, we can determine polynomials coefficients $R, S$ and $T$ using the Diophantine identity equation. Thus, we obtain:

$$
\begin{align*}
    s_0 & = 1 \\
    s_1 & = \frac{3}{2}pL_m\phi^*_r \\
    r_0 & = \omega_n^2 \\
    r_1 & = 2\zeta\omega_n - \frac{s_0M_L}{3pL_m\phi^*_r} \\
    t_0 & = \omega_n^2
\end{align*}
$$

The different parameters of the backstepping control and matrix converter are given in the tables 2 and 3. The simulated motor speed and current responses for the proposed controller are shown in figures 4 and 5 where the starting performances as well as the sudden load change impact are mentioned. The different parameters of the backstepping control and matrix converter are given in the tables 2 and 3.

The simulated motor speed and current responses for the proposed controller are investigated theoretically at different operating conditions. In this study, we make attention toward rotor field orientation, speed tracking performance and rotor resistance and the rotor inertia variations for wide range of reference speed.

We have illustrated the response of the machine under two types of speed reference sequence variations noted sequence 1 and sequence 2. The corresponding results are illustrated by figures 4 and 5. These Figures are obtained if the machine parameters are supposed to be constant in the interval from 0 to 6s.

Fig 4 gives the simulation results of control proposed for IM with nominal parameters: the controller is designed by using the same parameters that the motor model parameters. The motor is initially running under the load of 1N.m. To see the starting performance as well as the sudden load change impact, the drive system is started at a constant load of 1N.m with the speed reference which varies between 50 rad/s, 157 rad/s and -50 rad/s. Full-load torque is applied from $t=1.5$ s to $t=2$ s and from $t=4$ s to $t=4.5$ s.

This figure shows the reference and feedback speed, the stator current in the $(d-q)$ frame, the phase stator current, the rotor flux in the $(d-q)$ axis and the electromagnetic torque. Consequently, the simulation results show the efficiency and the feasibility of the proposed simultaneous control of rotor speed and stator current when a load torque is applied. For the two speed simultaneous control of rotor speed and stator current.
reference sequence variations, it can be seen that the dynamic speed response of the proposed system follows the reference model speed. The currents $i_{ds}$, $i_{qs}$ responses of the proposed system have good dynamic performances even with a torque changes ($T_s$) on a wide range. Indeed, the speed response is characterized by a strong dynamic so that the motor follows the imposed reference. In spite of the disturbances due to the load torque, the speed error does not exceed 0.3%, it illustrates the robust character of the control law. Note that the decoupling control is very quiet maintained with the wide speed range variations.

Also, both rotor speed and rotor flux converge perfectly to their reference value. Thus, in the rated case, the control gives good quality response. On the Fig.4, one observes that the system of speed control presents perfectly a dynamics of a second-order system. Indeed, the speed response to a step signal is optimal because the damping ratio is equal to 0.707.

In all these tests, the reference speed and reference rotor flux are maintained in sequence 1 and 2. We observed that rotor flux on the $q$-axis is fixed to zero. With the proposed algorithm of backstepping control we have recorded a good responses performance.

Since motor heating usually causes a considerable variation in the winding resistance, there is often a mismatch between the actual rotor resistance and its corresponding set value within the model used for flux estimation.

Now, in order to illustrate the robustness of the control scheme proposed, the influence of parameter deviations is investigated. Parameter deviations are intentionally introduced in the controller scheme. Fig.6 and Fig.7 show the responses for 25% increase of the rotor resistance and Fig.8 and Fig.9 show the responses for a 25% increase of the rotor inertia change.

We notice that for a change in the rotor resistance, the speed of the motor may be influenced for the reference speed given by the sequence 1. In Fig.6 and Fig.7 we observed a perturbation at the rotor field but the proposed control has maintained the dynamic system and imposed at the motor rotor field to follow the reference field.

The actual speed does not change during the disturbance and the rotor resistance variation while the rotor field swiftly reaches to its reference value.

Figs. 8 and 9 show the responses for +25% deviations for the rotor inertia $J$. The result on the speed tracking is good. However, these four later figures clearly confirm the effectiveness and the properties of robustness of the integral backstepping algorithm associated with the RST control that we introduced.

**Conclusion**

The successful application of the current control by the non-linear approach of the Backstepping type and of the speed control by the RST polynomial approach of the induction motor associated with a matrix converter is illustrated in this paper. It has been shown that the induction motor belongs to a non-linear system class for which the backstepping technique can be used effectively. Recursively, we have identified virtual control states of the induction motor and the stabilization control laws have been developed, in detail, subsequently using the Lyapunov stability theory.

In addition, the parameters of the speed polynomial RST were determined using an appropriate pole placement in order to fulfill two main objectives, namely to obtain a good tracking setup and a good rejection capacity of the load disturbances. Thus, the robustness of the drive system has been improved.

![Fig.4: Sequence 1: Sensitivity of the performances system to change in the speed reference and load torque.](image)
Fig. 5: Sequence 2: Sensitivity of the performance system to change in the speed reference and load torque.

Fig. 6: Simulation results under load torque condition and $R_r$ variation with sequence 1.

Fig. 7: Simulation results under load torque condition and $R_r$ variation with sequence 2.

Fig. 8: Simulation results under load torque condition and $J$ variation with sequence 1.

Fig. 9: Simulation results of RST-Backstepping control at ±157 rad/sec under load torque with inertia moment of 1.25*J.
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REFERENCES