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Modification of the stability and positivity of standard and descriptor linear electrical circuits by state feedbacks

***Abstract**. The modification of the stability and positivity of standard and descriptor linear electrical circuits by state feedbacks is investigated. It is shown that: 1) There is a class of nonpositive and unstable R, L, e circuits that can be stabilized and modified to positive ones by state feedback; 2) There is a class of nonpositive and stable R, L, e circuits that can be modified by state feedback to positive ones without loss of stability. The modification of stability and positivity of linear descriptor electrical circuits is addressed. Considerations are illustrated by examples of linear electrical circuits.*

***Streszczenie.** W pracy rozpatrzono problem modyfikacji stabilności i dodatniości standardowych i deskryptorowych liniowych obwodów elektrycznych poprzez sprzężenie zwrotne od wektora stanu. Pokazano, że: 1) Istnieje klasa niedodatnich i niestabilnych obwodów typu R, L, e, które mogą zostać ustabilizowane i zmodyfikowane do obwodów dodatnich; 2) Istnieje klasa niedodatnich i stabilnych obwodów typu R, L, e, które mogą zostać zmodyfikowane do obwodów dodatnich bez utraty stabilności. Rozpatrywany problem uogólniono dla klasy układów deskryptorowych. Rozważania zilustrowano przykładami obwodów elektrycznych. (**Modyfikacja stabilności i dodatniości standardowych i deskryptorowych liniowych obwodów elektrycznych poprzez sprzężenie zwrotne od wektora stanu**).*

Keywords: descriptor, electrical circuit, linear, positivity, stability, state feedback.
Słowa kluczowe: deskryptorowy, obwód elektryczny, liniowy, dodatniość, stabilność, sprzężenie zwrotne od wektora stanu.

Introduction

A dynamical system is called positive if its trajectory starting from any nonnegative initial state remains forever in the positive orthant for all nonnegative inputs. An overview of state of the art in positive theory is given in the monographs [6, 14]. Variety of models having positive behavior can be found in engineering, especially in electrical circuits [22], economics, social sciences, biology and medicine, etc. [6, 14].

The positive electrical circuits have been analyzed in [7-9, 12-17]. The constructability and observability of standard and positive electrical circuits has been addressed in [8], the decoupling zeros in [9] and minimal-phase positive electrical circuits in [12]. A new class of normal positive linear electrical circuits has been introduced in [13]. Positive fractional linear electrical circuits have been investigated in [16] and positive unstable electrical circuits in [17]. Zeroing of state variables in descriptor electrical circuits has been addressed in [19]. Controllability of dynamical systems has been investigated in [23].

Descriptor (singular) linear systems have been investigated in [1-5, 11, 20, 21, 24, 25, 27]. The eigenvalues and invariants assignment by state and input feedbacks have been addressed in [10, 11]. The computation of Kronecker's canonical form of a singular pencil has been analyzed in [26].

In this paper the modification of the stability and positivity of standard and descriptor linear electrical circuits by state feedbacks is investigated.

The paper is organized as follows. In section 2 some definitions and theorems concerning positive and stable linear electrical circuits are recalled. In section 3 the modification of the stability and positivity of standard linear electrical circuits by state feedbacks is investigated. The same problem for descriptor linear electrical circuits is addressed in section 4. Concluding remarks are given in section 5.

The following notation will be used: $\Re$ - the set of real numbers, $\Re^{n\times m}$ - the set of $n\times m$ real matrices, $\Re_+^{n\times m}$ - the set of $n\times m$ real matrices with nonnegative entries and $\Re_+^n = \Re_+^{n\times 1}$, M_n - the set of $n\times n$ Metzler matrices (real matrices with nonnegative off-diagonal entries), I_n - the $n\times n$ identity matrix, $\mathbf{C}$ - the field of complex numbers.

Preliminaries

Consider the linear continuous-time electrical circuit described by the state equation

$$\dot{x}(t) = Ax(t) + Bu(t)\,, \tag{1}$$

where $x(t)\in\Re^n$, $u(t)\in\Re^m$ are the state and input vectors and $A\in\Re^{n\times n}$, $B\in\Re_+^{n\times m}$. It is well-known [22] that any standard linear electrical circuit composed of resistors, coils, capacitors and voltage (current) sources can be described by the equation (1). Usually as the state variables $x_1(t)$, ..., $x_n(t)$ (the components of the vector $x(t)$) the currents in the coils and voltages on the capacitors are chosen.

Definition 1. [22] The electrical circuit (1) is called (internally) positive if $x(t)\in\Re_+^n$, for any initial condition $x(0)\in\Re_+^n$ and every $u(t)\in\Re_+^m$, $t\in[0,+\infty)$.

Theorem 1. [22] The electrical circuit (1) is positive if and only if

$$A\in M_n\,,\ B\in\Re_+^{n\times m}\,. \tag{2}$$

Theorem 2. [22] The *R*, *L*, *e* electrical circuit is positive for any values of the resistances, inductances and source voltages if the number of coils is less or equal to the number of its linearly independent meshes and the direction of the mesh currents are consistent with the directions of the mesh source voltages.

Theorem 3. [22] The *R*, *C*, *e* electrical circuit is not positive for almost all values of its resistances, capacitances and source voltages if each its branch contains resistor, capacitor and voltage source.

Definition 2. [22] The positive electrical circuit (1) is called asymptotically stable if

$$\lim_{t\to\infty} x(t) = 0 \ \text{ for all } \ x_0\in\Re_+^n\,. \tag{3}$$

Theorem 4. [22] The positive electrical circuit (1) is asymptotically stable if and only if

$$\mathrm{Re}\,\lambda_k < 0 \ \text{ for } \ k = 1,...,n\,, \tag{4}$$

where λ_k is the eigenvalue of the matrix $A\in M_n$ and

$$\det[I_n\lambda - A] = (\lambda-\lambda_1)(\lambda-\lambda_2)...(\lambda-\lambda_n)\,. \tag{5}$$

Consider the positive linear electrical circuit (1) with the state feedback

(6) $$u(t) = v(t) + Kx(t),$$

where $v(t) \in \Re_+^m$ is the new input vector and $K \in \Re^{m \times n}$.
Substitution of (6) into (1) yields

(7) $$\dot{x}(t) = A_C x(t) + Bv(t),$$

where

(8) $$A_C = A + BK, \; A_C \in M_n, \; B \in \Re_+^{n \times m}.$$

Modification of stability and positivity of standard linear circuits

We shall show the essence of the problem with simple examples of electrical circuits. We assume that there is maximum one voltage source per branch of the circuit.

Example 1. Consider the electrical circuit shown in Fig. 1 with given resistance R, inductances L_1, L_2 and source voltage e.

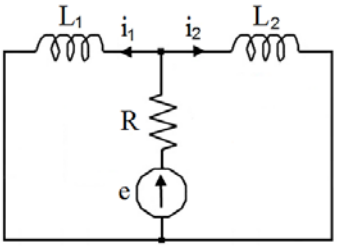

Fig.1. Electrical circuit of Example 1

Using the Kirchhoff's laws we may write the equations

(9a) $$e = R(i_1 + i_2) + L_1 \frac{di_1}{dt},$$

(9b) $$e = R(i_1 + i_2) + L_2 \frac{di_2}{dt}$$

which can be written in the form

(10a) $$\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = A_1 \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + B_1 e,$$

where

(10b) $$A_1 = \begin{bmatrix} -\frac{R}{L_1} & -\frac{R}{L_1} \\ -\frac{R}{L_2} & -\frac{R}{L_2} \end{bmatrix}, \; B_1 = \begin{bmatrix} \frac{1}{L_1} \\ \frac{1}{L_2} \end{bmatrix}.$$

From (10b) it follows that the electrical circuit is nonpositive since $A_1 \notin M_2$ and unstable since $\det A_1 = 0$. With the state feedback

(11) $$e = K_1 \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = [k_1 \quad k_2] \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

the closed-loop system matrix has the form

(12) $$A_{1C} = A_1 + B_1 K_1 = \begin{bmatrix} -\frac{R}{L_1} & -\frac{R}{L_1} \\ -\frac{R}{L_2} & -\frac{R}{L_2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ \frac{1}{L_2} \end{bmatrix} [k_1 \quad k_2] = \begin{bmatrix} \frac{k_1 - R}{L_1} & \frac{k_2 - R}{L_1} \\ \frac{k_1 - R}{L_2} & \frac{k_2 - R}{L_2} \end{bmatrix}.$$

From (12) it follows that for $k_1 > R$, $k_2 > R$ the electrical circuit is positive but still unstable. It is also possible to choose k_1, k_2 so that the electrical circuit is nonpositive but stable.

Conclusion 1. For the nonpositive and unstable *R*, *L*, *e* circuit with the number of source voltages less than the number of coils it is possible to solve only one of the following problems using the state feedback: 1) modification of the circuit from nonpositive to positive; 2) stabilization of the circuit.

Now let us consider the electrical circuit shown in Fig. 2 with given resistance R, inductances L_1, L_2 and source voltages e_1, e_2.

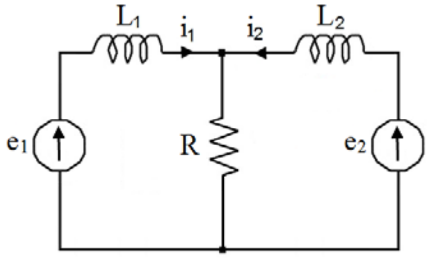

Fig.2. Electrical circuit of Example 1

Using the Kirchhoff's laws we may write the equations

(13a) $$e_1 = R(i_1 + i_2) + L_1 \frac{di_1}{dt},$$

(13b) $$e_2 = R(i_1 + i_2) + L_2 \frac{di_2}{dt}$$

which can be written in the form

(14a) $$\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = A_2 \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + B_2 \begin{bmatrix} e_1 \\ e_2 \end{bmatrix},$$

where

(14b) $$A_2 = \begin{bmatrix} -\frac{R}{L_1} & -\frac{R}{L_1} \\ -\frac{R}{L_2} & -\frac{R}{L_2} \end{bmatrix}, \; B_2 = \begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{1}{L_2} \end{bmatrix}.$$

From (14b) it follows that the electrical circuit is nonpositive and unstable since $A_2 \notin M_2$ and $\det A_2 = 0$. With the state feedback

(15) $$e = K_2 \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

the closed-loop system matrix has the form

(16) $$A_{2C} = A_2 + B_2 K_2 = \begin{bmatrix} -\frac{R}{L_1} & -\frac{R}{L_1} \\ -\frac{R}{L_2} & -\frac{R}{L_2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{1}{L_2} \end{bmatrix} \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} \frac{k_{11} - R}{L_1} & \frac{k_{12} - R}{L_1} \\ \frac{k_{21} - R}{L_2} & \frac{k_{22} - R}{L_2} \end{bmatrix}.$$

From (16) it follows that for $k_{11} < R$, $k_{22} < R$, $k_{12} > R$, $k_{21} > R$ the electrical circuit is positive and stable.

Conclusion 2. For the nonpositive and unstable *R*, *L*, *e* circuit with the number of source voltages at least equal to the number of coils it is possible to stabilize the circuit and to convert it to the positive one using the state feedback.

There has to be at least one source voltage per coil in every mesh of the circuit.

Remark 1. If in the system the number of linearly independent inputs is at least equal to the number of its state variables then it is possible to modify by the state feedback every element of the state matrix arbitrarily.

From the above considerations we have the following theorem.

Theorem 5. For the nonpositive and unstable *R*, *L*, *e* circuit it is possible to stabilize the circuit and to modify it to the positive one by the state feedback if and only if there is at least one source voltage per coil in every mesh of the circuit.

Proof. The proof follows immediately from Remark 1.

Example 2. Consider the electrical circuit shown in Fig. 3 [22] with given resistances R_1, R_2, R_3, inductances L_1, L_2 and source voltage e.

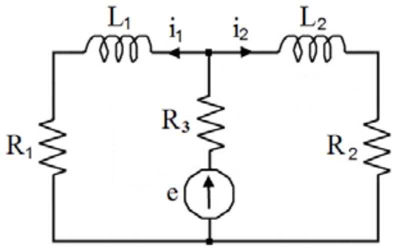

Fig.3. Electrical circuit of Example 2

Using the Kirchhoff's laws we obtain the equations

(17a) $$e = (R_1 + R_3)i_1 + R_3 i_2 + L_1 \frac{di_1}{dt},$$

(17b) $$e = R_3 i_1 + (R_2 + R_3)i_2 + L_2 \frac{di_2}{dt}$$

which can be written in the form

(18a) $$\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = A_3 \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + B_3 e,$$

where

(18b) $$A_3 = \begin{bmatrix} -\frac{R_1 + R_3}{L_1} & -\frac{R_3}{L_1} \\ -\frac{R_3}{L_2} & -\frac{R_2 + R_3}{L_2} \end{bmatrix}, \quad B_3 = \begin{bmatrix} \frac{1}{L_1} \\ \frac{1}{L_2} \end{bmatrix}.$$

The electrical circuit is not positive since $A_3 \notin M_2$. It is well-known [22] that the *R*, *L*, *e* circuit is stable if and only if the number of the inductances is less or equal to the number of linearly independent meshes and each independent mesh contains at least one resistance. Therefore, the considered circuit is stable. With the state feedback

(19) $$e = K_3 \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = [k_1 \quad k_2] \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

the closed-loop system matrix has the form

(20) $$\begin{aligned} A_{3C} &= A_3 + B_3 K_3 \\ &= \begin{bmatrix} -\frac{R_1 + R_3}{L_1} & -\frac{R_3}{L_1} \\ -\frac{R_3}{L_2} & -\frac{R_2 + R_3}{L_2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ \frac{1}{L_2} \end{bmatrix} [k_1 \quad k_2] \\ &= \begin{bmatrix} \frac{k_1 - (R_1 + R_3)}{L_1} & \frac{k_2 - R_3}{L_1} \\ \frac{k_1 - R_3}{L_2} & \frac{k_2 - (R_2 + R_3)}{L_2} \end{bmatrix}. \end{aligned}$$

From (20) it follows that for $R_3 < k_1 < R_1 + R_3$ and $R_3 < k_2 < R_2 + R_3$ the electrical circuit is positive and stable. Therefore, we can modify the nonpositive and stable *R*, *L*, *e* circuit to the positive and stable one by the state feedback with only one source voltage. Note that it is also possible for $R_3 = 0$ but not possible for $R_1 = 0$ or $R_2 = 0$.

Theorem 6. For the nonpositive and stable *R*, *L*, *e* circuit it is possible to modify the circuit to the positive and stable one by the state feedback if the number of source voltages is less than the number of the inductances.

By duality we can obtain similar results for the *R*, *C*, *e* electrical circuits.

Modification of stability and positivity of descriptor linear circuits

Consider the linear continuous-time electrical circuit

(21) $$E\dot{x}(t) = Ax(t) + Bu(t),$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$ are the state and input vectors and $E, A \in \Re^{n \times n}$, $B \in \Re^{n \times m}$. It is assumed that $\det E = 0$, $\operatorname{rank} E = r$ and

(22) $\det[Es - A] \neq 0$ for some $s \in \mathbf{C}$.

Definition 3. The linear electrical circuit described by (21) and satisfying the assumption (22) is called a descriptor electrical circuit.

Theorem 7. [22] Linear electrical circuit is descriptor if it contains at least one mesh consisting of only ideal capacitors and voltage sources or at least one node with branches with coils.

It is well-known [11] that if (22) holds, then there exist nonsingular matrices $P, Q \in \Re^{n \times n}$ such that

(23) $$P[Es - A]Q = \begin{bmatrix} I_{n_1}s - A_1 & 0 \\ 0 & Ns - I_{n_2} \end{bmatrix}.$$

where $n_1 = \deg\{\det[Es - A]\}$, $n_2 = n - n_1$, $A_1 \in \Re^{n_1 \times n_1}$ and $N \in \Re^{n_2 \times n_2}$ is the nilpotent matrix with the index μ, i.e. $N^{\mu - 1} \neq 0$, $N^{\mu} = 0$.

The matrices P and Q can be computed using procedures given in [11, 18, 26]. Premultiplying (21) by the matrix P, introducing the new state vector

(24) $$\bar{x}(t) = \begin{bmatrix} \bar{x}_1(t) \\ \bar{x}_2(t) \end{bmatrix} = Q^{-1}x(t), \; \bar{x}_1(t) \in \Re^{n_1}, \; \bar{x}_2(t) \in \Re^{n_2}$$

and using (24) we obtain

(25) $$PEQQ^{-1}x(t) = PAQQ^{-1}x(t) + PBu(t)$$

and

(26a) $$\dot{\bar{x}}_1(t) = A_1\bar{x}_1(t) + B_1u(t),$$

(26b) $$N\dot{\bar{x}}_2 = \bar{x}_2(t) + B_2u(t).$$

Definition 4. [20] The descriptor electrical circuit (21) (or equivalently (26)) is called (internally) positive if $x(t) \in \Re_+^n$, for any initial condition $x(0) \in \Re_+^n$ and every $u(t) \in \Re_+^m$, $t \in [0, +\infty)$.

Definition 5. The matrix $Q \in \Re^{n \times n}$ is called monomial if in each row and column only one entry is positive and the remaining entries are zero.

It is well-known [14] that $Q^{-1} \in \Re_+^{n\times n}$ if and only if the matrix $Q \in \Re_+^{n\times n}$ is monomial.

It is assumed that for positive system (21) the decomposition (23) is possible for monomial matrix Q. In this case $x(t) = Q\bar{x}(t) \in \Re_+^n$ if and only if $\bar{x}(t) \in \Re_+^n$. It is also well-known that premultiplication of the equation (21) by the matrix P does not change its solution $x(t)$.

Theorem 8. [20] Let the decomposition (23) of the system (21) be possible for a monomial matrix $Q \in \Re_+^{n\times n}$. The descriptor electrical circuit (21) (or equivalently (26)) is positive if and only if

(27) $$A_1 \in M_{n_1}, \; B_1 \in \Re_+^{n_1\times m}, \; -B_2 \in \Re_+^{n_2\times m}.$$

It is well-known [20] that the stability of the positive descriptor electrical circuit (26) depends only of the stability of the subsystem (26a).

Definition 6. [20] The positive descriptor electrical circuit (26) is called asymptotically stable if

(28) $$\lim_{t\to\infty} \bar{x}_1(t) = 0 \text{ for all } \bar{x}_{10} = \bar{x}_1(0) \in \Re_+^{n_1}.$$

Theorem 9. [20] The positive electrical circuit (26) is asymptotically stable if and only if

(29) $$\mathrm{Re}\,\bar{\lambda}_k < 0 \text{ for } k = 1,...,n_1,$$

where $\bar{\lambda}_k$ is the eigenvalue of the matrix $A_1 \in M_{n_1}$ and

(30) $$\det[I_{n_1}\bar{\lambda} - A_1] = (\bar{\lambda} - \bar{\lambda}_1)(\bar{\lambda} - \bar{\lambda}_2)...(\bar{\lambda} - \bar{\lambda}_n).$$

Consider the positive descriptor electrical circuit (26) with the state feedback

(31) $$u(t) = v(t) + K_1\bar{x}_1(t) = v(t) + [K_1 \;\; 0]\begin{bmatrix} \bar{x}_1(t) \\ \bar{x}_2(t) \end{bmatrix},$$

where $v(t) \in \Re_+^m$ is the new input vector and $K_1 \in \Re^{m\times n_1}$. Substitution of (31) into (26) yields

(32) $$\begin{bmatrix} I_{n_1} & 0 \\ 0 & N \end{bmatrix} \frac{d}{dt}\begin{bmatrix} \bar{x}_1(t) \\ \bar{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_1 + B_1K_1 & 0 \\ B_2K_1 & I_{n_2} \end{bmatrix}\begin{bmatrix} \bar{x}_1(t) \\ \bar{x}_2(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} v(t)$$

which can be also written in the form

(33a) $$\dot{\bar{x}}_1(t) = A_{C1}\bar{x}(t) + B_1v(t),$$

(33b) $$N\dot{\bar{x}}_2 = A_{C2}\bar{x}_1(t) + \bar{x}_2(t) + B_2v(t),$$

where $A_{C1} = A_1 + B_1K_1$, $A_{C2} = B_2K_1$.

Theorem 10. [20] Let the decomposition (23) of the system (21) be possible for a monomial matrix $Q \in \Re_+^{n\times n}$. The closed-loop subsystem (33a) is positive if and only if

(34) $$A_{C1} \in M_{n_1}, \; B_1 \in \Re_+^{n_1\times m}.$$

Theorem 11. Let the decomposition (23) of the system (21) be possible for a monomial matrix $Q \in \Re_+^{n\times n}$. The closed-loop subsystem (33b) is positive if and only if

(35) $$-A_{C2} = -B_2K_1 \in \Re_+^{n_2\times n_1}, \; -B_2 \in \Re_+^{n_2\times m}.$$

Proof. The solution of (33b) can be obtained in a similar way as it was shown in [11] and has the form

(36) $$\bar{x}_2(t) = -\sum_{k=1}^{\mu-1} N^k \delta^{(k-1)}(t)\bar{x}_{20} - \sum_{k=0}^{\mu-1} N^k \left(B_2 \frac{d^k u(t)}{dt^k} + A_{C2}\frac{d^k \bar{x}_1(t)}{dt^k} \right).$$

Let $\bar{x}_{20} = \bar{x}_2(0) = 0$. From (36) we have $\bar{x}_2(t) \in \Re_+^{n_2}$ if and only if $-B_2 \in \Re_+^{n_2\times m}$ and $-A_{C2} \in \Re_+^{n_2\times n_1}$. □

Theorem 12. The positive closed-loop system (33) is asymptotically stable if and only if the closed-loop subsystem (33a) is asymptotically stable.

Proof. The characteristic equation of (33) is given by

(37) $$\det\begin{bmatrix} I_{n_1}s - A_{C1} & 0 \\ -A_{C2} & Ns - I_{n_2} \end{bmatrix} = \det[I_{n_1}s - A_{C1}] \times \det[Ns - I_{n_2}] = 0.$$

From (37) we have

(38) $$\det[I_{n_1}s - A_{C1}] = 0$$

since $\det[Ns - I_{n_2}] = (-1)^{n_2}$. Therefore, the stability of the positive closed-loop system (33) depends only of the eigenvalues of the matrix $A_{C1} \in M_{n_1}$. □

Example 3. Consider the electrical circuit shown in Fig. 4 [22] with given resistance R, capacitances C_1, C_2 and the voltage source e.

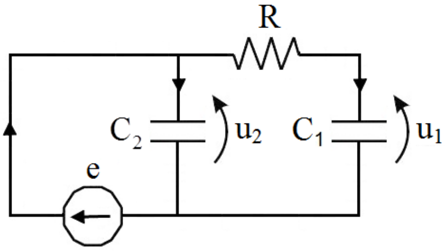

Fig.4. Electrical circuit of Example 3

By Theorem 7 the electrical circuit is a descriptor one. Using the Kirchhoff's laws we may write the equations

(39a) $$e = RC_1\frac{du_1}{dt} + u_1,$$

(39b) $$e = u_2$$

which can be written in the form

(40a) $$E\frac{d}{dt}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = A\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + Be,$$

where

(40b) $$E = \begin{bmatrix} RC_1 & 0 \\ 0 & 0 \end{bmatrix}, \; A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \; B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

The pencil is regular since

(41) $$\det[Es - A] = \det\begin{bmatrix} RC_1s + 1 & 0 \\ 0 & 1 \end{bmatrix} = RC_1s + 1 \neq 0.$$

In this case

(42) $$P = \begin{bmatrix} \frac{1}{RC_1} & 0 \\ 0 & -1 \end{bmatrix}, \; Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$\begin{bmatrix} I_{n_1} & 0 \\ 0 & N \end{bmatrix} = PEQ = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$

(43) $$\begin{bmatrix} A_1 & 0 \\ 0 & I_{n_2} \end{bmatrix} = PAQ = \begin{bmatrix} -\frac{1}{RC_1} & 0 \\ 0 & 1 \end{bmatrix},$$

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = PB = \begin{bmatrix} \frac{1}{RC_1} \\ -1 \end{bmatrix}.$$

The matrix Q defined by (42) is monomial and the conditions (27) are met since

(44) $$A_1 = -\frac{1}{RC_1} \in M_1, \; B_1 = \frac{1}{RC_1} \in \Re_+^{1\times 1}, \quad -B_2 = 1 \in \Re_+^{1\times 1}.$$

Therefore, by Theorem 8 the electrical circuit is positive. It is also asymptotically stable since the eigenvalue of the matrix A_1 is $s = -\frac{1}{RC_1}$.

With the state feedback

(45) $$e = [k \quad 0]\begin{bmatrix} \bar{u}_1 \\ \bar{u}_2 \end{bmatrix}$$

the closed-loop system is described by the equation

(46) $$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \bar{u}_1 \\ \bar{u}_2 \end{bmatrix} = \begin{bmatrix} \frac{k-1}{RC_1} & 0 \\ -k & 1 \end{bmatrix} \begin{bmatrix} \bar{u}_1 \\ \bar{u}_2 \end{bmatrix}.$$

By Theorems 10-12 the closed-loop system (46) is: unstable and positive for $k > 1$, stable and positive for $0 < k < 1$, stable and nonpositive for $k < 0$.

Concluding remarks

The modification of the stability and positivity of standard and descriptor linear electrical circuits by state feedbacks has been investigated. It has been shown that:

1) For the nonpositive and unstable *R*, *L*, *e* circuit it is possible to stabilize the circuit and to modify it to the positive one by the state feedback if and only if there is at least one source voltage per coil in every mesh of the circuit (Theorem 5).

2) For the nonpositive and stable *R*, *L*, *e* circuit it is possible to modify the circuit to the positive and stable one by the state feedback if the number of source voltages is less than the number of the inductances (Theorem 6).

The modification of stability and positivity of linear descriptor electrical circuits has been addressed.

The considerations have been illustrated by examples of linear electrical circuits. The presented approach can be extended to fractional linear electrical circuits.

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