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Discrete wavelet transformation in spectral analysis of vibration processes at hydropower units

***Abstract**. The paper presents application of wavelet transform in vibration diagnostics of hydropower units. It lies in the fact that, in view of a considerable nonstationarity of vibroacoustic signals, the spectral analysis thereof comprises discrete wavelet transformation so that spectral information both in frequency and time domain could be obtained and applied for hydropower unit diagnosis.*

***Streszczenie.** Artykuł prezentuje zastosowanie transformaty falkowej do wibracyjnej diagnostyki hydrogeneratorów. Biorąc pod uwagę niestacjonarny charakter sygnałów wibroakustycznych i dokonując analizy za pomocą dyskretnej transformaty falkowej uzyskuje się informację potrzebne do diagnostyki hydrogeneratora (**Dyskretna transformata falkowej w analizie spektralnej zjawisk wibracyjnych hydrogeneratorów**).*

Keywords: vibroacoustic signal, discrete wavelet transformation, hydropower unit.
Słowa kluczowe: sygnał wibroakustyczny, dyskretne przekształcenie falkowe, hydrogenerator.

Introduction

The future of the world's economics is closely related to accelerated development of power sources, hydropower industry being one of them. At the same time, hydraulic power plants (HPP), especially high-power ones, may in case of an emergency represent a significant hazard to environment and people, as it was the case with Nurekskaya HPP (Tajikistan, 1983), Sayano-Shushenskaya HPP (Russia, 2009), as well as in Switzerland (2000), USA (2005), India (2009, 2013) etc.

That is why timely diagnostics and particularly forecasting of hydropower units' defects are of great significance. Vibration diagnostics is one of the most widespread diagnostics types, since vibration signal's almost instantaneous reaction to change in equipment condition is a very important property in case o an emergency, when the speed of diagnosis establishment and decision making is a determinant factor.

Vibration diagnostics is a discipline that comprises theory and methods for arrangement of processes associated with machine identification and assessment of its technical condition that is based on the data contained in vibroacoustic signal. Vibroacoustic signal is the main physical carrier of information about condition of elements of operating equipment during vibration diagnostics such signal being a collective concept that contains information on oscillating processes (vibrational, hydro- or gas-dynamic etc.) and respective mechanism's acoustic noise in the environment.

The main approaches of information analysis for diagnostics and forecasting of hydropower unit's defect are: power analysis - based on measurement of controlled signal's amplitude (or power); frequency (spectral) analysis; phase-time technology based on comparison of the form of signals measured at fixed time intervals. In many existing automated diagnostic systems, several of the foregoing technologies are used at the same time, though the overwhelming majority of systems are based on spectral analysis of vibration signals.

Spectral analysis of vibration processes at hydropower units

The essence of spectral technology of diagnostics lies in the fact that certain spectrum components frequently conform to certain factors giving rise to vibration and, when analyzing the amplitudes of these components, one can make conclusions relating to the degree of each such factor's influence on vibration signal.

The main weak point of many diagnostic systems lies in the fact that signal spectrum is obtained through conventional Fourier transformation, which may only have an adequate effect with stationary signals, while hydropower unit's vibration signal is nonstationary. In such a case, conventional Fourier transformation is inadequate, and frequency-time transformations should be applied.

Frequency-time transformations do essentially differ from frequency ones by the fact that physical Heisenberg uncertainty principle works in their respect, which principle in relation to this transformation type may be formulated as follows: at no fixed moment in time one can determine, which spectral components are contained in a signal.

It follows from this principle that we can only determine the time intervals, during which a signal contains frequency bands. It in turn follows from the foregoing that, should the window size (i.e. time interval) be small, meaning that the spectrum's time localization is high, the frequency band will be very diffused and vice versa, more precise determination of spectral component will require a large window.

One of such transformations is represented by the so-called short-time Fourier transformation (STFT), which is also called sometimes the weighted Fourier transformation [1, 2].

Wavelet transform

Provided that mother wavelet is designated as $\psi(t)$, The direct continuous wavelet transformation (CWT) of signal $f(t)$ with s scale parameter and τ time displacement is defined as:

$$Wf(s,\tau)=\frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty} f(t)\psi^*\left(\frac{t-\tau}{s}\right)dt \tag{1}$$

where ψ^* – the associated mother wavelet (with $\psi(t)$ being a real function, $\psi^*(t)=\psi(t)$).

$Wf(\tau,s)$ domain is represented by the product of all possible combinations s and τ. Scale parameter s is in its essence a value inverse to frequency. Since it is contained in the denominator, $s>1$ means stretching the signal, while $s<1$ shrinking. CWT results in the so-called wavelet ratio matrix (WRM).

The $f(t)$ signal may be reproduced pursuant to WRM obtained using inverse CWT

(2) $$f(t)=\frac{1}{C_\Psi}\int_{R^2} Wf(s,\tau)\frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)\frac{dsd\tau}{|s|^2}$$

where C_Ψ denotes constant determined by the mother wavelet.

At the same time, actual vibroacoustic signals cannot be represented in analytical form. They enter the system for further analysis in the form of a finite numeric group. In such cases, a direct discrete wavelet transformation (DWT) is applied. DWT results in a triangular wavelet ratio matrix, each successive row of which is by k times shorter than the previous one, i.e. row lengths are terms of geometrical progression, compression ratio k being its indicator.

Given that one element should be left in the last row, the length of the signal's input vector should equal to k^M, where M is a natural number. Such being the case DWT implementation results in wavelet ratio matrix of size:

(3) $$k^{M-1}\times M .$$

DWT when $k = 2$ is used most frequently, but sometimes it may be advisable to use other whole (or even rational) compression coefficients.

This allows increasing the speed of algorithms, though aggravates the definition of frequency-time signal transformation. The mathematical models and algorithms of discrete wavelet transformation are based on the mathematical apparatus of the so-called multiple-scale analysis [1, 2, 3].

The multiple-scale analysis is based on ONB comprising the two functions: scaling function $\phi(x)$ and mother (basic) wavelet $\psi(x)$.

At compression coefficient 2, scaling function must conform to the following formula:

(4) $$\varphi(x)=\sqrt{2}\sum_{i=0}^{N-1}h(i)\varphi(2x-i),$$

where $h(N)$ – the coefficient vector that characterizes the scaling function unambiguously.

This function ensures double zooming at each step. Based on $\phi(x)$ function, mother wavelet $\psi(x)$ is built using the formula:

(5) $$\psi(x)=\sqrt{2}\sum_{i=0}^{N-1}g(i)\varphi(2x-i),$$

where $g(N)$ the coefficient vector that characterizes the mother wavelet unambiguously. Coefficient vector $g(N)$ is determined as follows:

(6) $$\forall i=0,\, N-1\left(g(i)=(-1)^i h(N-1-i)\right)$$

At compression coefficient 2, scaled and displaced scaling function and mother wavelet are written as:

(7) $$\varphi_{j,i}(x)=2^{\frac{j}{2}}\varphi\left(2^j x-i\right),$$

(8) $$\psi_{j,i}(x)=2^{\frac{j}{2}}\psi\left(2^j x-i\right).$$

In order to implement DWT algorithm, the scaling function $\phi(x)$ and mother wavelet $\psi(x)$ are not used actually; they are entirely replaced by coefficient vectors $h(N)$ and $g(N)$.

Technically, there are algorithms of building mother wavelets for arbitrary rational compression coefficient instead of 2, though this only compression coefficient allowed demonstrating that there is an algorithm, under which the smoothness of mother wavelet increases in a linear fashion with increase of its support. For example, at compression coefficient 3 this smoothness increases in a logarithmic fashion with increase in definition domain. At the same time, it sometimes may be advisable, for various reasons, to use wavelets wit other (particularly, fractional) compression coefficients.

Algorithms for obtaining $h(N)$ and $g(N)$. vectors are in most cases complicated. Among them, the simplest ones are the algorithms for determination of Daubechies wavelets, such algorithms coming down to solution of algebraic equation systems [3, 4].

Let us consider the principles for building the required algebraic equation system and its solution by the example of determination of coefficient vector $h(N)$ for mother Daubechies wavelet D^8 (i.e. $N = 8$) at compression coefficient 2. It follows from the orthogonal property of scaling functions

(9) $$\int\varphi(x)\varphi(x-m)dx=\delta_{0m} .$$

where δ_{0m} – Kronecker delta, and (4) equation that $\sum_i h(i)h(i+2m)=\delta_{0m}$, and since $N = 8$, consequently $m = 1, 2, 3$, this expression is broken down into subsystem consisting of three equations:

(10) $$\begin{aligned}&h_0h_2+h_1h_3+h_2h_4+h_3h_5+h_4h_6+h_5h_7=0;\\&h_0h_4+h_1h_5+h_2h_6+h_3h_7=0;\\&h_0h_6+h_1h_7=0.\end{aligned}$$

The condition of mother wavelet's orthogonality to polynomials up to L - 1 degree that determines its smoothness,

(11) $$\forall n=0,\, L-1\left(\int x^n\psi(x)dx=0\right)$$

is confined to the relation:

(12) $$\sum_i i^n g(i)=0 ,$$

or, allowing for (6)

(13) $$\sum_i (-1)^i i^n h(i)=0 .$$

Since $L=\frac{N}{2}=4$, then $n = 0,1,2,3$ and expression (13) is broken down into the subsystem composed of the four equations:

(14) $$\begin{aligned}&h_0-h_1+h_2-h_3+h_4-h_5+h_6-h_7=0;\\&-h_1+2h_2-3h_3+4h_4-5h_5+6h_6-7h_7=0;\\&-h_1+2^2h_2-3^2h_3+4^2h_4-5^2h_5+6^2h_6-7^2h_7=0;\\&-h_1+2^3h_2-3^3h_3+4^3h_4-5^3h_5+6^3h_6-7^3h_7=0.\end{aligned}$$

And finally, the condition of the scaling function standardization:

(15) $$\int\varphi(x)dx=1 ,$$

gives rise to another equation:

(16) $$\sum_i h(i)=\sqrt{2} .$$

Solving the general equation system

(17a) $$\begin{aligned}&h_0h_2+h_1h_3+h_2h_4+h_3h_5+h_4h_6+h_5h_7=0;\\&h_0h_4+h_1h_5+h_2h_6+h_3h_7=0;\\&h_0h_6+h_1h_7=0;\\&h_0-h_1+h_2-h_3+h_4-h_5+h_6-h_7=0;\end{aligned}$$

(17b)
$$-h_1+2h_2-3h_3+4h_4-5h_5+6h_6-7h_7=0;$$
$$-h_1+2^2h_2-3^2h_3+4^2h_4-5^2h_5+6^2h_6-7^2h_7=0;$$
$$-h_1+2^3h_2-3^3h_3+4^3h_4-5^3h_5+6^3h_6-7^3h_7=0;$$
$$h_0-h_1+h_2-h_3+h_4-h_5+h_6-h_7=\sqrt{2}.$$

Similarly, one can form the coefficient vectors $h(N)$ for other mother Daubechies wavelets.

Results of using the discrete wavelet transformation for vibration signal analysis at actual hydropower unit

DWT input data are as follows: $h(N)$ – the coefficient vector that characterizes the scaling function unambiguously; $g(N)$ – the coefficient vector that characterizes the mother wavelet unambiguously; input signal vector $f(N)$. $k = 2$ is the one most frequently used in DWT algorithms: with this, the length of each WRM row is twice as little as the previous one's length. At the same time, when investigating complicated nonstationary signals, it would be advisable to consider the options of DWT algorithms with other compression coefficients, these not necessarily to be whole numbers.

The choice of compression coefficient value is quite a complicated problem. The point is that, with k growing and given that the same input signal vector length the number of WRM rows reduces, which results in definition aggravation and reduction in WRM information content level, but with DWT speed growing, which may turn out to be a considerable advantage when it is necessary to ensure a fast signal diagnostics. Evidently, when it is necessary to ensure a more detailed diagnostics, one should choose lesser k values and agree to reduction of algorithmic speed. Since nonintegral k values cause certain difficulties algorithm development and implementation thereof, let us restrict ourselves to consideration of algorithm with whole natural k.

As already mentioned above, DWT with compression coefficient k results in triangular wavelet ratio matrix, each successive row of which is by k times shorter than the previous one, i.e. row lengths are terms of geometrical progression, the indicator of which equals to k [5, 6, 7].

DWT algorithm contains the following steps:

- the first row of intermediate calculations matrix $\lfloor a_{j.n} \rfloor$ is formed:

(18) $$a(0,n)=f(n).$$

- then, subsequent rows of intermediate calculations matrix $\lfloor a_{j.n} \rfloor$ are formed one-by-one:

(19) $$a(j,n)=\sum_{i=0}^{N-1}h(i)a(j-1,\,kn+i),$$

as well as the rows of wavelet ratio matrix $\lfloor d_{j.n} \rfloor$

(20) $$d(j,n)=\sum_{i=0}^{N-1}g(i)a(j-1,\,kn+i).$$

This algorithm was implemented and used for analysis of vibration signals in hydropower units of Dnistrovska HPP-2 (in the example set forth in the paper, DWT with compression coefficient 2 was taken). At the same time, provision was made for the specificity of measurement channels, from which the information on vibration signals comes, and namely:

Numerical data come from the channel to the system for further processing in separate stacks sized up to 32768 values. In this case, each stack conforms to a separate time interval, and it is inadmissible, for a correct analysis of one stack's data, to use another stack's data. Therefore, maximum data vector length that may be set in an application should not exceed 32768, which corresponds to 2^{15}.

It follows from the foregoing expressions that, upon condition of a finite size of a numerical data set and inadmissibility of using another stack's data, we can only use half of a stack in the algorithm, i.e. 16384 = 2^{14}. On the basis of (4), maximum WRM size with compression coefficient 2 may equal to $2^{13}\times14$ = 8192×14.

Set forth below in Fig. 1 is the diagram of WRM obtained for one time interval of vibration signal at a hydropower unit. This diagram is three-dimensional, with vertical axis containing WRM values and the horizontal one – time values, and located in the deep of the diagram are WRM rows corresponding to DWT scale ratios (i.e. frequency bands). The bands are arranged in such a way that low frequency bands are located in the diagram's deep, approaching to the viewer with frequency growth. The width of each frequency band, in accordance with Heisenberg principle, is reducing with the increase in time interval length that is why low frequency bands are quite narrow, which ensures a high definition of DWT. For better visualization of WRM diagram, the width of each frequency band is depicted at a logarithmic scale.

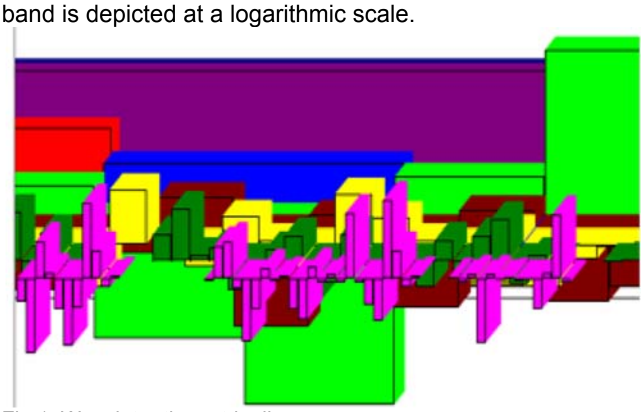

Fig.1. Wavelet ratio matrix diagram

Frequency identification of scale coefficient of discrete wavelet transformation

It is common knowledge that DWT results in WRM, the diagram of which is built in "time–scale coefficient" coordinates rather than in "time–frequency" coordinates.

That is why, to ensure the opportunity of diagnosing the defects of hydropower unit, one should first of all determine, which frequency band conforms to each of scale coefficients (i.e. to each WRM row) [9]. Recall that WRM is a triangular matrix, in which the length of each matrix row typically reduces exponentially. The progression's exponent is compression coefficient k.

The number of wavelet ratios naturally represents the scale coefficient of a certain WRM row. It is evident that, given a whole compression coefficient k for M rows as well, the scale coefficient of the last WRM row equals to 1, the one of the second-to-last row – to k, the one of the first row k^{M-1}. The length of the input data vector in this case (based on DWT algorithm) equals to k^M.

Since WRM sets the frequency-time spectrum of vibroacoustic signal, the row's scale coefficient will essentially equal to the number of frequency intervals, into which the input signal is broken down. Hence, each subsequent WRM row has the time interval length by k times longer than the same of previous one, and in accordance with Heisenberg uncertainty principle, the width of each subsequent WRM row (frequency band width) is by k times lesser than the same of the previous one.

Let us mark the scale coefficient of the i_{th} row as m_i (for the sake of convenience, let us number the rows, beginning from the last one). Then:

(21) $$m_i = k^{i-1},$$

with the frequency band width of i_{th} row to be directly proportionate to m_i.

Let us now determine the frequency range ΔF obtained as a result of DWT of vibroacoustic signal's frequency-time spectrum. Assume that vibration signal's discretization interval (being one of parameters of the vibration channel being measured) equals to F_∂. According to Shannon-Kotelnikov theorem,

(22) $$\Delta F = \frac{F_\partial}{2}.$$

It is evident that frequency range ΔF corresponds to the sum of all scale coefficients m_i, i.e. being the total of geometrical progression S_m elements. Thus, we may write:

(23) $$S_m = \sum_{i=1}^{M} m_i = \frac{k^M - 1}{k - 1}.$$

It follows from the foregoing that the frequency band width of the i_{th} row is Δf_i, which conforms to scale coefficient m_i, may be obtained using the formula:

(24) $$\Delta f_i = m_i \frac{\Delta F}{S_m} = \frac{F_\partial (k-1) k^{i-1}}{2\left(k^M - 1\right)}.$$

Naturally, WRM row with one wavelet ratio conforms to the narrowest frequency band, which begins from 0.

It is apparent from (24) expression that frequency bandwidth depends on discretization frequency, compression coefficient and on the length of input vibration signal stack. The foregoing algorithm was implemented in Microsoft Excel environment with the following input data: discretization interval of measurement channels equals to 913.92 Hz.; for compression coefficient 2, maximum possible input signal vector length is assumed as k^M = 16382, M = 14. Microsoft Excel spreadsheet is shown in Figure 2 below.

	A	B	C	D	E
1	Fd	Line MVC	Width of a strip	The strip beginning	The strip end
2	913,92	1	0,027892327	0	0,027892327
3	dF	2	0,055784655	0,027892327	0,083676982
4	456,96	3	0,11156931	0,083676982	0,195246292
5	k	4	0,223138619	0,195246292	0,418384911
6	2	5	0,446277239	0,418384911	0,86466215
7	M	6	0,892554477	0,86466215	1,757216627
8	14	7	1,785108954	1,757216627	3,542325581
9		8	3,570217909	3,542325581	7,11254349
10		9	7,140435818	7,11254349	14,25297931
11		10	14,28087164	14,25297931	28,53385094
12		11	28,56174327	28,53385094	57,09559421
13		12	57,12348654	57,09559421	114,2190808
14		13	114,2469731	114,2190808	228,4660538
15		14	228,4939462	228,4660538	456,96

Fig.2. Frequency bands of vibration signal's amplitude-frequency-time spectrum

Analyzing the results obtained it can be noticed that it is common for the existing vibration diagnostics practice to call vibration in the range lower than rotor spinning frequency a low-frequency one, the vibration conforming to rotor spinning frequency up to its 20th harmonic – a medium-frequency one, and if higher – a high-frequency one.

It is known that rotor spinning frequency at hydropower units of Dnistrovska HPP-2 equals to 1.785 Hz. Hence, frequency bands from the first to the sixth ones will approximately cover the low-frequency range, the frequency bands from the seventh to the tenth contained in the medium-frequency range, with the eleventh band covering the end of medium-frequency and the start of high-frequency range, with frequency bands from the twelfth to the fourteenth ones wholly belonging to the high-frequency range.

Conclusions

Application of wavelet transformation for spectral analysis of vibration signals allows obtaining information of vibration signal both in time and frequency domains with variable window width that ensures proper definition of spectral characteristics.

When choosing the compression coefficient value one should note that its increase results (with the same input signal vector length) in reduction in the number of WRM rows, definition aggravation and reduction of WRM information content level, however DWT speed increases in this case, which may turn out to be a considerable advantage where there is a need to ensure a fast signal diagnostics. It is evident that, in case of the need to ensure a more detailed diagnostics, one should choose lesser compression coefficient values and agree to reduction in algorithm speed.

Furthermore, it is important to choose a mother wavelet optimal in size, since with increase in coefficient vector $h(N)$ order, $\psi(x)$ function density grows, enabling the improvement of DWT quality. At the same time resulting in reduction in the speed of the application that has to analyze the vibration signal.

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