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New non-system physical quantities for vibration monitoring of transient processes at hydropower facilities, integral vibratory accelerations

Abstract. *Proposed are new non-system quantities – order 1 and order 2 integral vibratory accelerations, the use of which allows increasing the response time of vibration monitoring system at hydropower facilities, completed were theoretical substantiations of their application adequacy and obtained were generalized mathematical mapping operators of these quantities' reference values.*

Streszczenie. *W artykule zaproponowano nowe wielkości niesystemowe – zintegrowane przyspieszenia drgań 1 i 2 rzędu, których stosowanie pozwala na wydłużenie czasu reakcji systemu monitorowania drgań na obiektach wodnych. Wypełnione zostały założenia teoretyczne ich adekwatności aplikacji i uzyskały uogólnione operatory matematyczne do mapowania wartości referencyjnych tych wielkości. (**Nowe niesystemowe wielkości fizyczne monitorowania drgań nieustalonych procesów w elektrowniach, zintegrowane przyspieszenia drgań**).*

Keywords: dynamic vibration monitoring, transient processes, hydrogenerators, vibratory acceleration, vibration velocity, vibration displacement, order 1, order 2 and order n integral vibratory accelerations
Słowa kluczowe: monitoring wibracji dynamicznych, procesy przejściowe, hydrogeneratory, przyspieszenie drgań, prędkość wibracji, przemieszczenie drgań, zintegrowane przyspieszenia drgań rzędu 1, 2 i n.

Starting provisions

Monitoring of vibration parameters of hydropower units is an important condition to ensure their operational reliability. Specifically important in this respect is vibration monitoring of hydropower unit subassemblies in wake-up mode, during which, even for bug-free equipment, typical is vibration level excess, which is achieved in stabilized mode, by several times [1]. Such vibration increase will inevitably cause growing mechanical stress in structural elements of both hydropower unit itself and in its foundations and fixtures, thus providing grounds for consideration of wake-up mode as the mode of increased risk of damage to equipment and support structures. And since operating conditions of the vast majority of hydropower units, and particularly speedup process provide for control of vibration parameters at rotor's minor instantaneous speeds, for which the use of vibration speed sensors is quite restricted [1], the development of new methods for analytic calculation of vibration speed based on known vibratory acceleration values suitable or use at hydropower unit's wake-up mode is a relevant scientific mission of significant practical value.

Under the effective standards, the primary parameters that describe vibration condition of a system on qualitative and quantitative terms are *vibration displacement*, *vibration velocity* and *vibratory acceleration*. The first one of the said parameters is defined as a vibration component of movement, the other two – as its first and second time differentials, respectively [1-3].

As of today, the theory and practice of vibration monitoring at low-frequency hydropower facilities [1, 2, 4-6] has been settled, and so far there is a predominant paradigm that clearly defines and regulates the procedure of measuring transformation of the said vibration parameters. When you disregard other physical quantities (of both mechanical and non-mechanical origin) associated with them mathematically, this order may compactly be represented by the simplified diagram:

vibratory acceleration → {vibration velocity; vibration displacement}.

The reason for the above is that the frequency of vibration signal's first harmonic coincides with the frequency of hydropower unit's rotor spinning, being Hertz units [1]. When monitoring the vibration in this frequency band, the use of the vast majority of vibration sensor types (except accelerometers) is strongly restricted. Hence, the typical approach to defining the parameters of vibration speed and vibration displacement in hydropower unit's structural elements lies in integration and double integration of vibratory acceleration signal per rotor spinning period [1, 3]. However, such approach cannot be used to define vibration speed and vibration displacement parameters in transitional modes of operation, since on the one side it only enables to obtain one, time-averaged value of vibration speed and vibratory acceleration, which is insufficient for vibration monitoring in transitional operating mode, and on the other side, it requires a fixed value of rotor spinning period, which in transitional operating mode is no constant value. Therefore it is evident that, for the purposes of monitoring the vibration speed and vibration displacement in hydropower units in nonstationary operational modes, one is required to develop new approaches that would enable, on the one side, to increase the speed of these parameters' numerical calculation, and on the other side – would not be bound to rotor frequency [7-10].

An integrated analysis of this task allows concluding that there is now a possibility of its modification that enables noticeably to reduce the calculation load on the primary path of vibration monitoring method and accordingly guarantees the possibility for hardware-based implementation of this procedure.

However, the implementation of the foregoing requires the introduction of a number of new non-system physical quantities, which due to their information content may serve the equivalents of vibration speed and vibration displacement, while by total volume of calculation-related, temporal and hardware resources required for determination of their values, being capable to serve an efficient alternative.

The newly introduced physical quantities are hereinafter referred to as *n-order integral vibratory accelerations*, where, n = 0, 1, 2, … since being formed based on a sequence of reference values of time-discretized vibratory acceleration. It is essential that the number of mathematical operations required for determination of *n*-order integral vibratory acceleration at each time interval will be less than required in case of conducting a *n*-multiple integration of vibratory acceleration. At the same time, such vibratory acceleration, subject to certain conditions, is capable of being directly proportional to the integral, for this reason having the same informational properties as the integral itself.

Therefore the paper provides for solution of developing the mathematical fundamentals or the procedure of control of vibration condition of HPP and PSPP based on displacement parameters alternative to vibration speed and vibration – integral order 1 and order 2 vibration accelerations, which is the main *objective* of this paper.

It should be emphasized that the relevance of the goal assumes an exceptional significance in case of the dynamic type of vibration monitoring with hydrogenerator being in transitional condition, for instance, during start-up and acceleration of a system. The need in this very type of vibration monitoring of hydrogenerators is now overwhelming, with this very type of monitoring the said facilities being poorly investigated.

Integral vibratory accelerations as new non-systemi physical quantities

First of all, let us formulate the reference condition of the problem. As of today, such condition is represented by calculated ordered sequence $\{a_0, a_1, ..., a_k, a_{k+1}, ...\}$ of reference values of time-discretized signal $a_k = a(t_k)$, where $t_k = k \cdot \Delta t$, $k = 0,1,2,...$ generated by sensor-type accelerometer [7-9] of vibration monitoring method based on current vibratory acceleration values $a(t)$ of the unit under monitoring, starting with a certain initial point in time t_0, for instance, $t_0 = 0$, which undergoes the following single-valued reflections into the sequence order, the elements of which have linear mathematical ties with current reference values of vibration speed and vibration displacement of the said unit under monitoring (figure 1). The assigned task requires determination of the law for reflection and disclosure of its practical possibilities.

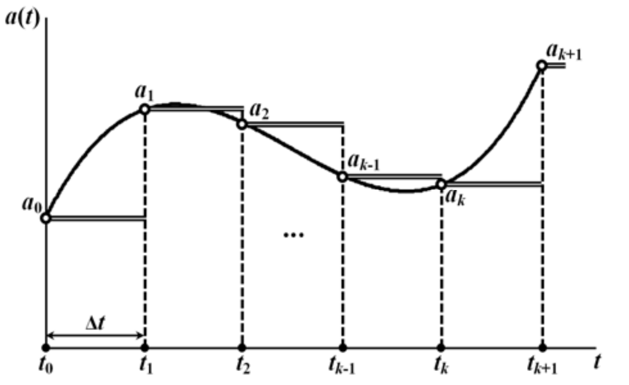

Fig.1. The sequence the reading of vibration acceleration values a (*t*)

Let us also note that the foregoing variable Δt means discretization interval, the value of which will hereinafter be considered arbitrary, though constant.

Hence, $\xi^{(n)}$ physical quantity will be referred to as *n-order integral vibratory acceleration*, where $n = 0,1,2,...$ provided that its reference values will be determined under the rule

(1)
$$\xi_k^{(n)} = \sum_{i_1=0}^{k} \xi_{i_1}^{(n-1)} = \sum_{i_1=0}^{k} \sum_{i_2=0}^{i_1} \xi_{i_2}^{(n-2)} = = \sum_{i_1=0}^{k} \sum_{i_2=0}^{i_1} ... \sum_{i_n=0}^{i_{n-1}} a_{i_n},$$

with the ordered totality of these values forming the order: $\{\xi_0^{(n)}, \xi_1^{(n)}, ..., \xi_k^{(n)}, \xi_{k+1}^{(n)}, ...\}$.

The recursion formula (1) manifests itself as the operator of single-valued reflection of $\{a_0, a_1, ..., a_k, a_{k+1}, ...\}$ sequence into $\{\xi_0^{(n)}, \xi_1^{(n)}, ..., \xi_k^{(n)}, \xi_{k+1}^{(n)}, ...\}$ sequence.

Formula (1) has a generalized character, determining a number of related physical quantities of the same dimension as the very $a(t)$ initial value has.

Based on (1) formula and due to a generalized character thereof, one can easily obtain integral vibratory accelerations deductively with particular $n = 0,1,2,...$ values.

As of today, order 1 and order 2 integral vibratory accelerations are of significance. That is why let us determine them on the basis of generalized (1).

For order 1 integral vibratory acceleration $\xi^{(1)}$, the rule for determination of *k* reference value, pursuant to (1) formula, is represented by operator

(2)
$$\xi^{(1)} = \sum_{i_1=0}^{k} a_{i_1} = a_0 + a_1 + ... + a_k.$$

Reference values of order 2 integral vibratory acceleration $\xi^{(2)}$ pursuant to (1), where $n = 2$ should be calculated using the formula

(3)
$$\xi^{(1)} = \sum_{i_1=0}^{k} \sum_{i_2=0}^{i_1} a_{i_2} = = a_0 + (a_0 + a_1) + ... + (a_0 + ... + a_k),$$

or taking into account (2)

$$\sum_{i_1=0}^{k} \xi_{i_1}^{(1)} = \xi_0^{(1)} + \xi_1^{(1)} + ... + \xi_k^{(1)}.$$

Let us add in the end that, due to its cyclic character, practical calculation of reference values of integral vibration accelerations of both orders 1 and 2 is quite simple (figure 2).

Replacement informational equivalents of physical quantities

The term *replacement informational equivalents* of a physical quantity means all other physical quantities, the reference values of which bear a linear dependence on the reference values of the original. Such being the case, the latter may be presented as

(4)
$$Y_k = c_1 X_{k-s} + c_2,$$

where; $k = 0,1,2,..., c_1, c_2$ are invariants to X reference values, for instance, constant or time-dependent values, where $s \le k$ is constant; X – one of equivalents to physical quantity Y.

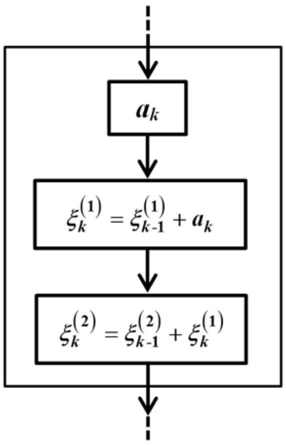

Fig.2. Recursive calculation operator $\xi_k^{(1)}$ i $\xi_k^{(2)}$

During the monitoring procedure, replacement informational equivalents are able to replace the originals at various stages of secondary transformations. Under certain circumstances, such a replacement may appear efficient in the context of reduction of calculation-related, temporal and hardware resources:

$$Z_k = Y_k^{(m)} = c_1^{(m)} Y_k^{(m-1)} + c_1^{(m)} = \ldots =$$

$$= c_1^{(m)} \left\{ \begin{array}{c} c_1^{(m-1)} \left[\begin{array}{c} c_1^{(m-2)} \ldots \left(c_1^{(1)} X_k + c_2^{(1)} \right) + \\ + c_2^{(m-2)} \end{array} \right] + \\ + c_2^{(m-1)} \end{array} \right\} + c_2^{(m)}$$

or

$$Z_k = C_1 X_k + C_2$$

where $C_1 = \prod_{p=1}^{m} c_1^{(p)};\ C_2 = \sum_{s=1}^{m} \left[c_2^{(s)} \times \prod_{p=s+1}^{m} c_1^{(p)} \right].$

Order 1 and order 2 integral vibratory accelerations as replacement informational equivalents of vibration velocity and vibration displacement

1) In order to find and identify possible replacement informational equivalents for vibration speed and vibration displacement, let us build generalized recursive operators, which disclose the rules for one-for-one reflection of sequence reference values of time-discretized vibratory acceleration signal $\{a_0, a_1, \ldots, a_k, a_{k+1}, \ldots\}$ into the sequence of vibration speed $\{v_0, v_1, \ldots, v_k, v_{k+1}, \ldots\}$ and vibration displacement $\{s_0, s_1, \ldots, s_k, s_{k+1}, \ldots\}$ values, respectively.

For vibration speed, such operator is represented by:

$$v_{k+1} = v_k + \Delta v_{k+1} = v_k + a_k \Delta t.$$

Consistently we rewrite the last formula

$$v_{k+1} = \left(v_{k-1} + a_{k-1}\Delta t\right) + a_k \Delta t =$$
$$= \left[\left(v_{k-2} + a_{k-2}\Delta t\right) + a_k \Delta t\right] = \ldots =$$
$$= \left[\left(v_0 + a_0 \Delta t\right) + a_1 \Delta t\right] + \ldots + a_k \Delta t,$$

where $v_0 = v(t_0)$ – the initial value of of vibration velocity.

Hence we have

(5) $$v_{k+1} = v_0 + \Delta t \sum_{i_1=0}^{k} a_{i_1}.$$

Pursuant to (2) expression, the sum total in (5) formula is k reference value of order 1 integral vibratory acceleration.

$$\sum_{i_1}^{k} a_{i_1} = a_0 + a_1 + \ldots + a_k = \xi_k^{(1)}.$$

Then (5) expression acquires (4) form

(6) $$v_{k+1} = \Delta t \xi_k^{(1)} + v_0,$$

where $c_1 = \Delta t,\ c_2 = v_0$, and $\xi^{(1)}$ it is substitute informative equivalent of vibration velocity v.

to (6) formula, with zero $v_0 = 0$ value, direct proportionality will be formed between reference values of vibration speed and order 1 integral vibratory acceleration

(7) $$v_{k+1} = \Delta t \xi_k^{(1)}.$$

In (7) formula,

$$\xi_{\min}^{(1)} = \frac{v_{\min}}{\Delta t}$$

and $$\xi_{\max}^{(1)} = \frac{v_{\max}}{\Delta t}$$

will serve the borders of tolerance interval.

2) Reference value of vibration displacement s at discretization interval $(k+1)$

$$s_{k+1} = s_k + \Delta s_{k+1} = s_k + v_k \Delta t = s_0 + \Delta t v_0 + \sum_{i_1=1}^{k} v_{i_1}.$$

Rewrite:

$$s_{k+1} = \left(s_{k-1} + v_{k-1}\Delta t\right) + v_k \Delta t =$$
$$= \left[\left(s_{k-2} + v_{k-2}\Delta t\right) + v_{k-1}\Delta t\right] + v_k \Delta t = \ldots =$$
$$= \left[\left(s_0 + v_0 \Delta t\right) + v_1 \Delta t\right] + \ldots + v_k \Delta t,$$

or

$$s_{k+1} = s_0 + \Delta t v_0 + \sum_{i_1=1}^{k} v_{i_1},$$

where $s_0 = s(t_0)$ – the initial value of the vibration displacement.

In accordance with (6)

$$v_{i_1} = v_0 + \Delta t \sum_{i_2=0}^{i_1-1} a_{i_2}.$$

Then

$$s_{k+1} = s_0 + (k+1)\Delta t v_0 + \Delta t^2 \sum_{i_1=1}^{k} \sum_{i_2=0}^{i_1-1} a_{i_2},$$

Taking into account

$$\sum_{i_1=1}^{k} \sum_{i_2=0}^{i_1-1} a_{i_2} = \sum_{i_1=0}^{k-1} \sum_{i_2=0}^{i_1} a_{i_2}.$$

After some simple mathematical transformations we finally obtain

(8) $$s_{k+1} = s_0 + (k+1)\Delta t \cdot v_0 + \Delta t^2 \sum_{i_1=0}^{k-1} \sum_{i_2=0}^{i_1} a_{i_2}.$$

It is easy to note that the double sum in (8) formula is $(k-1)$th reference value of order 2integral vibratory acceleration.

$$\sum_{i_1=0}^{k-1} \sum_{i_2=0}^{i_1} a_{i_2} = a_0 + (a_0 + a_1) + ... + (a_0 + ... + a_{k-1}) =$$

$$= \xi_0^{(1)} + \xi_1^{(1)} + ... + \xi_{k-1}^{(1)} = \sum_{i_1=0}^{k-1} \xi_{i_1}^{(1)} = \xi_{k-1}^{(2)}.$$

That is why the formula (8) may be rewritten allowing for (3):

$$s_{k+1} = s_0 + (k+1)\Delta t \cdot v_0 + \Delta t^2 \, \xi_{k-1}^{(2)}. \tag{9}$$

The obtained relation (9) is the operator for reflection of sequence reference values of vibratory acceleration $\{a_0, a_1, ..., a_k, a_{k+1}, ...\}$ of the unit under measurement or monitoring into reference values of vibration displacement $\{s_0, s_1, ..., s_k, s_{k+1}, ...\}$. This operator is distinguished for a number of positive properties.

Operator's generality is one of such properties that enables mathematical description of the process of the said transformation not only at stationary operational modes of the facility under vibration monitoring, but also during its transient processes, and to do so both under zero and non-zero initial conditions of vibration speed v_0 and vibration displacement s_0. Another significant property of the said operator is represented by its linearity, since reflection equation (9) may be written in (4) form.

Should all initial conditions for the unit under vibration monitoring be simultaneously taken as zero, i.e. $s_0 = 0,\ v_0 = 0,\ a_0 = 0$, the reflection operator (9) undergoes considerable and significant simplifications. It should be noted that such a situation is quite frequent in manufacturing practice. For instance, these very preconditions are observed during start-up and acceleration of hydrogenerator system in the event of its vibration state monitoring. As of today, such type of vibration monitoring at hydropower facilities is in demand, though poorly investigated.

Hence, under zero initial conditions in (9) formula, qualitative changes are observed, where not just linear dependence, but also direct proportionality is formed $s_{k+1} = \Delta t^2 \, \xi_{k-1}^{(2)}$ between reference values of vibration displacement and order 2 integral vibratory acceleration.

$$s_{k+1} = \Delta t^2 \xi_{k-1}^{(2)},$$

where $c_1 = \Delta t^2,\ c_1 = 0$, and $\xi^{(2)}$ Are replacing informative equivalents of vibration displacement s.

Conclusions

The paper allowed obtaining mathematical generalized mapping operators for reference values of sequence vibration accelerations in sequence vibration speed and vibration displacement, and on this basis developed is a new class of non-system physical quantities – integral vibration accelerations, the practical use of some of which – order 1 and order 2 integral vibration accelerations – as against conventional approaches enable the reduction of the total volume of calculation load on the primary path of vibration monitoring method and ensure the procedure for dynamic monitoring of vibration state of HPP and PSPP hydropower units, as well as other related hydropower facilities during their transient processes.

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