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The synthesis of optimum current obtained by mathematical models for an electrically propelled truck drive electromotor

***Abstract**. This paper synthesizes the models of the current to be applied to the armature of a drive direct-current series electric motor used within an electric drive system of an electrically propelled truck, such models being optimal in terms of battery power expenditure minimization. The paper offers a parameter identification method for the synthesized models to be used either for a loaded truck or for an empty one, either for a horizontal run or for a downhill/uphill run.*

***Streszczenie.** W artykule przedstawiony został sposób modelowania prądu płynącego przez obwód silnika, stosowany w układzie napędowym wózka pojazdu ciężarowego. Przedstawiony został dobór modelu optymalnego z punktu widzenia minimalizacji wymaganego nakładu energii akumulatora. W artykule zawarto również metodę identyfikacji parametrów dla przedstawionych modeli, w tym przeznaczone zarówno dla pustego jak i obciążonego wózka pojazdu, jadącego po powierzchni płaskiej i pochylonej. (**Synteza optymalnego prądu silnika elektrycznego wózka pojazdu ciężarowego za pomocą modeli matematycznych**).*

Keywords: mathematical model, truck, electric drive, DC electric motor, battery, power expenditure minimization.
Słowa kluczowe: model matematyczny, pojazd ciężarowy, napęd elektryczny, silnik prądu stałego, akumulator, minimalizacja wydatku energii.

Introduction

Every day witnesses an increased number of electrically-propelled battery-powered vehicles on the roads in many countries. So far, these are the cars driven mostly by AC electric motors where the battery DC power is converted into AC power applicable to an asynchronous electric motor. However, as is known from any electric drive theory textbook, e.g. [1, 2], the speed-torque curve of an asynchronous electromotor (which is a relationship $\omega = f(T)$, where ω is the speed of the electromotor shaft, and T is the shaft tractive effort torque of such motor) looks as shown in Fig. 1 below.

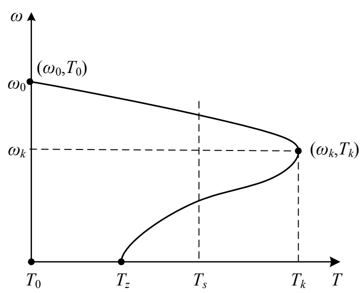

Fig.1. Asynchronous electromotor speed-torque curve

It is clear from Fig. 1 that as soon as the load torque T_s applied to an asynchronous electromotor shaft rises to its tractive effort torque critical value T_k, the electric motor behavior shifts over to an unstable segment of the speed-torque curve between the points (ω_k, T_k) and T_z, and, upon a further increase of the load torque, the motor's tractive effort torque goes to the point T_z, where the driving electric motor stops and where the vehicle driven by such electric motor has to stop, too. Meanwhile, it is noteworthy that a speed-torque curve of a synchronous AC electric motor differs from a speed-torque curve of an asynchronous electric motor, as shown in Fig. 1, only in the fact that its operating performance, which lies between the points (ω_0, T_0) and (ω_k, T_k) is a straight-line segment parallel to the torque axis [11].

Inasmuch as a car does not weigh much and the passengers inside and any luggage in the trunk are not so heavy, we can choose a right drive electric motor to avoid the situation where the load torque T_s exceeds the tractive effort torque critical value T_k even when a car runs uphill. Therefore, for a car, it is quite possible to use an AC asynchronous or synchronous electric motor as a propelling motor.

However, due to a great weight of trucks and even a greater weight of the load they carry, when a truck goes uphill, the load torque T_s may surpass the electric motor's tractive effort torque critical value T_k of the electric motor, if such motor is an AC electromotor. So, while selecting drive electric motors for trucks, preference is given to DC series electric motors, whose speed-torque curve, as is known [1, 2], looks as shown in Fig. 2 below.

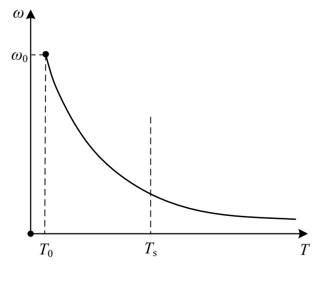

Fig.2. Speed-torque curve of a drive direct-current series electric motor

It is clear from Fig. 2 that the speed-torque curve of a drive direct-current series electric motor is stable within the whole range of load torque potential values, so if such motor is used as a drive electric motor for a truck, there cannot be a case when the tractive effort torque becomes lower than the load torque value; hence such motor-driven truck will never stop even on a very steep rise of a road.

Since the longest distance coverable by an electrically-propelled truck without recharging the battery depends, first and foremost, on the battery capacity and on the electric power consumption per kilometer of the run course, then, in view of the battery capacity limits set by the manufacturers of such batteries, the vital problem to be solved is to find out such rate of electric power consumption which would allow the truck to cover the longest distance possible. A solution to the problem starts with a synthesis of the mathematical models simulating the running process of a vehicle driven by a direct-current series electric motor; such models should be optimum in terms of minimized consumption of the battery power used to energize the electric drive system of an electrically propelled vehicle.

Problem formulation for named units

Let us start solving the problem with tying it up to the equation [3], known from the theory of mechanics academic course

(1) $$m\frac{dV}{dt}=F_T-F_S$$

$$V(t_j)=V_j,\quad V'(t_j)=V'_j,\quad j=0,1,2,\ldots,$$

which describes, in the course of time *t*, the motion of a vehicle of the weight m at the speed V under the motor-induced tractive effort F_T which has to overcome the resistance force F_S, during the period of time T_i under the initial conditions that, at the start of such period of time, the said vehicle was moving at the speed V_j and acceleration V'_j.

As is known from the abovementioned theory of mechanics [3] and aerodynamics [4], the resistance force F_S consists of three components, among which the first component F_O, which denotes the friction of the vehicle wheels against the road pavement, is proportional to the weight of the vehicle F_G; the second one F_1, which denotes the friction of the vehicle side surface against the air, is proportional to the vehicle speed; and the third one F_2, which denotes the front air pressure on the vehicle cross-section, is proportional to the squared speed of the vehicle, i.e.

(2) $$F_S=F_O+F_1+F_2=k_0F_G+k_1V+k_2V^2,$$

where the value of the coefficient k_0 is a tabular value, which depends on the material of the road surface; the value of the coefficient k_1 depends on the vehicle side surface area; and the value of the coefficient k_2 depends on the cross-sectional area of the vehicle.

By inserting, for F_S, the expression from (2) into (1), we derive the vehicle dynamic equation such as

(3) $$m\frac{dV}{dt}=F_T-k_0F_G-k_1V-k_2V^2,$$

In case of this differential equation and further on, we will omit, for the sake of brevity, any boundary conditions, while remembering that such conditions are certainly required to achieve a specific solution to such equation.

However, recalling that

(4) $$F_T=\frac{T_I}{R},$$

and taking into account that the expression for a drive direct-current series electric motor is as follows

(5) $$T_I=k_DI\Phi(I),$$

where k_D is a coefficient derivable from the nameplate data of the electric motor in question, I is the armature current of such electric motor, and $\Phi(I)$ is the induction coil magnetic flux, which, according to the magnetization curve, is a function of the armature current [1, 2], so, taking into consideration the relations (4) and (5), we arrive at the dynamic equation (3) for a vehicle such as

(6) $$m\frac{dV}{dt}=\frac{k_D}{R}I\Phi(I)-k_0F_G-k_1V-k_2V^2,$$

However, it is worth mentioning that the equation (6), as presented herein, describes the dynamics of a vehicle only on the way along a horizontal road.

Yet, if a vehicle is moving downhill, as is shown in Fig. 3a, or uphill, as is shown in Fig. 3b, then, the term ($-k_0F_G$) in the equation (6) should be substituted for the binominal ($-k_0F_G\cos\alpha+F_G\sin\alpha$) for the downhill motion, or for the binominal ($-k_0F_G\cos\alpha-F_G\sin\alpha$) for the uphill motion, i.e. the expression becomes as follows:

(7)

$$m\frac{dV}{dt}=\frac{k_D}{R}I\Phi(I)-k_0F_G\cos\alpha+F_G\sin\alpha-k_1V-k_2V^2,$$

– for the downhill motion,

(8)

$$m\frac{dV}{dt}=\frac{k_D}{R}I\Phi(I)-k_0F_G\cos\alpha-F_G\sin\alpha-k_1V-k_2V^2,$$

– for the uphill motion.

Now, let us consider the criteria, whose values should be minimized to solve the problem of optimization.

For the problem of optimization of the motion of a vehicle propelled by a drive direct-current series electric motor, the optimization criterion shall be a functional such as

(9) $$E_I=\int_0^{t_I}UIdt,$$

which characterizes the quantity of the electrical power E_I, spent within the run time t_I by a vehicle propelled by a DC-series-electromotor, the armature of which bears the current I created by the voltage U applied to its terminals.

It is evident from the diagram, as shown in Fig. 4 (where U_B is the voltage of the battery B with the internal resistance r_B) that the criterion (9) is easily reducible to the form such as formula (10)

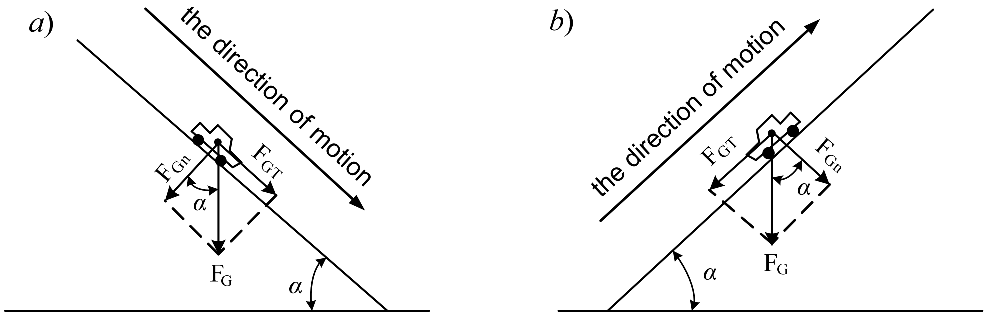

Fig.3. Vector diagrams of forces created by the weight force of a vehicle moving downhill (a) and uphill (b)

(10)
$$E_I = \int_0^{T_I} UIdt = \int_0^{T_I} (U_B - \Delta U)Idt = \int_0^{T_I} U_B(1 - \frac{r_B}{U_B} I)Idt$$

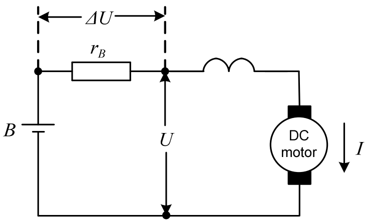

Fig.4. The wiring diagram for connection of a drive DC series electric motor to the battery B with the internal resistance r_B

Normally, while driving a vehicle, we are concerned not only about the amount of the electric power consumable over the run time t_I, but also about the distance S coverable within such time. Apparently, knowing the speed of the vehicle V, we can determine such distance using the functional:

(11)
$$S = \int_0^{t_I} Vdt,$$

However, considering the functional (11), we thereby attribute our problem of optimization to the isoperimetric class of problems [5, 6], for we have to find out such law of the vehicle speed variation which would minimize the functional (10) on condition that the dynamics of the vehicle be describable by the relevant equation from the set (6)-(8), and the distance, covered by the vehicle, be set by the functional (11).

Problem formulation for relative units

In order to generalize the obtained results, it makes sense to switch over to the relative values, using the following basic value: for the propelling force F_T, its value F_R in the rated duty mode; for the torque T on the shaft, its rated value T_R; for the vehicle speed V, its rated value V_R; for the drive motor armature current I, its rated value I_R; for the drive motor magnetic flux $\Phi(I)$, its value under the rated current $\Phi_R = \Phi(I_R)$; for the voltage U, applied to the drive motor terminals, the voltage U_B of the fully-charged battery. Also, we will use the basic value derivatives such as: for the time t, the electromechanical time constant t_M, which relates to the basic values through the expression –

(12)
$$t_M = \frac{mV_R}{F_R},$$

for the distance S, the value S_R, which relates to the basic values through the expression –

(13)
$$S_R = V_R t_M,$$

for the battery direct-current energy E, the value E_R, which relates to the basic values through the expression –

(14)
$$E_R = U_B I_R t_M,$$

Having taken into account the input basic values and the derivative basic values:

– as a relative value equivalent to equation (6), we obtain the equation –

(15)
$$\frac{d\nu}{d\tau} = i\phi(i) - f_0 - f_1\nu - f_2\nu^2,$$

where

(16)
$$\nu = \frac{V}{V_R},\ \tau = \frac{t}{t_M},$$

(17)
$$i = \frac{I}{I_R},\ \phi(i) = \frac{\Phi(I)}{\Phi(I_R)},$$

(18)
$$f_0 = \frac{k_0 F_G t_M}{mV_R},\ f_1 = \frac{k_1 t_M}{m},\ f_2 = \frac{k_2 V_R t_M}{m},$$

(19)
$$\frac{k_D I_R \Phi(I_R) t_M}{RmV_R} = 1,$$

– as a relative value equivalent to the dynamic equation (7) of downhill motion, we obtain the equation –

(20)
$$\frac{d\nu}{d\tau} = i\phi(i) + f_0^* \sin\alpha - f_0 \cos\alpha - f_1\nu - f_2\nu^2,$$

where, in addition to the above-defined variables and coefficients, we have one more coefficient

(21)
$$f_0^* = \frac{F_G t_M}{mV_R},$$

– as a relative value equivalent to the dynamic equation (8) of uphill motion, we obtain the equation

(22) $$\frac{d\nu}{d\tau} = i\phi(i) - f_0^* \sin\alpha - f_0 \cos\alpha - f_1\nu - f_2\nu^2 ,$$

– as a relative value equivalent to the optimization criterion (10), we obtain the criterion functional –

(23) $$e = \int_0^{\tau_i} (1-\beta i) i d\tau ,$$

where

(24) $$e = \frac{E}{E_R} = \frac{E}{U_B I_R t_M} ,$$

(25) $$\tau_i = \frac{t_I}{t_M} ,$$

(26) $$\beta = \frac{r_B I_R}{U_B} ,$$

– as a relative value equivalent to the constraint (11), we obtain the criterion functional –

(27) $$s = \int_0^{\tau_i} \nu d\tau ,$$

where

(28) $$s = \frac{S}{V_R t_M} ,$$

To complete the formalization of the problem, which we are to solve using the variational method [5, 6], we have only to formalize, in the relative units, the magnetization curve $\phi(i)$, which, as shown in the work [7, 8], is best generated for the variational method by the model

(29) $$\phi(i) = \begin{cases} -a_2 i^2 + b_2 i, & i = [0, i_*), \\ a_1 + b_1 i, & i = [i_*, \infty), \end{cases} ,$$

which is a combination of a straight line and part of parabola that come together when the argument value is i_*. It is quite clear that while deciding on the electric motors to be used for an electrically propelled truck, we should choose such motors which would function within the straight-line segment of the magnetization characteristic under full load and within the parabolic segment of the magnetization characteristic without load.

Finally, for the equations (6), (7) and (8), we will specify the initial conditions, as mentioned above near the equation (1) only, in the relative units as follows:

(30) $$\begin{cases} \nu(\tau_j) = \dfrac{V(t_j)}{V_R} = \nu_j, \quad j = 0,1,2,... \\ \nu'(\tau_j) = V'(t_j)\dfrac{t_M}{V_R} = \nu'_j. \end{cases} ,$$

Now, we have got all formal relations in relative units, which are required to solve the problem of synthesis of vehicle optimum motion mathematical models.

Synthesis of optimum current mathematical models for a truck drive electromotor

At first, let us synthesize the optimum current mathematical model for a drive electric motor of a fully loaded truck running along a horizontal segment of a road. In this case, we will use the linear component in the equation (15) in order to achieve approximation of the magnetization curve $\phi(i)$ expressed in relative units and described by the expression (29). So, let us assume that

(31) $$\phi(i) = a_1 + b_1 i ,$$

As far as the problem contains the constraint (27) in the form of a functional, i.e. this problem belongs to the isoperimetric class of problems, the Lagrange function adjusted for the criterion (23) and constraint (27) will be as follows:

(32)
$$L(i,s,\nu,i',s',\nu',\tau) = i - \beta \cdot i^2 + \lambda_0(\tau)\cdot(s' - \nu) + ,$$
$$+ \lambda_1(\tau)\cdot\left(\nu' - a_1 \cdot i - b_1 \cdot i^2 + f_0 + f_1 \cdot \nu + f_2 \cdot \nu^2\right)$$

and the system of Euler equations will be as follows

(33) $$\left.\begin{aligned} L_i - \frac{d}{d\tau} L_{i'} = 0, \\ L_s - \frac{d}{d\tau} L_{s'} = 0, \\ L_\nu - \frac{d}{d\tau} L_{\nu'} = 0. \end{aligned}\right\} ,$$

Having the Lagrange function (32) inserted into the system of Euler equations (33), i.e. having taken all required partial and ordinary derivatives of $L(i,s,\nu,i',s',\nu',\tau)$ with respect to i,s,ν,i',s',ν',τ, we will obtain the system of equations such as

(34) $$\left.\begin{aligned} & 1 - 2\cdot\beta\cdot i - a_1\cdot\lambda_1(\tau) - 2\cdot b_1\cdot i\cdot\lambda_1(\tau) = 0, \\ & -\frac{d\lambda_0(\tau)}{d\tau} = 0, \\ & -\lambda_0(\tau) + f_1 \cdot\lambda_1(\tau) + 2\cdot f_2\cdot\nu\cdot\lambda_1(\tau) - \frac{d\lambda_1(\tau)}{d\tau} = 0. \end{aligned}\right\}$$

The second equation of the system (34) implies that

(35) $$\lambda_0(\tau) = -C_1 ,$$

where C_1 is the unknown constant.

Let us rewrite the third equation of the system (34) as

(36) $$\frac{d\lambda_1(\tau)}{d\tau} - \lambda_1(\tau)\cdot(f_1 + 2\cdot f_2\cdot\nu) = C_1 .$$

Now, we will solve the differential equation (36) with respect to the undetermined multiplier $\lambda_1(\tau)$, while taking into account that the solution consists of two components, viz. a free component and a forced component

(37) $$\lambda_1(\tau) = \lambda_{1free}(\tau) + \lambda_{1forced}(\tau) .$$

The free component of the solution of the equation (26) will be as follows

(38) $$\lambda_{1free}(\tau) = C_2 \cdot e^{(f_1 + 2\cdot f_2\cdot\nu)\tau} ,$$

where C_2 is the integration constant, and the forced component is –

(39) $$\lambda_{1forced}(\tau) = -\frac{C_1}{f_1 + 2\cdot f_2\cdot\nu} .$$

Having inserted the expressions (38) and (39) into the equation (37), we obtain

$$\lambda_1(\tau) = C_2 \cdot e^{(f_1 + 2\cdot f_2 \cdot \nu)\tau} - \frac{C_1}{f_1 + 2 \cdot f_2 \cdot \nu} . \tag{40}$$

Then, having inserted the expression (40) into the first equation of the system (34) and having solved this equation with respect to i, we will have

$$i(\tau) = \frac{1 - a_1 \cdot \left(C_2 \cdot e^{(f_1 + 2\cdot f_2 \cdot \nu)\cdot\tau} - \frac{C_1}{f_1 + 2 \cdot f_2 \cdot \nu} \right)}{2 \cdot \beta + 2 \cdot b_1 \cdot \left(C_2 \cdot e^{(f_1 + 2\cdot f_2 \cdot \nu)\cdot\tau} - \frac{C_1}{f_1 + 2 \cdot f_2 \cdot \nu} \right)} = i^*(C_1, C_2, \nu, \tau). \tag{41}$$

It is the expression (41) which defines the current law $i(\tau)$ for the armature of a drive motor used for a loaded electric vehicle. Having implemented such law, we will minimize the expenditure of the battery energy e for the linear speed $\nu(\tau)$ of the electric vehicle at a given moment of time.

Having analyzed the expression (41), we can see that it does not structurally contain the parameter f_0 peculiar to a horizontal segment of a road, meaning that if the optimization problem is solved according to a similar procedure both with the use of the equation (20) for the motion of a vehicle downhill and with the use of the equation (22) for the motion of a vehicle uphill, while such equations contain a sinusoidal component and a cosinusoidal component for f_0, such components will be absent in the optimum current model, too. Therefore, the expression (41) will be true both for the downhill motion and for the uphill motion. As is shown below, such components influence only the numerical values of the constants C_1, C_2, definable with the use of the optimum current model parametric identification method (41), as offered below.

Now, let us synthesize an optimum current mathematical model for an electric motor, which drives an empty vehicle running along a horizontal segment of a road. In this case, we will use the parabolic component in the equation (15) in order to achieve approximation of the magnetization curve $\phi(i)$ expressed in relative units and described by the expression (29). So, let us assume that

$$\phi(i) = -a_2 i^2 + b_2 i . \tag{42}$$

In such a case, the Lagrange function will be as follows:

$$L(i, s, \nu, i', s', \nu', \tau) = i - \beta \cdot i^2 + \lambda_0(\tau) \cdot (s' - \nu) + , \tag{43}$$

$$+ \lambda_1(\tau) \cdot (\nu' + a_2 \cdot i^3 - b_2 \cdot i^2 + f_0 + f_1 \cdot \nu + f_2 \cdot \nu^2)$$

and the system of Euler equations will be, obviously the same as in the expression (33), discussed above.

Having the Lagrange function (43) inserted into the system of Euler equations (33), i.e. having taken all required partial and ordinary derivatives of $L(i, s, \nu, i', s', \nu', \tau)$ with respect to $i, s, \nu, i', s', \nu', \tau$, we will obtain the system of equations such as

$$\left. \begin{aligned} & 1 - 2 \cdot \beta \cdot i + 3 \cdot a_2 \cdot i^2 \cdot \lambda_1(\tau) - 2 \cdot b_2 \cdot i \cdot \lambda_1(\tau) = 0, \\ & -\frac{d\lambda_0(\tau)}{d\tau} = 0, \\ & -\lambda_0(\tau) + f_1 \cdot \lambda_1(\tau) + 2 \cdot f_2 \cdot \nu \cdot \lambda_1(\tau) - \frac{d\lambda_1(\tau)}{d\tau} = 0. \end{aligned} \right\} \tag{44}$$

As in the preceding case, using the second and the third equations of the system (44), we find the undetermined multipliers, $\lambda_0(\tau)$ and $\lambda_1(\tau)$, respectively, which take the form, identical to that, as represented by the expressions (35) and (40).

Then, having solved the first equation of the system (44) with respect to the current i, we obtain

$$i_{1,2}(\tau) = \frac{2 \cdot \beta + 2 \cdot b_2 \cdot \lambda_1(\tau) \pm \sqrt{(2 \cdot \beta + 2 \cdot b_2 \cdot \lambda_1(\tau))^2 - 12 \cdot a_2 \cdot \lambda_1(\tau)}}{6 \cdot a_2 \cdot \lambda_1(\tau)} = i^*_{1,2}(C_1, C_2, \nu, \tau), \tag{45}$$

where $\lambda_1(\tau)$ should be replaced with the value thereof, as given by the expression (40), but we will refrain from doing so in order to preclude the expression (45) from overloading.

It is the mathematical model (45), as adjusted for the expression (40), which defines the current law $i(\tau)$ for the armature of a drive electric motor used for an empty truck. Having implemented such model, we will minimize the expenditure of the battery energy e for the linear speed $\nu(\tau)$ of the vehicle at a given moment of time.

Having analyzed the expression (45), we can see that it does not structurally contain the parameter f_0 peculiar to a horizontal segment of a road, meaning that if the optimization problem is solved according to a similar procedure both with the use of the equation (20) for the motion of a vehicle downhill and with the use of the equation (22) for the motion of a vehicle uphill, while such equations contain a sinusoidal component and a cosinusoidal component for f_0, such components will be absent in the optimum current model, too. Therefore, the expression (45) holds true both for the downhill motion and for the uphill motion. As to the *plus* and *minus* signs in the expression (45), the *plus* is to be used to denote acceleration whereas the *minus* denotes deceleration.

The mathematical models (41) and (45) contain the unknown constants, viz. C_1 and C_2; so, to make such models fit for practical application, we have to define such constants, using the specified initial conditions (30), i.e. we have to perform a parametric identification of such models.

Parametric identification of optimum current models

In the process of parametric identification of the mathematical model (41), we need to define the two parameters, viz. C_0 and C_1. This implies that the method to be used for definition thereof should be based on the solution of a system of two equations where such parameters are unknowns. Therefore, the synthesis method for parametric identification of the mathematical model (41) actually boils down to setting up such equations and to developing an algorithm to solve them.

We will set up one of the equations with the use of the electric circuit, as shown in Fig. 4.

As is known form the electric drive theory [1, 2], the voltage balance in the electric power circuit, as shown in Fig.4, can be described as

$$U_B = Ir_B + Ir_A + E_\omega + L_A \frac{dI}{dt}, \tag{46}$$

where r_A, L_A are active resistance and inductance, respectively, of the armature winding of a drive electric motor, and

(47) $$E_{\omega} = k_{\omega}\omega\Phi(I)$$

is the rotational electromotive force, which is generated in the armature and is proportional to the product of the angular velocity of the armature rotation ω times the magnetic flux, which, in turn, is a function of the armature current.

Having divided the equation (5) by U_B and having performed some additional conversions, we obtain its counterpart in relative units –

(48)
$$1 = \beta i + \sigma i + \delta v\phi(i) + \gamma\frac{di}{d\tau} = \beta i + \sigma i + \delta v(a_1 + b_1 i) + \gamma\frac{di}{d\tau}$$

where

(49) $$\sigma = \frac{r_A I_R}{U_B},\quad \gamma = \frac{I_R L_A}{U_B T_M},\quad \delta = \frac{k_{\omega} V_R \Phi(I_R)}{U_B R}$$

while the other parameters and variables have been found out before.

By inserting the expression (41) into the equation (48), and then by inserting the initial conditions (30) into the result of the first substitution and having transposed all terms of the equation to one side, we obtain a functional equation of the two variables, C_1, C_2, such as

(50) $$\varphi_1(C_1, C_2, a_1, b_1, f_1, f_2, v_j, v_j', \beta, \sigma, \delta, \gamma, \tau_j) = 0$$

where all other parameters are known numbers.

So, the equation (50) will one of those required by us to develop a parametric identification method for the mathematical model (41) of the optimum armature current for a drive electric motor of a loaded vehicle running along a horizontal segment of a road.

To set up the other equation required, first, let us insert the expression (41) and then the initial conditions (30) into the dynamic equation (15) for a vehicle running along a horizontal span of a road. As a result of such substitution and re-grouping of the terms, we obtain another functional equation of the two variables, C_1, C_2, such as

(51) $$\varphi_2(C_1, C_2, a_1, b_1, f_0, f_1, f_2, v_j, v_j', \beta, \sigma, \delta, \gamma, \tau_j) = 0$$

Having solved the system of the nonlinear equations (50) and (51) with one of the approximation methods available in any computer software base, we arrive at the numerical values C_1^*, C_2^* of the parameters C_1, C_2 of the mathematical model (41), thereby completing the implementation of the algorithm of the proposed method for parametric identification of the mathematical model (41).

It is quite clear that the parametric identification algorithm for the mathematical model (45), which specifies the optimum armature current for a drive electric motor of an empty vehicle running along a horizontal segment of a road will differ from the above-said procedure only in the fact that, in deriving the functional equations (50) and (51), the expression (45) will be always inserted instead of the expression (41).

As to the parametric identification algorithm for the mathematical models of the optimum armature current for a drive electric motor of a loaded or empty vehicle running downhill or uphill, in the course of deriving the second functional equation, which is to be adjusted for the sinusoidal or cosinusoidal component, the expressions (20) and (21) are to be used instead of the expression (15).

Finally, it should be noted, that some fragments of this article were published in other works [9, 10] of the authors hereof.

Conclusions

1. This paper presents the solution to the optimization problem in terms of minimized consumption of the battery power by a loaded or empty vehicle propelled by DC series electric motors and running along a horizontal segment of a road or downhill/uphill.
2. This paper includes synthesized the optimum current mathematical models for a drive direct-current series electric motor propelling an electrical vehicle, whether loaded or empty.
3. The paper offers the parameter identification method for the synthesized mathematical models of the optimum armature current in a drive electric motor.
4. It has been proven that implementation of the synthesized mathematical models will allow minimizing the battery energy consumption by a loaded or empty electric vehicle under any law of its speed variation.

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