An Algorithm for transformer hot spot temperature determining

Abstract. This paper presents an algorithm for the winding temperature change estimation and the hot spot temperature determining of an oil filled power transformer. The proposed algorithm is based on the functional relation between the change of winding resistance and the measured oil temperature. The temperature changes approximation is based on the method of least squares. The proposed method is applied in the software for a field test instrument and verified by the field experiments.


Keywords: algorithm, extrapolation, hot spot, temperature measurement, transformers.

Słowa kluczowe: algorytm, ekstrapolacja, najgorzejzony punkt, pomiar temperatury, transformatory.

Introduction

The information about the transformer over-temperature events and hot spot temperature is a key factor for preventing premature aging, loss of the insulation properties, increased operating losses and preventing damage of paper-oil insulation [1]. The data obtained from oil temperature measuring and field tests are used to estimate possible overheated locations inside and outside windings, and also to predict actual transformer loading capability. Temperature of the transformer hot spot should be within the range of allowed values required by the international standards [2, 3]. This temperature is most often calculated from the results of the heat run tests [4, 5].

The hot spot areas are generally located close to the upper top of the transformer, what is discussed in [6, 7] and, in some cases, are not accessible for direct temperature measurement. One efficient way to estimate the hot spot temperature is the use of the measured winding resistance during the transformer cooling. The main problem in this approach is the influence of winding cooling from the moment of shutdown from the power to the moment of starting the resistance measurement. Thus, the algorithm for transformer hot spot determining must estimate initial value of resistance (and temperature) before the resistance measurement, at the instant of shutdown. The resistance change function must take into account both thermal properties, of the winding and of the oil. After the approximated resistance change curve is extrapolated and initial resistance is determined, the hot spot temperature can be calculated according to the IEC 60076-2 standard [2], by using the information of the oil temperature at the bottom and on the top of the transformer.

A Hot Spot Temperature - HST algorithm for transformer hot spot temperature determining presented in the paper is based on the approximation of the resistance curve by two different functions, which depend on winding and oil temperature, respectively. The conclusion in [8] recommends that an oil cooling curve can be used to determine the oil thermal time constant. The shape and parameters of the resistance function in presented algorithm are determined and calculated by the least squares method [9], which uses measured data of winding resistance. For this purpose, resistance measurements was obtained using voltmeter-ammeter method [10], discussed in [11].

The hot spot temperature algorithm

The main idea of the proposed algorithm

Transformer hot spot temperature can be determined from the mean winding resistance $R_{wm}$ which is extrapolated from the resistance change function $F(t)$, (Figure 1). $R_{wm}$ represents the mean winding resistance, considering the temperature resistance change along the winding height.

The transformer is switched off from the power supply at the moment $t_0$. At the moment $t_1$, resistance measurements starts in equidistant time samples $\Delta t$. Measured values of the resistance $R(k)$, for $k = 0, 1, 2, ..N$, at the instant $k\Delta t$, are shown by dots in Figure 1. Index $k$ represents the ordinal number of the measurement. In order to determine the initial winding resistance $R_{wm}$, the function $F(t)$ should be fitted from measured data and extrapolated to the moment $t_0$ in which $R_{wm} = F(t_0)$.

The cooling process can be divided into three parts. In the first segment $A$ in figure 1 measured data are not available. In the second segment $B$, the shape of the function $F(t)$ is defined by simultaneous cooling of the winding (function $F_1(t)$) and the oil (function $F_2(t)$). In this segment the winding resistance function $F(t)$ is represented as the sum of these two functions. In the third segment $C$ the shape of the function $F(t)$ is defined by the oil cooling only, and has a slowly changing character [2].

To fit the function $F(t)$ using measured resistance data and extrapolate it to the instant $t_0$, functions $F_1(t)$ and $F_2(t)$ must be found [12]. The main idea consists in determining

Fig. 1. Approximation of the winding resistance change

\[ R(t) = R(t_0) \star F(t) \]

\[ R_{wm} = \text{extrapolation} \]

\[ R(t) = \text{measured} \]

\[ F(t) = \text{function} \]

\[ t_0 \]

\[ t_1 \]

\[ \Delta t \]

\[ R_{wm} \]

\[ F_1(t) \]

\[ F_2(t) \]

\[ \text{function} \]

\[ \text{measurement} \]

\[ \text{period} \]

\[ \text{extrapolation period} \]

\[ \text{function} F_1(t) \]

\[ \text{function} F_2(t) \]

\[ \text{Measurement} \]

\[ \text{period} \]
Thus, the coefficients for all three types of the function that both segments known in advance, it can be assumed in the first iteration resistance data. Since the beginning of segment the corresponding scaled values transformer rating and cooling system: constant case selected by one of three possible types, depending on the measured resistance change, for \( t > t_2 \) the form of this function is assumed to be an linear case, \( F(t) = \exp(\frac{t - t_0}{T_0}) \). The main problem in this approach is that the instant and index \( N \) corresponds to the total number of the resistance data. Since the beginning of segment \( C \) is not known in advance, it can be assumed in the first iteration that both segments \( B \) and \( C \) have equal length \( (j = N/2) \). Thus, the coefficients for all three types of the function \( F(t) \) are as follows:

**Interpolation of the function \( F(t) \) in segment \( C \)**

In the segment \( C \), function \( F(t) \) consists only of the function \( F_2(t) \). Considering that the function in that segment may take any of the above mentioned forms, it is necessary to define all possible types and determine the form that interpolates measured values with lowest error.

The coefficients \( A_{02C} \), \( A_{02L} \), \( A_{02E} \), \( m \) and \( T_0 \) are determined from the measured resistance data \( R(k) \) at the instants \( k = j, j + 1, j + 2, ..., N \), in the segment \( C \) (figure 1). Index \( j \) corresponds to the measurement at the instant \( t_0 \), and index \( N \) corresponds to the total number of the resistance data. Since the beginning of segment \( C \) is not known in advance, it can be assumed in the first iteration that both segments \( B \) and \( C \) have equal length \((j = N/2)\). Thus, the coefficients for all three types of the function \( F_2(t) \) are as follows:

**Case 1: Constant shape of the function:**

\[
F_2 = A_{02C}.
\]

The sum of the least squares is given by:

\[
S_{2C} = \sum_{k=j}^{N} \left[ A_{02C} - R(k) \right]^2.
\]

The coefficient \( A_{02C} \) is found by differentiating \( \frac{\partial S_{2C}}{\partial A_{02C}} = 0 \), so that:

\[
A_{02C} = \frac{\sum_{k=j}^{N} R(k)}{N - j}.
\]

**Case 2: Linear shape of the function:**

\[
F_2 = A_{02L} \cdot m \cdot t.
\]

The sum of the least squares is given by:

\[
S_{2L} = \sum_{k=j}^{N} \left[ A_{02L} - m \cdot k \cdot \Delta t - R(k) \right]^2.
\]

After differentiating \( \frac{\partial S_{2L}}{\partial A_{02L}} = \frac{\partial S_{2L}}{\partial k} = 0 \), the coefficients are found as:

\[
A_{02L} = \frac{(N - j) \sum_{k=j}^{N} (k \cdot \Delta t)^2 - \sum_{k=j}^{N} R(k) + \sum_{k=j}^{N} R(k) \sum_{k=j}^{N} (k \cdot \Delta t)^2}{(N - j)^2 \sum_{k=j}^{N} (k \cdot \Delta t)^2 - \sum_{k=j}^{N} (k \cdot \Delta t)^2}
\]

**Case 3: Exponential shape of the function:**

\[
F_2 = A_{02E} \cdot \exp\left(\frac{t - t_0}{T_0}\right).
\]

In order to simplify the calculation, logarithmic transformation of this function is used:

\[
\ln F_2 = \ln A_{02E} - \frac{1}{T_0} \cdot k \cdot \Delta t.
\]

The sum of least squares is given by:

\[
S_{2E} = \sum_{k=j}^{N} \left[ \ln A_{02E} - \frac{1}{T_0} \cdot k \cdot \Delta t - \ln R(k) \right]^2,
\]

and by differentiating \( \frac{\partial S_{2E}}{\partial \ln A_{02E}} = \frac{\partial S_{2E}}{\partial \ln t_0} = 0 \) follows:

\[
A_{02E} = \exp \left[ \frac{\sum_{k=j}^{N} \ln R(k) \cdot \frac{N}{k} \cdot (k - \Delta t)^2 - \sum_{k=j}^{N} \ln R(k) \cdot k \cdot (k - \Delta t) \cdot \sum_{k=j}^{N} (k - \Delta t)}{N \cdot \sum_{k=j}^{N} (k - \Delta t)^2 - \sum_{k=j}^{N} (k - \Delta t) \cdot \sum_{k=j}^{N} \ln R(k)} \right].
\]

After the coefficients \( A_{02C} \), \( A_{02L} \), \( A_{02E} \) and \( T_0 \) are determined, the proper type of the function \( F_2(t) \) (Case 1, Case 2 or Case 3) should be selected by the minimum value of sums \( S_{2C} \), \( S_{2L} \) and \( S_{2E} \). The selected type of the function \( F_2(t) \) should remain the same for assuming shorter lengths of segment \( C \). Thus, the length of segment \( C \) should be split by setting \( j = N/4 \) and the calculation (1)-(12) repeated. When the type (constant, linear or exponential) obtained from two successive iterations is the same, the type of function \( F_2(t) \) is found and the iteration can be stopped.

**Interpolation of the function \( F_2(t) \) in segment \( B \)**

The function \( F_2(t) \) is found in segment \( B \) based on the interpolation of scaled values \( R(t) \), calculated from the measured data and a previously adopted function \( F_2(t) \):

\[
R_0(t) = R(t) - F_2(t).
\]

**Case 1: The shape of the function is exponential:**

\[
F_1(t) = A_{01E} \cdot \exp\left(\frac{t - t_0}{T_0}\right).
\]
Coefficients $A_{01P}, B_{01P}, C_{01P}$ are calculated by the least squares method using values $R_i(k)$. This procedure is the same as given in (8)-(12), and results are presented by equations (11), (12).

- Case 2: The shape is logarithmic parabola:

$$\log R_i(k) = A_{01P} + B_{01P} \cdot (k \cdot \Delta t) + C_{01P} \cdot (k \cdot \Delta t)^2.$$  

By the least squares method, coefficients $A_{01P}, B_{01P}, C_{01P}$ are determined from the equations:

$$A_{01P} = \frac{S_2^2 + S_4 \cdot S_5 - S_3 \cdot S_7 - S_0 \cdot S_1 + S_1 \cdot S_2 + S_3 \cdot S_4 - S_5 \cdot S_1}{S_2^3 + S_4^2 \cdot S_5 - S_3 \cdot S_7 - (2 \cdot S_2 \cdot S_4 + N \cdot S_1)},$$

$$B_{01P} = \frac{S_2 \cdot S_4 + S_4 \cdot S_5 - S_3 \cdot S_7 - S_0 \cdot S_1 + S_1 \cdot S_2 + S_3 \cdot S_4 - S_5 \cdot S_1}{S_2^3 + S_4^2 \cdot S_5 - S_3 \cdot S_7 - (2 \cdot S_2 \cdot S_4 + N \cdot S_1)},$$

$$C_{01P} = \frac{S_2^2 \cdot S_5 + S_4 \cdot S_5 - S_3 \cdot S_7 - S_0 \cdot S_1 + S_1 \cdot S_2 + S_3 \cdot S_4 - S_5 \cdot S_1}{S_2^3 + S_4^2 \cdot S_5 - S_3 \cdot S_7 - (2 \cdot S_2 \cdot S_4 + N \cdot S_1)},$$

where are:

$$S_0 = \sum_{k=1}^{N} (k \cdot \Delta t); \quad S_2 = \sum_{k=1}^{N} (k \cdot \Delta t)^2; \quad S_3 = \sum_{k=1}^{N} (k \cdot \Delta t)^3$$

$$S_4 = \sum_{k=1}^{N} (k \cdot \Delta t)^4; \quad S_5 = \sum_{k=1}^{N} \ln R(k); \quad S_6 = \sum_{k=1}^{N} \ln R(k) \cdot (k \cdot \Delta t)$$

$$S_7 = \sum_{k=1}^{N} \ln R(k) \cdot (k \cdot \Delta t)^2.$$  

After the coefficients $A_{01P}, T_m$ from the case 1 and $A_{01P}, B_{01P}, C_{01P}$ for the logarithmic parabola in the case 2 are determined, the proper type of function $F(t)$ should be selected by the minimum value of sums of the least squares for the functions from these two cases.

Extrapolation of the function $F(t)$ in segment $\Delta t$

After the functions $F_1(t)$ and $F_2(t)$ are determined, the function $F(t)$ is calculated. The resistance $R_{wm}$ is found at the moment $t_i$ by extrapolation, as $R_{wm} = F(t_i)$, which is the mean winding resistance at the instant of transformer shutdown.

Determining of the hot spot temperature

From the mean resistance $R_{wm}$ at the instant of shutdown, the mean winding temperature $T_{wm}$ is calculated by using:

$$T_{wm} = \frac{(1 + \alpha T_a)R_{wm} - R_a}{\alpha \cdot R_a},$$

where $\alpha = 1/235^\circ C$ is the temperature coefficient, and $T_a$ and $R_a$ are the ambient temperature and resistance value measured at that temperature [13]. This temperature is further used to determine the transformer hot spot temperature pursuant to [2].

Distribution of the winding temperature and the oil temperature as the function of the transformer height is shown in figure 2 by dashed and solid lines, respectively.

If the temperature of the oil is known at the bottom $T_{ob}$ and on the top $T_{ot}$, the mean oil temperature is:

$$T_{om} = \frac{T_{ob} + T_{ot}}{2},$$

and then, the temperature gradient $g$ is:

$$g = \frac{T_{om} - T_{om}}{L}.$$  

The gradient $g$ is constant along the entire height of the transformer, when the effect of the oil viscosity can be neglected [14, 15] and without taking winding thermal anisotropy into account, described in [16, 17]. Finally, the transformer hot spot temperature is found from the expression:

$$T_{hs} = T_{om} + H \cdot g,$$

where $H$ is the hot spot factor which depends on the additional transformer losses and the type of the transformer cooling. This factor is given in [2], and also implemented in [18]. It is supposed that the time change between the hot spot and top oil temperature can be neglected [19].

![Fig. 2. Temperature change with the transformer height](image)

![Fig. 3. Detailed flowchart of the proposed algorithm](image)
Practical implementation

The proposed algorithm for the hot spot temperature determining is implemented in the software module for controlling the field test device. The instrument itself is capable of measuring the transformer resistance and oil temperature. The hot spot temperature is calculated according to the [2]. The detailed flowchart of the algorithm described in the previous section is shown in the figure 3.

The algorithm inputs are the time duration of the test $T$, sampling time $\Delta t$ for the resistance measurement, switch-off time $t_1$ from the instant of shutdown of the transformer to the starting of the test and the oil temperatures at the bottom and on the top of the transformer, which can also be measured automatically, using temperature probes. Winding resistance at the ambient temperature is an optional parameter.

The software gives the following output data: overview of measured resistances during the test; curves of winding resistance and temperature change from the instant of transformer shutdown until the end of the test, obtained by the proposed approximation; hot spot temperature for each winding under the test. Additional transformer data are used for scaling the obtained hot spot temperature to the rated winding according to the [2], and are also analyzed as a load factor in [15].

Experimental results

Verification of the proposed algorithm and approximation method was conducted by two experiments. The aim was to show the accuracy of the implemented algorithm and approximated method. Input parameters for the both tests are summarized in Table 1.

Table 1. Input parameters for the test 1 and test 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The duration of the test [min]</td>
<td>30 20</td>
</tr>
<tr>
<td>Time from the instant of shutdown [s]</td>
<td>30 30</td>
</tr>
<tr>
<td>Sampling interval [s]</td>
<td>5 5</td>
</tr>
<tr>
<td>Oil temperature at the top of the tank [°C]</td>
<td>35 32</td>
</tr>
<tr>
<td>Oil temperature at the bottom of the tank [°C]</td>
<td>32 31</td>
</tr>
<tr>
<td>Ambient temperature [°C]</td>
<td>19 22</td>
</tr>
</tbody>
</table>

Test 1

The first test is performed on the transformer with the following nameplate data:

- nominal power: $S_n = 630$ kVA;
- vector group: $Dyn5$;
- voltage ratio: 10 kV/0.4 kV;
- nominal current on HV side: 36.4 A.

The transformer heating is carried out with current of 15 A DC on the second transformer limb.

For the purpose of verifying accuracy of the extrapolated resistance value, it was possible to measure winding resistance at the instant of the end of transformer heating, in case of this distribution transformer. Registered mean winding resistance value, with transformer winding resistance meter is $R_0 = 1,510 \, \Omega$. Otherwise, the inability of measurement at the moment of shutdown is the main limitation in operational conditions.

According to [2], for power transformers up to 100 MVA the recommended time duration from the instant of shutdown (the end of heating) until the start of resistance measurement and first valid point is up to two minutes. Figure 4 represents measured and approximated value of winding resistance during cooling of the transformer from this test. The mean winding temperature change in monitored period is represented in the same figure. Because of comparative analysis of the proposed approximation, the fundamental parabolic and exponential approximations on the whole measurement period are also displayed.

Test 2

In the second test, the proposed algorithm is used for approximation of the temperature and resistance curve of the three-phase, two winding power transformer with nominal power of 6 kVA and vector group $Dyn5$. The test is performed on the second transformer limb. Measured and approximated mean winding resistance and mean winding temperature change of the middle transformer limb are given in figure 5.

Winding resistance was measured at 240 points. Lower sampling interval then proposed, using the least squares method, usually does not lead to better accuracy [2]. Approximated values during transformer cooling are also close to the measured value. As in the previous test, for comparative analysis, the parabolic and exponential approximations are shown. In addition to resistance change, figure 5 shows the transformer mean winding temperature change during cooling period of 20 minutes.
It can be seen that the mean value at the instant of shutdown is 69.6 °C. The calculated transformer hot spot temperature is 73.9 °C, and is within the temperature rise limit for this type of transformer insulation.

Algorithm also checks the measurement’s accuracy at the beginning of the resistance test. If the calculated temperature for the successive resistance measurements differs by more than 1 °C, these results are discarded. Effect of the first valid time point is presented in [20].

Conclusion

The proposed Hot Spot Temperature algorithm is used to interpolate windings resistance values and detect a hot spot temperature of a power transformer. The given procedure is based on estimation of the winding resistance change as function of the temperature. The expression for this function is obtained from measured resistance data from the moment of transformer shutdown during transformer cooling. From the approximated cooling function, the initial resistance value is found by extrapolation. Based on the estimated resistance, transformer hot spot temperature according to IEC standard is calculated. The algorithm takes into account two thermal time constants, thermal constant of insulating system and thermal constant of a transformer winding itself. In addition, the algorithm can be used for determining of the tested transformer hot spot temperature value, i.e. at the moment of the transformer shutdown. The Hot Spot Temperature algorithm can also be used for non-standard tests, for example if the heating current value is lowers than nominal current and for values between 70% and 100% of nominal value, as required by the standard.

The results of practical tests appeared as quite precise for both types of transformers. The accuracy of the estimation increases with the shortening of the time from the transformer shutdown to the start of resistance test.

Further development of the Hot Spot Temperature algorithm will be based on integration in a monitoring system of a transformer losses and calculation of a transformer transmission capability.

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