

## Correlation between weight, error and type of measurements in WLS State estimation of a Real Network

**Abstract** This paper presents a study of the effect of measurement's weight and error on the accuracy of WLS power system state estimation and the correlation between those two parameters and measurement's type. Different simulation cases are tested on a real Network of 14 bus, part of the electrical transmission network of the Moroccan National Office for Electricity and Potable Water (ONEE).

**Streszczenie.** W artykule zaprezentowano analize wpływu pomiaru wagi i błędu na dokładność metody ważonej najmniejszych kwadratów WLS w systemie energetycznym. Analizowano też korelację tymi dwiema wielkościami. **Korelacja między wagą, błędem i rodzajem pomiaru w określaniu WLS (weighted least square) w realnej sieci energetycznej**

**Keywords:** measurement's weight, WLS power system state estimation, measurement's type, real network .

**Słowa kluczowe:** WLS –metoda ważonych najmniejszych kwadraty, dokładność, sieć energetyczna

### Introduction

State estimation represents an essential tool for monitoring the power system. In energy control centers, power system state estimation is carried out in order to provide best estimates of what is happening in the system based on real-time measurement and a predetermined system model. It is required in the critical operational functions of a power grid such as real-time security monitoring, load forecasting, economic dispatch, and load frequency control. This need is particularly more in focus today due to deregulated and congested systems and smart grid initiatives[1]. Therefore, the accuracy and the reliability of the state estimator represent great concern for electrical engineering researchers . There are many papers studying the impact of different factors on the accuracy of state estimation but rare are the works approaching the effect of measurement's weight and its relation with measurement's error and type .

Most state estimation programs in practical use are formulated as over determined systems of non-linear equations and solved as weighted least-squares(WLS) problems [2].

It's observed that conventional weighted least squares (WLS), when applied to the power system state estimation considering the same weight, causes large state estimation errors [3]. That's why usually, The reciprocals of measurement error variances are chosen as weights of measurements . However measurement error varies due to variation of operating conditions of telecommunication systems and aging of the instruments, the weights need to be continuously updated. Furthermore, without considering the type of measurements, the results may be inaccurate enough because it's noted that for measurements of different type, same measurement error has different influence on the accuracy of state estimation [4].

This paper presents a study of the effect of measurement's weight and error on the accuracy of state estimation, and the correlation between those two parameters and measurement's type. Section 1, presents a description of the weighted least squares algorithm. Section 2, discuss the simulation results tested on real Network .

### Weighted Least Squares Algorithm

The starting equation for the WLS state estimation algorithm is:

$$(1) \quad z = h(x) + e$$

where: z is the (mx1) measurement vector; x is an (nx1) state vector to be estimated; h is a vector of nonlinear functions that relate the states to the measurements; and e is an (mx1) measurement error vector. Clearly, m must be grater then n in order to have measured the n states and have additional information to provide redundancy, m>n.

The solution to the state estimation problem can be formulated as a minimization of the following objective function:

$$(2) \quad J(x) = \sum_{i=1}^m \frac{(z_i - h_i(x))^2}{R_{ii}} = [z - h(x)]^T R^{-1} [z - h(x)]$$

Where R is the covariance matrix  $R_{ii} = \sigma_i^2$ .

To find the minimization of this objective function the derivative should be set to zero. The derivative of the objective function is denoted by g(x):

$$(3) \quad g(x) = \frac{\partial J(x)}{\partial x} = -H^T(x) R^{-1} [z - h(x)] = 0$$

Where:  $H(x) = \partial h(x) / \partial x$  called the measurement Jacobian matrix. Ignoring the higher order terms of the Taylor series expansion of the derivative of the objective functions yields an iterative solution as shown below:

$$(4) \quad x^{k+1} = x^k + [G(x^k)]^{-1} [[H(x^k)]^T R^{-1} [z - h(x^k)]]$$

Where the gain matrix, G, is defined as:

$$(5) \quad G(x^k) = \frac{\partial g(x)}{\partial x} = H^T R^{-1} H$$

The flowchart of WLS method is shown in figure1[7]:

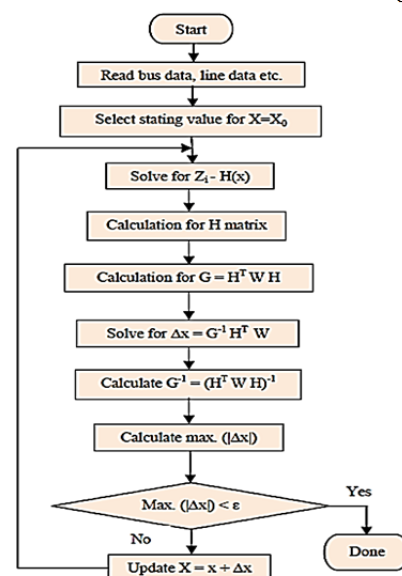


Fig.1. The flow chart of WLS Method

For the first iteration of the optimization the measurement function and measurement Jacobian should be evaluated at flat voltage profile, or flat start. A flat start refers to a state vector where all of the voltage magnitudes are 1.0 per unit and all of the voltage angles are 0 degrees. In conjunction with the measurements, the next iteration of the state vector can be calculated again and again until a desired tolerance is reached [5,6].

### Choice of weight

The gain matrix,  $H^T R^{-1} H$ , which consists of the Jacobian matrix  $H$  and the weighted matrix  $R^{-1}$ , has a great influence on the modification in the process of iteration.

The measurement errors are defined as :

$$(6) \quad e_i = z_i - h_i(x), \quad i=1,2,\dots,m$$

The measurement errors  $e_i$  are assumed to satisfy the following statistical properties:

First, the errors have zero mean:  $E(e_i) = 0, i = 1, \dots, m$

Second, the errors are assumed to be independent, such that the covariance matrix is diagonal.

$$(7) \quad \text{Cov}(e) = E(e, e^T) = R = \text{diag}\{\sigma_1^2, \sigma_2^2, \dots, \sigma_m^2\}$$

The probability density function of  $z$  is given as :

$$(8) \quad f(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-\mu)^2}{2\sigma^2}} \quad -\infty < z < \infty$$

The mean of the measurement is :

$$(9) \quad \mu = \int_{-\infty}^{\infty} z f(z) dz$$

The measurement variance is as follow :

$$(10) \quad \sigma^2 = \int_{-\infty}^{\infty} (z - \mu)^2 f(z) dz$$

WLS is derived from maximum likelihood estimation. When the weight takes inverse variance of the measurement error, we can get ideal results.

In fact, the uncertainty exists in the process of measurement, and it is difficult to achieve the ideal state, so the weight setting should be adjusted accordingly[3].

### Simulation results

This paper presents a study of the effect of measurement's weight and error on the accuracy of state estimation, and the correlation between those two parameters and measurement's type. Simulations are tested on a real Network of 14 bus, part of the electrical transmission network of the Moroccan National Office for Electricity and Potable Water (ONEE).

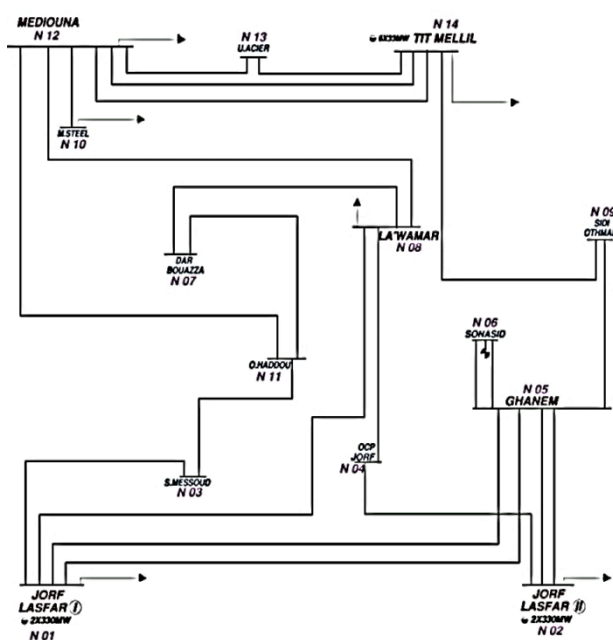


Fig.2. 14 bus system part of the Moroccan transmission Network

### Case Study

The Moroccan transmission network is interconnected with the Spanish and Algerian electricity grids, with a total length of 24,508 km in 2015 [8], covering 648 bus.

For our study, we have chosen a part of the global network composed of 14 bus.

The network data are shown in Table 1, below :

Table 1. Network data of the case study

From Bus	To Bus	R (P.U)	X (P.U)	B (P.U)
1	5	0.00254	0.01634	0.02643
1	5	0.00254	0.01634	0.02643
1	8	0.01212	0.08435	0.16273
1	3	0.02109	0.13573	0.21951
2	5	0.00245	0.01576	0.02549
2	5	0.0025	0.01607	0.02599
2	4	0.002	0.01286	0.02079
5	9	0.01868	0.12022	0.1944
5	6	0.00174	0.0112	0.01811
5	6	0.00163	0.0105	0.01704
8	12	0.01154	0.08053	0.15646
8	7	0.00372	0.02396	0.03874
8	4	0.0131	0.0695	0.11772
11	12	0.00335	0.02157	0.03488
11	7	0.00259	0.01666	0.02693
11	3	0.0007	0.00451	0.0073
9	14	0.00086	0.00554	0.00896
14	12	0.0019	0.01224	0.01979
14	12	0.0019	0.01224	0.01979
14	13	0.0013	0.00839	0.01357
12	13	0.0014	0.00901	0.01456
10	12	0.00297	0.01908	0.03086

From the supervisory software, we collected 79 measurements of the studied network : 14 voltage measurements, 36 real power flow measurements and 29 reactive power flow measurements.

This state is taken on 09/01/2017 at 10h31min56s.

The true values of voltage magnitude are obtained from a load flow calculation.

To compare the state estimate accuracy of the following simulations, mean absolute percentage error (MAPE) is introduced as follows [9]:

$$(11) \quad MAPEV = \frac{1}{n} \sum_{t=1}^n \left| \frac{V_t - V_e}{V_t} \right| \times 100\%$$

Where,  $V_t$  is the true value of voltage magnitude and  $V_e$  is the estimated value. A smaller value of MAPE indicates a more accurate state estimation result.

### Effect of measurements weights

The WLS algorithm was tested on 4 different cases of combinations of measurements weights, presented in table1.

Table 2. Different Cases of combinations of measurements weights

Simulation Case	Measurements Error Variance [V, Pflow/Qflow]			
	Case1 [0.1, e-6]	Case 2 [e-6, 0.1]	Case 3 [e-6,e-6]	Case4 [0.1, 0.1]
MAPEV (%)	15	0,2	1,4	1,4
Number of Iterations	6	3	4	4

As observed , for cases 3 and 4 , the result doesn't change . We conclude that Whatever the value of the weight as it is the same for all measurements, the result doesn't change.

Comparison between Case 1 and Case 2 : when voltage measurements has a lower weight ( higher error variance ) compared to power flow measurements : A large deviation from the true value is obtained (15%) , and the execution of the program requires more iterations number . On the other hand , when the highest weight is affected to voltage measurements , we note a smaller deviation and the program converge in fewer iterations. Therefore , the influence of voltage measurements weight is more important compared to power flow measurements weight.

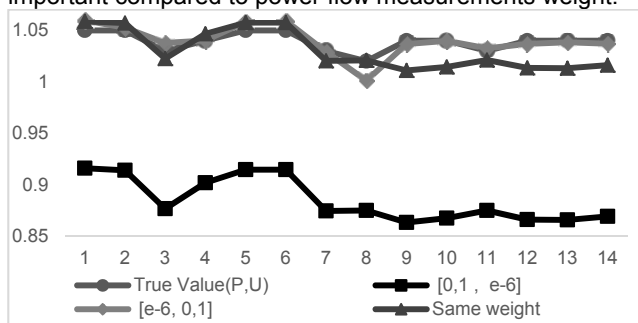


Fig.3. Estimated value of voltage magnitude for different combinations of measurements weights

#### Effect of level of noise

In practice , the measurements are not always accurate. And the detection of bad data is not evident because noise can be added to the known instrument metering error , so measurements become corrupted .

The aim of this point is to study the response of WLS program to different level of noise affecting measurements and observe the relation between measurements error and their type.

Two simulation cases are tested: one assuming different level of noise unified for all measurements and the second assuming different level of noise changing according to the type of measurements :

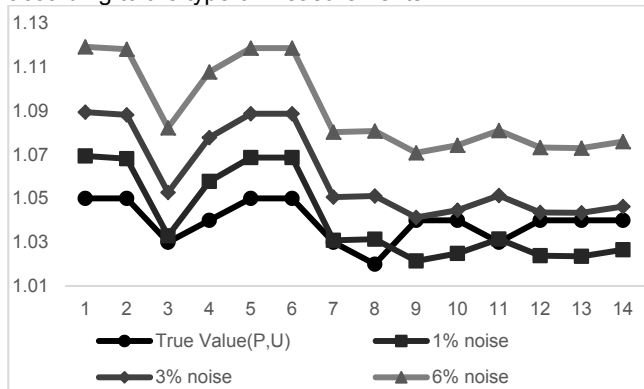


Fig.4. Estimated value of voltage magnitude with different level of noise unified for all measurements

As seen in figure4 , the accuracy of state estimation is inversely proportional to the level of noise. This is evident , indeed more measurement's error is big , more the deviation from the true value is important.

With the same level of noise (error) , the influence of voltage and power flow measurements is different . As the results of weight's effect study , we note that voltage measurement error prevails and provides a big deviation from the true values .

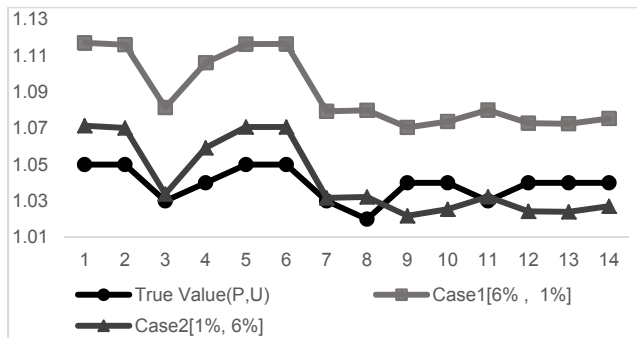


Fig.5. Estimated value of voltage magnitude with different level of noise changing according to measurements type [V,Pflow]

#### Correlation between weight , error and type of measurements

In this point , we will put in evidence the relation between weight , error and type of measurements observed in the previous simulation results .Therefore , other simulation cases were tested presented in table2, taking into account those three factors.

Table 2. Simulation cases taking into account the correlation between weight, error and type of measurements

	Case1	Case2	Case3	Case4
Measurement error variance [V,Pflow]	[1%, 6%]	[1%, 6%]	[6%,1%]	[6%,1%]
Level of noise [V,Pflow]	[1e-6, 0.1]	[0.1, 1e-6]	[e-6,0.1]	[0.1, e-6]
MAPEV (%)	0.2	12.4	0.2	18.5
Iterations Number	3	5	3	6

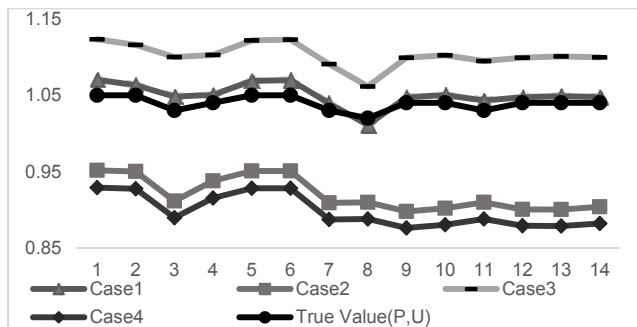


Fig.6. Estimated value of voltage magnitude for 4 simulation cases taking into account the correlation between weight, error and type of measurements

As observed , voltage measurement's weight has the biggest influence. Therefore , it is interesting to use accurate instruments for this type : voltage measurements' should be the more accurate as possible and have the highest weight.

For other measurements types , weight and error should be combined : It's interesting to affect high weights to accurate measurements while low weights should be affected to bad measurements.

#### Conclusion

This paper presents a study of the effect of measurement's weight and error on the accuracy of power system state estimation , and the correlation between those two parameters and measurement's type.

Different simulation cases were tested , confirming the importance of voltage measurement compared to other types. Indeed , voltage measurement's should be accurate as possible and have a big weight. In practice , this can be obtained , by ensuring the acquisition of accurate device meters for voltage measurements.

For other measurements types , weight and error should be combined : It's interesting to affect high weights to accurate measurements while low weights should be affected to bad measurements.

This study completes and confirms the results obtained in a previous paper [10] , it takes into consideration the random nature of measurements because the simulations are applied to a real network with a real state from the supervisory software of the Moroccan national dispatching centre in Casablanca.

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