Vasyl MALYAR [1], Andriy MALYAR [1]

Lviv Polytechnic National University (1)

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Mathematical model of single-phase induction motors with auxiliary winding resistance

Abstract. A method and algorithm for computing static curves of single-phase induction motors with an auxiliary winding resistance are proposed. The method relies on a mathematical model of the induction motor that takes into account the magnetic path saturation and current displacement in the rotor bars. The problem is solved as a boundary-value one for the system of differential equations describing the processes in the motor in the differential coordinates axes.

Streszczenie. *Opisano metodę I algorytm oblicznia zależności statycznej jednofazowego silnika indukcyjnego z pomocniczym uzwojeniem. W modelu matematycznym uwzględniono nasycanie sie obowdu magnetycznego I rozkład prądu w prętach wirnika.* ***Model matematyczny jednofazowego silnika indukcyjnego z pomocniczym uzwojeniem***
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Keywords: Induction motor, single-phase power supply, start winding, mathematical model, boundary-value problem, static curve.
Słowa kluczowe: silnik indukcyjny, uzwojenia startowe, model matematyczny

Introduction

Due to being reliable and inexpensive, three-phase induction motors (IM) are most widely used in various electric drives. Their correct performance is ensured by a three-phase power supply, but for various reasons, many low-power consumers do not possess it. Therefore, it becomes necessary to supply power to IM from a single-phase network. IM designed to operate in such a mode are called single-phase IM. They are mostly used in uncontrolled low-power electric drives (home appliances, tools, etc.) and generally come with a squirrel-cage rotor [1–3].

The single-phase winding on the stator creates a magnetic field pulsating along its axis, which can be decomposed into two identical counter-rotating circular fields. Therefore, such a motor does not produce a starting electromagnetic torque, and, therefore, it cannot be started without taking additional measures. To be started, a single-phase IM must have a major (operating) winding and an auxiliary (starting) winding that is switched on during the start. It helps create an elliptical magnetic field, due to which a starting torque is generated. For the operation from a single-phase power supply network, three-phase induction motors can also be used [4]. In this case, two series phase windings perform the function of the operating winding, and the winding of the third phase is used as the start winding. There are also universal IM optimized to operate both from a three-phase network and a single-phase one. Such motors ensure the required performance curves for both modes and are applicable in different electric drives.

In single-phase IM, the start winding is displaced with reference to the main winding by the angle $\pi/2$, and the current phase displacement in it is implemented by connecting the phase-displacing elements, whose function in practice is performed by capacitors [1–4] or resistors [5]. Capacitors enable the achievement of a significant driving torque, but, despite the simplicity of their use, there are serious drawbacks. For instance, switching on the capacitors can cause resonance phenomena [5], which are hazardous both for the capacitors and for the start winding. The capacitors' resistance depends on the frequency, and their dimensions are quite large. Therefore, they are used for drives that need a significant driving torque, as using capacitors makes it possible to obtain a circular magnetic field [7]. Besides, there are drives in which the starting torque is small, for example, in fan units, compressors, etc. In such electric drives motor with the starting resistance in the start winding are used. [5]. This can be implemented not only by connecting an additional resistance, but also by implementing the start winding with a small wire cross sectional area, i.e., the start winding itself should have an increased active resistance.

Research problem

Methods for studying IM powered from the single-phase network have been developed and improved for many years in accordance with the level of the computing facilities. However, the problem of mathematical modelling of electromagnetic and electromechanical processes has not been solved in step with modern requirements.

An insight into the phenomena of the operation processes in the single-phase induction motor can be gained using the theory of two counter-rotating magnetic fields [3], but the computing methods based on this theory, in particular the method of symmetrical components [1, 3], have been theoretically substantiated for linear systems only. The common standard software has been developed on these same theoretical bases [8, 9]. Therefore, despite the methodological value of the method of symmetrical components, the results of computing modes and curves using this method should be regarded as approximate only.

In spite of the magnetic paths of single-phase IM being symmetrical, they belong to the category of the so-called non-symmetrical electrical machines [3], irrespective of their design: either as three-phase motors or as special ones with an auxiliary winding. Therefore, their analysis requires a specific approach, as designing of the start winding, determination of the starting resistance and computing of their curves cannot be performed with a high accuracy using the conventional [3, 10] classical equivalent circuits. This is due to the fact that not only transients, but also steady-state electromagnetic and electromechanical processes occurring in them are dynamic and are described by a system of differential equations (DE) [10–12]. Thus, static curves cannot be computed with a high accuracy using the approximate approaches described in the classical literature. Specifically, using equivalent circuits can unsure obtaining accurate results only for symmetrical three-phase machines, and in case of power supply to IM by single-phase voltage the obtained results cannot always be verified in practice.

The research aims at developing the mathematical model and algorithm for computing static starting curves for single-phase IM with an increased start winding resistance.

Mathematical model of the motor

The idea of the proposed computing method and algorithm can be presented on the example of a three-phase IM powered from the single-phase power supply network, with the star connection of the stator winding and an additional resistance in one of the phases (Fig. 1). Evidently, the active resistance in the start winding can be computed only on an IM mathematical model that takes into consideration all major factors having an effect on the processes in the motor. Here belong the saturation of the IM magnetic system and the skin effect that occurs during the starting in the bars of the squirrel cage rotor. Besides, as in the single-phase IM the stator circuit is always non-symmetrical, the processes can be considered only in the real phase coordinates of the stator [12].

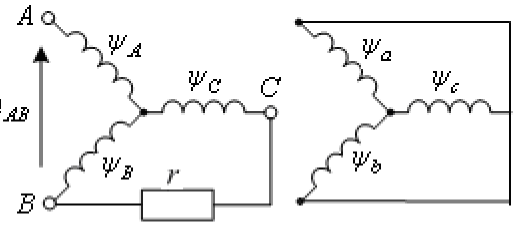


Fig.1. Computational electrical circuit of IM with an additional resistance connected in one of the phases

Studying the processes in IM requires a fairly accurate evaluation of the motor’s parameters, including the active resistances of the circuits, which are variable due to the displacement of the current in the squirrel-cage rotor bars, and the self-inductances and mutual inductances of the electrical circuits, which depend on the magnetic path saturation.

In order to account for the current saturation, each rotor bar along with the squirrel-cage rings is divided into *k* elementary sections heightwise, as a result of which the rotor winding is represented by *k* windings, each of which is reduced to a three-phase winding [15].

To take into account the saturation of the IM magnetic path, for determining a full matrix of differential inductances of the motor’s circuits as the derivative $L = d\vec{\psi} / d\vec{i}$ of the flux linkage vector with respect to the current vector, the major characteristic of magnetization and dependencies of flux linkage losses in the circuits of the stator and rotor windings on the respective currents are used [15, 20]:

$$\psi_\mu = \psi_\mu(i_\mu)\,;\;\; \psi_{\sigma s} = \psi_{\sigma s}(i_1)\,,\;\; \psi_{\sigma r} = \psi_{\sigma r}(i_2)\,,$$

where i_μ, i_1, i_2 are the moduli of the imaging vectors of the magnetization current (μ), of the stator (*s*) and the rotor (*r*) currents, which are computed based on the geometric and winding data.

In the fixed three-phase coordinate system, the processes in IM powered from a single-phase network, in case of connecting an active resistance *r* into the start winding of the stator, are described by a non-linear system of DE of electric equilibrium

(1)
$$\frac{d\psi_A}{dt} - \frac{d\psi_B}{dt} = -r_A i_A + r_B i_B + u_{AB}\,;$$

$$\frac{d\psi_B}{dt} - \frac{d\psi_C}{dt} = -r_B i_C + (r_C + r) i_C\,;$$

$$i_A + i_B + i_C = 0\,; \qquad (1)$$

$$\frac{d\psi_a}{dt} - \frac{d\psi_b}{dt} = -r_a i_a + r_b i_b - \alpha\left(\psi_b - 2\psi_c + \psi_a\right);$$

$$\frac{d\psi_b}{dt} - \frac{d\psi_c}{dt} = -r_b i_b + r_c i_c - \alpha\left(\psi_c - 2\psi_a + \psi_b\right);$$

$$i_a + i_b + i_c = 0\,.$$

where $u_{AB} = U_m \sin(\omega_0 t - \psi_0)$ is the applied voltage $\alpha = \omega_0(1 - s)$, where ω_0 is the power supply voltage frequency, *s* is the sliding; ψ_ξ, i_ξ, r_ξ (ξ = *A, B, C, a, b, c*) are the flux linkages, currents and active resistances of the circuits, respectively.

For the skin effect to be taken into account, the number of equations should be equal to the number of segments which each rotor bar was divided into. These equations have identical forms and differ only in the parameters, which is why further on, to make the discussion of the material shorter, we will deal with one three-phase winding of the rotor.

Computation method and algorithm

Let us consider the problem of computing the starting resistance that corresponds with the maximum value of the electromagnetic torque. To do this, it is necessary to calculate the static curve as the function of the coordinates vs. the stating winding resistance.

As at the instance of starting the sliding *s* = 1, then the factor α = 0, which significantly simplifies the system of equations (1). It can be written in the form of the vector equation

(2)
$$D\frac{\partial \vec{y}(\vec{x},t)}{\partial \vec{x}} = \vec{f}(\vec{y},\vec{x},t) + \vec{u}_{AB}\,,$$

where

$$\vec{y} = \begin{array}{|c|}\hline \psi_A \\ \hline \psi_B \\ \hline \psi_C \\ \hline \psi_a \\ \hline \psi_b \\ \hline \psi_c \\ \hline \end{array}\,;\quad \vec{i} = \begin{array}{|c|}\hline i_A \\ \hline i_B \\ \hline i_C \\ \hline i_a \\ \hline i_b \\ \hline i_c \\ \hline \end{array}\,;\quad \vec{f} = \begin{array}{|c|}\hline -r_A i_A + r_B i_B \\ \hline -r_B i_B + (r_C + r) i_C \\ \hline i_A + i_B + i_C \\ \hline -r_a i_a + r_b i_b \\ \hline -r_b i_b + r_c i_c \\ \hline i_a + i_b + i_c \\ \hline \end{array}$$

$$D = \begin{array}{|c|c|c|c|c|c|}\hline 1 & -1 & & & & \\ \hline & 1 & -1 & & & \\ \hline & & & & & \\ \hline & & & 1 & -1 & \\ \hline & & & & 1 & -1 \\ \hline & & & & & \\ \hline \end{array}\,;\quad \vec{u} = \begin{array}{|c|}\hline 0 \\ \hline u_{AB} \\ \hline 0 \\ \hline 0 \\ \hline 0 \\ \hline 0 \\ \hline \end{array}\,.$$

The effect of the active resistance in the start winding of the stator can be studied on the basis of computing the static curves as a sequence of steady-state modes that correspond with the constant values of the sliding *s*. However, as the system of equations (2) comprises the time coordinate *t,* a static curve can be computed only by multiple calculation of the steady-state process till its reaching the steady state. Obviously, such an approach is inefficient and unacceptable for many reasons. If the sliding is constant, the system of equations (2) describes a steady-state asynchronous operation of IM, and since the voltage u_{bc} changes under the periodic law, in the steady-state mode of operation the solution of the non-linear system of

DE (2) is periodic dependencies of the coordinates in the period T

$$\vec{x}(t) = \vec{x}(t+T),$$

which can be estimated with little computing efforts, using the method of the boundary-value problem [15].

Out of the wide range of the methods for solving boundary-value problems, the most effective are projection methods theoretically based upon the approximation of the state coordinates on the grid of n nodes of the period by basis functions. As a result, an approximate solution of the differential equations describing the periodic mode is obtained as a projection of an infinite function space onto a finite subspace defined by a linear combination of the basis functions. Their application makes it possible to switch from DE to algebraic equations. In our case, spline functions were used [17–18], as they have advantages before other approximations when building a computation algorithm.

To do this, each coordinate of the vector $\vec{y}$ is approximated on a grid of nodes of the period T by a third degree spline. As a result, we obtain a continuous function, which in each j^{th} time segment $h = t_j - t_{j-1}$ is described by the equation

(3) $$y(t) = a_j + b_j(t_j - t) + c_j(t_j - t)^2 + d_j(t_j - t)^3,$$

where a_j, b_j, c_j, d_j are the spline factors; $j = (\overline{1, n})$ is the segment number.

The relationships between the spline factors are determined by the properties of the spline function [17]. For instance, as seen from (3), for $t = t_j$

(4) $$y(t_j) = y_j = a_j,\; dy/dt\big|_{t=t_j} = \vec{f}_j = -\vec{b}_j,$$

and the equation (2) written for the j^{th} node (2) has the form

(5) $$D\frac{d\vec{y}}{d\vec{x}}\bigg|_j = \vec{f}_j + \vec{u}_j,$$

Given the continuity of the spline (3), its first and second derivatives and periodic boundary value conditions, taking into account (4) and (5), we obtain a non-linear system of nm^{th} order algebraic equations [16, 17]

(6) $$H\vec{Y} - \vec{F} = \vec{U},$$

where $\vec{Y} = \left(\overline{\vec{y}_1, \vec{y}_n}\right)^*$, $\vec{F} = \left(\overline{\vec{f}_1, \vec{f}_n}\right)^*$, $\vec{U} = \left(\overline{\vec{u}_1, \vec{u}_n}\right)^*$ are column vectors, each of which consists of n vectors with m nodal values of the respective variables, where m is the order of the system (2), and the matrix H is block diagonal. Its coefficients are defined by the step h of the grid of nodes, which can be regular.

The system of algebraic equations (6) is a discrete presentation of DE (2), and, as the initial system of DE is non-linear, the obtained system is non-linear, too. The unknown variable in it is the vector $\vec{X} = \left(\overline{\vec{x}_1, \vec{x}_n}\right)^*$, which consists of the vectors $\vec{x}_j = \vec{i}_j$ of the nodal values of the currents in the IM circuits. For its calculation, Newton's iteration method combined with the method of continuation with respect to the parameter is used [19]. The parameter ε is introduced into the non-linear system (6) by multiplying the vector of the applied voltages $\vec{U}$ by the scalar parameter ε, the differentiation with respect to which results in the system of DE

(7) $$W\frac{d\vec{X}}{d\varepsilon} = \vec{U},$$

where W is the Jacobian matrix of the system (6).

In order to study the effect of the value of the active resistance in the auxiliary winding on the starting (for s = 1.0) electromagnetic torque, it is necessary to compute the multidimensional static curve

$$\vec{X} = \vec{X}(r).$$

The calculation is carried out using the differential method, by which the equation (6) is differentiated with respect to r, considering the vector $\vec{U}$ of the nodal values of the applied voltage to be constant. As a result, we obtain DE

(8) $$W\frac{d\vec{X}}{dr} = \frac{\partial\vec{F}}{\partial r},$$

which is different form DE (7) only by the vector of the right-hand parts $\partial\vec{F}/\partial r$, which consists of n vectors

$$\partial\vec{f}/\partial r = \left(0, ..., 0, -i_{Cj}\right)^*, \; (j = 1,\dots, n).$$

By integrating DE (8) with respect to r from $r=0$ to the pre-set value we obtain the dependencies of the coordinates $\vec{X} = \vec{X}(r)$ and, consequently, of the electromagnetic torque $M_e = M_e(r)$ on the additional resistance. Then we select the value of r, which ensures the maximum starting electromagnetic torque, which is calculated according to the formula [15]

$$M_e = p_0(\psi_{\mu A}(i_B - i_C) + \psi_{\mu B}(i_C - i_A) + \psi_{\mu C}(i_A - i_B))/\sqrt{3},$$

where $\psi_{\mu A}, \psi_{\mu B}, \psi_{\mu C}, i_A, i_B, i_C$ are the projections of the flux linkage vectors $\vec{\psi}_\mu$ and current $\vec{i}$ on the phase axes.

Therefore, computation of the static curve is performed in two steps. In the first step, we determine the value of the coordinates for s = 1.0 and r = 0 using the method of continuation with respect to the parameter by increasing the vector of the applied voltage from zero to the set value, while in the second step we calculate the characteristic $M_e = M_e(r)$ by integrating DE (8). The values of the coordinates found in the first step are used as the starting conditions for the second step.

Using the above-presented algorithm, one can study the effect of the value of the active resistance in the start winding of the single-phase IM on its characteristics and select the value needed for the starting. The developed algorithm can be used not only for the sliding s =1.0, but also for any other values of the siding, specifically for computing static starting curves. For this, in the equation (5) the function f is assumed to be

$$\vec{f}_j = \begin{bmatrix} -r_A i_{Aj} + r_B i_{Bj} \\ -r_B i_{Bj} + (r_C + r) i_{Cj} \\ 0 \\ -\alpha\left(\psi_{bj} - 2\psi_{cj} + \psi_{aj}\right) - r_a i_{aj} + r_b i_{bj} \\ -\alpha\left(\psi_{cj} - 2\psi_{aj} + \psi_{bj}\right) - r_b i_{bj} + r_c i_{cj} \\ 0 \end{bmatrix}$$

Taking into consideration the saturation by means of dividing the bars into elementary segments heightwise raises the order of the initial system of equations (1), as the rotor equations must be written for each element of the bar. As a result, the electromagnetic processes in IM will be described by the system having the form (1), which consists of 3 equations for the stator and $(3 \times k)$ equations for the rotor, where k is the number of elements that each bar is divided into.

Digital Simulation and Results

According to the presented algorithm, a PC program was developed, using which one can carry out computations of the steady-state modes of the single-phase IM for different values of the starting active resistance and of the respective static curves. The examples of computation results for the three-phase induction motor 4A80B2Y3 (P=2.2 kW, U=220 V, I=4.7 A) with an additional resistance in the phase C powered from a single-phase network are provided below.

Fig. 2 shows the dependence of the mean value of the IM electromagnetic torque on the relative value of the active resistance. It is seen that this dependence has a distinct maximum, which allows selecting a necessary value of the starting resistance.

Dependencies of the relative values of the starting currents (for s = 1.0) on the active resistance are presented in Fig. 3. These curves can be used to set the corresponding equipment into switching off the starting resistance.

Fig. 4 presents periodic dependencies of the current of the start winding (*a*) and electromagnetic torque (*b*) for two values of the starting resistance for the sliding s = 1.0.

Fig. 5 presents the results of computing the mechanical curve (*a*) and dependencies of the root-mean-square values of the phase currents on the sliding (*b*) for r^* = 1.5.

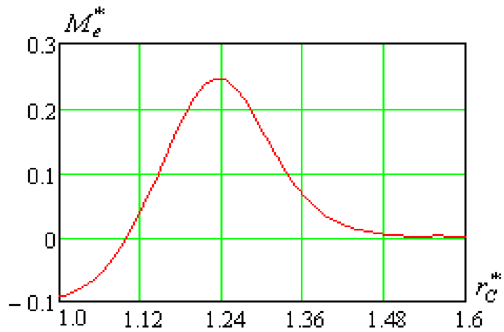


Fig. 2. Relative value of the starting electromagnetic torque of the motor vs. the active resistance of the start winding

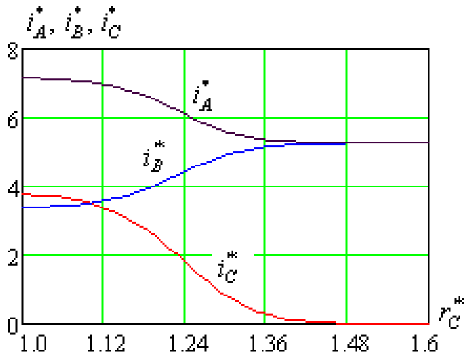
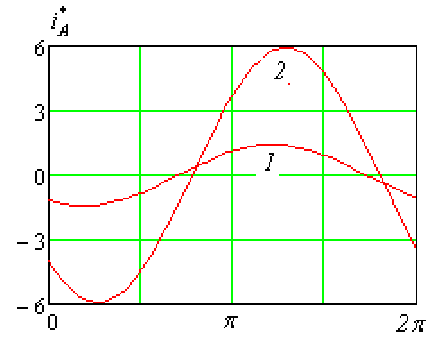
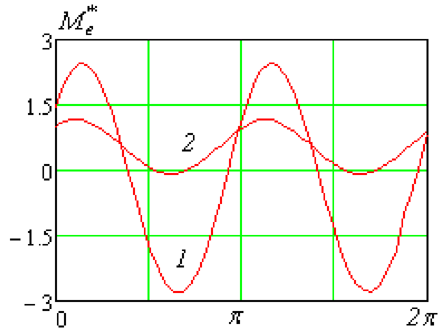


Fig. 3. Relative value of the root-mean-square phase current of the motor vs. the active resistance of the start winding for s = 1.0

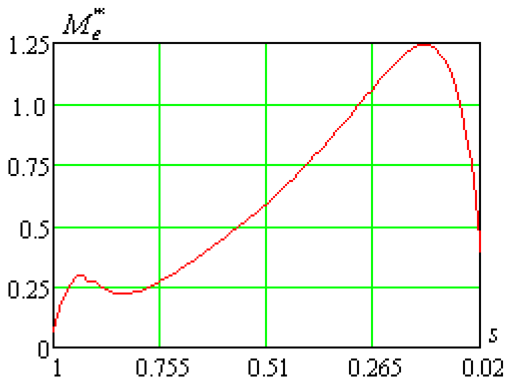


a)

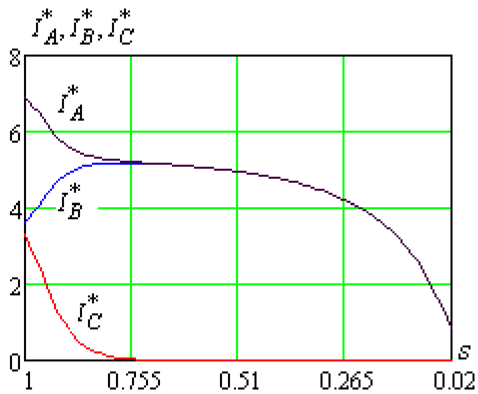


b)

Fig. 4. Periodic time dependencies of the consumed current (*a*) and electromagnetic torque (*b*) for s = 0.025 corresponding to different relative values of the start winding resistance: 1 – r_c^*= 0.9 i 2 – r_c^* = 1.8



a)



b)

Fig. 5. Electromagnetic torque (*a*) and phase current (*b*) vs. the sliding for the start winding resistance increase ratio r^* = 1.5.

Conclusions

The developed method of mathematical modelling of the starting static curves of the single-phased IM with a start winding relies on the projection method of solving the boundary-value problem, which enables periodical dependencies of the coordinates to be computed beyond time, without having to compute the transients. This approach allows computing steady-state modes in a time-efficient way and multidimensional static curves as dependencies of the IM currents and flux linkages on any coordinate assumed to be independent.

The proposed computation method was used as the basis for the development of the algorithm for determining the value of the additional active resistance which needs to be connected in series to ensure the maximum value of the driving electromagnetic torque.

It was shown that the dependence of the electromagnetic torque on the value of the total resistance of the start winding for the constant sliding has an extreme to which corresponds the optimal value of the resistance ensuring the maximum torque of the motor.

The method presented in the paper makes it possible to compute starting static curves both in the function of the active resistance for the constant sliding and for the cases of the constant resistance in the function of the sliding, which allows the motor starting control system to be set.

The efficacy of the developed algorithm is provided by using the high-adequacy mathematical model of IM, which gives consideration to the saturation of the magnetic system and displacement of the current in the bars of the squirrel cage rotor, as well as by applying the efficient mathematical method for computing periodic dependencies of the coordinates.

***Authors**: Prof, Dr. of Sc., Vasyl Malyar, Lviv Polytechnic National University, Institute of Power Engineering and Control Systems, 12 Bandera Str., Lviv, 79013, Ukraine, E-mail: mvs@polynet.lviv.ua.*

Prof, Dr. of Sc., Andriy Malyar, Lviv Polytechnic National University, Institute of Power Engineering and Control Systems, 12 Bandera Str., Lviv, 79013, Ukraine, E-mail: svandrew@i.ua.

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