Measurements and calculation of self inductance of testing coils used in physical transformer model construction and its frequency analysis

Streszczenie. W artykule dokonano przeglądu i porównania różnych metod i sposobów wyznaczania, najdokładniej jak to możliwe, indukcyjności własnej wielowarstwowej cewki spiralnej. Cewka poddana analizie stanowiła element wyposażenia pomiarowego w modelu fizycznym transformatora badanego pod względem charakterystyk częstotliwościowych. Uzyskane indukcyjności zostały porównane z wartością uzyskaną metodą pomiarową.

Abstract. The paper reviews and compares various methods of accurately determining the value of self-inductance of multilayer spiral coil, being the part of measuring equipment in physical model of the transformer tested for frequency characteristics. These values were compared to the inductance measured in the laboratory and the accuracy of calculation methods has been evaluated. (Pomiar i obliczanie indukcyjności własnej cewki testowej użytej do budowy fizycznego modelu transformatora oraz jego analizy częstotliwościowej)

Słowa kluczowe: indukcyjność własna, cewka spiralna, model transformatora

Keywords: self-inductance; spiral coil; transformer model

Introduction
The aim of the paper is to present calculations and measurements of self-inductance of the flat spiral coils with thin internal insulation used to build a physical transformer model and its frequency analysis. The exact knowledge of self-inductance was necessary for further use of the real model and its equivalent circuit diagram as well. The authors decided to use numerous formulas available in literature. The measure of this accuracy is the comparison of the results with the value obtained in the direct measurement.

Test coil
To build the transformer physical model [6], 5 coils were made of 20 mm wide copper tape with a thickness of 0.5 mm. Each coil (Fig. 1) had 30 turns separated by a thin insulation tape, what gave a square with 0.02 x 0.02 m side in the cross section. Other parameters were as follows: internal diameter $D_o = 0.295$ m; external $D_i = 0.335$ m and the mean diameter $D = 0.315$ m.

Fig. 1. The test coil on the measuring stand, sketch of its spiral and the cross-section with geometric size markings

The coils were wound manually on a specially built machine. It helped to keep the circular shape of the coil, its dimensions (internal diameter) and ensured proper tension of the copper tape during winding.

Review of methods and formulas for determining the inductance of current contours
Inductance is a property of an electrical conductor which opposes a change in current. It does that by storing and releasing energy from a magnetic field surrounding the conductor when current flows, according to Faraday’s law of induction. It is connected with the magnetic flux $\Phi$ and the current $I$ by the formula: $\Phi = LI$, which is most often used to calculate inductance as $L = \Phi/I$. Therefore, the calculation of inductivity becomes mainly the task of determining the magnetic flux (Fig. 1) with the induction vector $B$, excited through the conductor 1, leading current $I$, therefore the fundamental formula might be used; [4]:

$$\Phi = \int \frac{B ds}{S}$$

where: $S$ is any surface spread over another, but closed contour 2 (not necessarily leading the current), with a positive orientation of its circulation, $ds$ - elementary surface vector normal to the surface $S$.

Fig. 2. Magnetic flux $\Phi$ associated with a current-free contour 2 and conductor 1 with current $I$.

The gain $dB$ of induction at any point $P$ of the space, coming from the section $dl$, under the law of Biot-Savart [4], is given by the formula:

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl \times r}{r^3}$$

and integration over the entire length $l$ of conductor 1 with current $I$ gives induction $B$ at the point $P$. Usually, however, we do not deal with a single current filament, but with their bunch filling the cross-section $\Sigma$ of the conductor, and the current filament itself is considered to be closed, single turn of the coil (Fig. 3).
The mutual inductance then equals to:

\[ M_{21} = \frac{\mu_0}{4\pi} \int \frac{dl_1 \cdot dl_2}{r} \cos \theta \]

The interaction between current loops is symmetrical, so \( M_{12} = M_{21} \), that is why the double indexing is usually omitted, leaving one symbol \( M \). There is a special case to be considered when only one conductor of non-negligible cross-sectional area carries the current \( I \), for which the concept of self-inductance \( L \) (Fig. 5) needs to be explained.

In a general case, two separate loops 1 and 2 of the lengths \( l_1 \) and \( l_2 \) are being considered, respectively.

If in the first loop current \( I_1 \) flows, then we can specify the flux called \( \Phi_{21} \) associated with the second contour. The quotient of this flux by the excitation current \( I_1 \) determines [4] the mutual inductance \( M_{21} = \Phi_{21}/I_1 \). So we have:

\[ \Phi_{21} = \int \text{rot} A \, ds_2 = \int_{l_2} A \, dl_2 \]

where: \( dl_2 \) - unitary vector along the contour 2; \( ds_2 \) - unitary vector, normal to the surface \( S_2 \) limited by contour \( l_2 \), positively oriented relative to the direction of its circulation, i.e. in the direction of current flow \( I_2 \).

In turn, the vector potential \( A \) coming from the current \( I_1 \), at any point in the space [4] is defined by the formula:

\[ A = \frac{\mu_0 I_1}{4\pi r} \]

which can replace (4) by another one:

\[ \Phi_{21} = \frac{\mu_0}{4\pi} \int \frac{dl_1 \cdot dl_2}{r} = \frac{\mu_0 I_1}{4\pi} \int_{l_1} dl_1 dl_2/2 \cos \theta \]

where: \( I_1 \) - current in loop 1; \( dl_1 \), \( dl_2 \) - unitary lengths vectors of two current loops; \( \theta \) - angle between the directions of contour lengths.

The formula (6) will be referred to the \( k \)-th current filament of length \( l_k \) and will take form that respect the total share of all other current filaments \( l_{\neq k} \) of the same conductor. To avoid singularities, the influence of an elementary segment \( dl_k \) with current \( \Delta I_k \) on itself is omitted in integration, because then \( r = 0 \). For this reason, there is no formal integration, but sufficiently numerous summation of shares with exclusion of this one component.

\[ \Phi_{21} = \frac{\mu_0}{4\pi} \sum \left( \int_{l_k} dl_k dl_{\neq k} \cos \theta \right) \Delta I_k = \sum M_{km} \Delta I_k \]

where: \( \Delta I_k = I/l_k \), \( \Delta I_{\neq k} = I/l_{\neq k} \) – the current \( I \) of conductor referenced to the contour lengths (linear density).

In order to determine the linkage flux \( \Psi \) with the entire current (and hence the self-inductance \( L \)), one should apply formula (3) and integrate over the entire surface \( S \) of the conductor with current \( I \). In practice, the integral (3) is again replaced by the sum of shares, as a result of which we get:

\[ \Psi = \frac{1}{I} \int \Phi_x dl_k = \frac{1}{I} \sum \Phi_x \Delta I_k \]

and finally the self-inductance

\[ L = \frac{\Psi}{I} = \frac{1}{I} \sum \left( \frac{\Delta I_k}{\Delta I_{\neq k}} \sum M_{km} \Delta I_m \right) \]

or alternatively

\[ L = \frac{1}{\sum_{\Sigma} \Delta \sigma \sum_{\Sigma} M_{km} \Delta \sigma_m} \]

related to the elementary cross sectional area of fictitious current tube, assigning it a value resulting from the sequence of proportions: \( \Delta I_k/I = \Delta I_{\neq k}/I = \Delta \sigma_k/\Sigma = \Delta \sigma_{\neq k}/\Sigma \).

The general formulas quoted here, applied to the most common coreless coils, but varied in terms of their shape and winding arrangement, resulted in many specific cases. The paper recalls a few of them that could be considered as approximate modeling of the analyzed test coil.

**Inductance of rectangular cross-section ring**

It is assumed that the ring is built of one solid turn of winding (Fig. 1) with uniform current density. According to
[3], we can use the formula (in μH):

\[
L = \frac{\mu_0 R}{2\pi} \left( \ln \frac{8R}{a+b} - 0.5 \right)
\]

where: \(a, b\) – width and height of the ring; \(D/2\) – radius of the axial line of the conductor.

By applying the above mentioned data, we obtain (in μH):

\[
L = \frac{4\pi \cdot 10^{-7}}{2\pi} \left( \ln \frac{8 \cdot 0.315}{0.02 + 0.02} - 0.5 \right) = 0.584
\]

The result concerns a solid coil so fitting to the test coil requires a multiplication of this value by the square of number of turns \(w^2\). We then get \(L = 0.584 \cdot 30^2 = 525.5\) μH.

**Inductance of circular cross-section ring**

The replacement of a rectangular section with a circular one of radius \(r = a/2 = b/2\) is intended only to show how significant is this change in affecting the previous result. According to [4], in case of \(R >> r\) (here: 0.315 >> 0.2), we can use the formula

\[
L = \mu_0 R \ln \frac{8R}{r} - \frac{7}{4}
\]

and for \(r = 0.01/0.315 = 0.03175\) returns the value \(\Phi = 19,438\), so inductance of such a coil is equal to (in μH)

\[
L = \frac{4\pi \cdot 10^{-7}}{4\pi} \cdot 30^2 \cdot 0.315 \cdot 19,438 = 550,2
\]

The procedure presented so far was to find a coil of similar geometry and use a ready formula for its inductivity. However, none of the formulas took into account the spiral winding, i.e. the continuous increase of the coil radius rather than the stepwise coil, as for completely separate coils laid on each other.

**Inductance of the flat spiral coil wound with round wire**

There is a small amount of literature focussing directly to spiral coils. Most attention is paid by the author [4], where spiral coils are still treated in an approximate way as the concentricity of circular coils with a circular cross-section (Fig. 6).

![Fig. 6. Top view on a flat coil consisting of round circular wires](image)

Given there formula looks as follows

\[
L = \mu_0 \sum_{k=1}^{z_{w-1}} \sum_{l=1}^{z_{w-1}} \int_0^{\phi_w} \left[ \left( r_w + \frac{d_p}{2} \right) \left( r_w + h_l \right) \cos \theta \right] d\theta \rightarrow -2 \left( r_w + \frac{d_p}{2} + h_l \right) \frac{r_w + h_l}{r_w + h_l} \cos \theta
\]

Assuming that \(r_w = D_w/2 = 0.1475\); \(z_w = 30\) (number of spiral turns); \(d_p = 0.0005\); insulation thickness \(h_l = 1.67 \cdot 10^{-4}\) and using MathCad program for integral calculations, we get \(L = 506.4\) μH.

**Inductance of flat spiral coil**

The next method used to determine the inductance of spiral coil is taken from the monograph [1] (1926). The formulas and references to the tables contained there will be cited here.

![Fig. 7. Top and side view on spiral coil and its geometry](image)

Further calculations were performed with a coil of rectangular cross section and parameters \(b = w, c = nd, a = a_1 + 0.5 (n-1)d\), as shown in Figure 7.

According to [1], the following algorithm was applied:

a) calculate the inductance of a rectangular coil (in μH):
\[ L = 0.002(\pi n)^2 \left( \frac{2a}{b} \right) a \cdot (K-k) \]

with the values of parameters for the test coil \( a = 0.15725; \)
\( b = 0.02; d = 6(6) \cdot 10^{-4}; n = 30 \) and estimated parameters \( K = 0.215 \) and \( k = 0.0336 \) found in the Table 4 of [1].

b) calculate the adjustment \( \Delta L = 0.01257 \cdot n \cdot \left( A_i + B_i \right) \)
respecting the winding shape and presence of insulation, where \( d = 6(6) \cdot 10^{-4}; t = 0.0005 \):
\[ A_i = \ln \frac{b + d}{b + t} = 8.097 \cdot 10^{-3} \]
\[ B_i = -2 \left( \frac{n-1}{n} \delta_2 + \frac{n-2}{n} \delta_3 + \ldots + \frac{1}{n} \delta_n \right) = -3 \cdot 10^{-3} \]
coefficients \( \delta_2, \delta_3, \ldots, \delta_n \) = 0.001 and 0 for all others, what gives the adjustment \( \Delta L = 3.02 \cdot 10^{-4} \) μH.

c) calculate corrected value of \( L := L + \Delta L = 510.9 \) μH.

**Fields method**

Computer programs being currently in wide use for determining of fields distributions are equipped with tools supporting post-processing calculations. The Quickfield software [7] belongs to them and in the case of magnetic supporting post-processing calculations. The Quickfield software operation. Then multiplier \( z^2 \) appears, instead of dividing the entire current into \( n = z \) wires. The field type was selected as magnetostatic (\( I = \text{const} \)), the number of turns \( n = 30 \) and then the "Inductance Wizzard" was chosen. As we can see (Fig. 8) the inductances are as follows (in μH): \( L = \psi / I = 532.8 \) – from the linkage flux calculations and \( L = 0.5W_{\text{eff}} / I^2 = 500.2 \) – from the energy of magnetic field.

**Measurement by technical method**

As a criterion for assessing the accuracy of the methods used, a direct measurement of inductance by technical method was performed [8]. With DC supply, one can measure current \( I \), hence the resistance \( R = U / I \), while with the AC supply of frequency \( f \) the measurement will show the value of impedance:

\[ (12) \quad Z = \sqrt{R^2 + X_f^2} = \sqrt{R^2 + (\omega L)^2} \quad \text{so} \quad L = \frac{\sqrt{Z^2 - R^2}}{2\pi f} \]

The measurements gives following results: \( R = 0.059 \) Ω; \( Z = 0.18 \) Ω, what, for \( f = 50 \) Hz, leads to \( L = 549.8 \) μH.

**Conclusions**

The problem of determining the inductance of the spiral coil gave a chance to review various methods and formulas for coils geometrically similar to original one in shape. The results of calculations and measurements are given in Table 2.

Assuming that the self-inductance measurement obtained by the technical method will be referred to, it is concluded that all other methods did not exceed the error of more than 10%. The field method, despite quite a thick division into FEM elements, in the variant with the linkage flux - gives correct results (error of 3.1%), while on the basis of field energy - the biggest error, of 9%.

**REFERENCES**


[6] www.quickfield.com - the website of Quickfield software


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