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Influence of filters for numerical differentiation on parameter tuning of PI speed controllers

***Abstract**. The main purpose of this study was to describe a parameter tuning of PI speed controller when the speed is determined by differentiating the position. In the first part three methods of speed calculation was selected: moving average, low pass filter, and smooth derivative with FIR filter. Dynamic and static properties of presented methods were compared through simulation studies. The results can be helpful in choosing method for estimation the speed of the direct drive, especially for very low speeds. In the second part the relationship between the degree of smooth filter and controller setting is analysed. Simulation results show that the proposed control approach can provide good dynamic of the drive*

***Streszczenie.** W pracy przedstawiono dobór parametrów regulatora prędkości typu PI w układzie sterowania, w którym sygnał prędkości jest wyznaczany poprzez różniczkowanie sygnału położenia. W pierwszej części wybrano trzy metody wyznaczania sygnału prędkości: ruchoma średnia, filtr dolno-przepustowy, oraz gładka metoda różniczkowania z filtrem FIR. Na drodze badań symulacyjnych porównano właściwości dynamiczne i statyczne zaprezentowanych metod. Zaprezentowane wyniki mogą być pomocą w wyborze metody wyznaczania prędkości, szczególnie w zakresie bardzo niskich prędkości obrotowych. W drugiej części pracy przeanalizowano zależność nastaw regulatora prędkości od stopnia filtru wygładzającego. Wyniki badań symulacyjnych wskazują, że zaproponowane podejście zapewnia uzyskanie dobrych właściwości dynamicznych napędu. (**Wpływ cyfrowego różniczkowania z filtracją na nastawy regulatora PI prędkości**)*

Keywords: low-speed, speed measurement, PI controller, speed control, motion control
Słowa kluczowe: mała prędkość obrotowa, pomiar prędkości, regulator PI, regulator prędkości, sterowanie ruchem

Introduction

The problem to improve the low-speed performance of an electromechanical servo system, for example, a direct drive, has always been a hot topic in servo control [1]. There has been considerable interest in the approaches to improve low-speed performance, primarily including low-speed estimation, friction compensation, and disturbance observation.

In practice, the optical encoder is still the most popular position sensor used in technical fields because of its simple detection circuit, high resolution, high accuracy, and relative ease of adaptation in digital control systems. Therefore, it is necessary to reconstruct a high-quality velocity signal numerically from the noisy position measurement. Typically, the output from the encoder takes the form of pulses. Velocity sensing based on such optical encoder is usually done by the so-called M/T method [2,3]. The method selects the velocity with the number of the encoder pulses during the detection time, which generates accurate velocities over a wide range.

In the new sensors measuring position information is transmitted via a serial port. For example, high accuracy absolute angle encoder REXA by Renishaw uses BISS serial interface [4]. This way of data transmission excludes the use of methods of counting pulses or measuring the time between pulses. In this case, it is necessary to determine the speed based on successive position readings, with determined sampling interval.

The various speed estimation methods had been proposed, such as speed filter [5], speed observer [6], multiple sampling method and multi-sensor fusion method [7, 8]. Digital differentiators are one of the important signal processing systems for motion control [9, 10]. Work [11] suggests the possibility of such selection of smooth differentiation algorithm for reduce the influence of noise.

In section 2 of this paper the four groups of speed estimation methods, based only on position measurement, are compare for direct drive task. An often overlooked problem under some practical situation, the angular measurement quantization of the driver, is analyzed and modeled on mechanism. This factor has a certain effect on the performance of low-speed servo system. Section 3 presents the relationship between the degree of the smooth noise robust differentiators and the parameters of the speed controller, so as to ensure allowable reference current ripple. The simulations result and discussion are presented in section 4. Some concluding remarks are submitted in section 5.

Analysed method of speed calculation

Simple discrete derivative

Figure 1 shows a discrete control system. For analysis purposes, the closed current control loop was replaced by an inertial module with delay. For this module the input signal is the reference current i^{ref}, while the output signal is the current component i generating the electromagnetic torque. The *Electrical drive* block represents an electro-mechanical subsystem in which, for example, a complex mechanical structure and torque ripple can be taken into account. The speed controller system is presented as a discrete block with input sampling and output extrapolation. The discrete position measurement system is represented by the block *Quantizer*. The speed ω is defined as a derivative of position θ. In the discrete control system, derivative is replaced by change of position within the sample period (1):

$$\omega_{\mathrm{sim}}(n) = \frac{\theta(n) - \theta(n-1)}{\tau}, \tag{1}$$

where $\theta(n)$, $\theta(n-1)$ indicate the position in the current and previous sample, ω_{sim} is the simply calculated speed and τ is the sampling time. Assuming the resolution of the measurement system on *N* pulse per revolution, the absolute quantum of speed calculation is defined as:

$$|\Delta\omega| = \frac{2 \cdot \pi}{N \cdot \tau}. \tag{2}$$

Such an quantum is too large, especially for direct drives, rotating with low speeds. Direct entry of signal (2) to the speed controller causes a very large torque oscillations and unstable operation of the drive. Therefore, for example, averaging of the speed signal is used. An important part of the regulator block is the filter, which reduces the effects introduced by discrete differential algorithm.

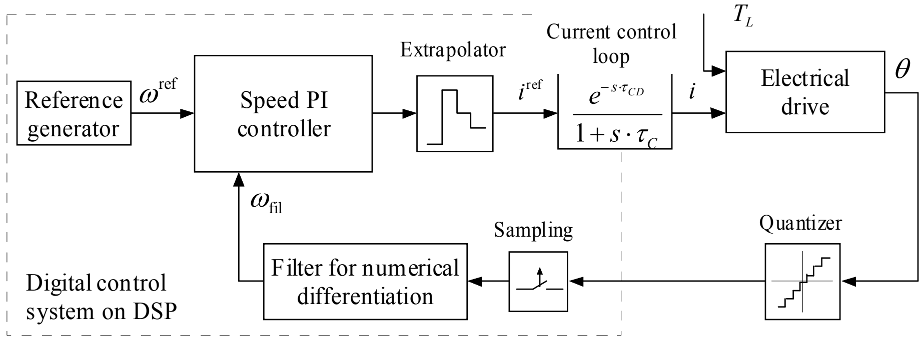

Fig. 1 Block diagram of the control system

Moving average

A moving average (MA) is commonly used with time series data to smooth out short-term fluctuations. Simple moving average is the unweighted mean of the previous L data, defined as (3):

$$\omega_{MA}(n) = \frac{1}{L}\sum_{i=0}^{L-1}\omega_{sim}(n-i) \,. \quad (3)$$

When calculating successive values of average speed ω_{MA}, a new value of speed ω_{sim} comes into the sum and an old value drops out (4)

$$\omega_{MA}(n) = \omega_{MA}(n-1) + \frac{\omega_{MA}(n) - \omega_{MA}(n-L)}{L} \,. \quad (4)$$

This calculation method provides efficient implementation, in particular by using delay line with cyclic buffers. If the speed has a periodic fluctuation, then applying an MA of that period will eliminate that variation.

Low pass filters

Another method to eliminate pulsation in the measurement speed is the use of low-pass filters. A popular method is to design a filter in the area of continuous transfer function, and then converting it to the discrete form. We suggest the use of a 2nd order filter [12], with a transmittance according to formula (5)

$$G_{LPF1} = \frac{\left(2\cdot\pi\cdot f_f\right)^2}{s^2 + 2\cdot\xi_f\cdot\left(2\cdot\pi\cdot f_f\right)\cdot s + \left(2\cdot\pi\cdot f_f\right)^2} \,. \quad (5)$$

The design frequency of the filter f_f should be chosen so as to match the pass band of torque control loop. The damping factor should be selected from the range (6):

$$\xi_f \in \left\langle \frac{2}{\pi}, \frac{\sqrt{3}}{2} \right\rangle . \quad (6)$$

A smaller value provides the lowest latency, while larger decreases oscillations in response. It is sometimes useful to use 4th order filter (7):

$$G_{LPF2} = \left(\frac{\left(2\cdot\pi\cdot f_f\right)^2}{s^2 + 2\cdot\xi_f\cdot\left(2\cdot\pi\cdot f_f\right)\cdot s + \left(2\cdot\pi\cdot f_f\right)^2}\right)^2 . \quad (7)$$

Discrete implementation of the filter (5) leads to a 2nd order dynamic system with infinite impulse response. A convenient way to get the discrete equations of state is to use a MATLAB c2d() function [13].

Smooth discrete derivative

The study [11] presented an original method of calculating the value of the derivative with guaranteed noise suppression and efficient computational structure. The numerical derivative can be written in general form as (8)

$$\omega(n) = \frac{1}{\tau}\cdot\sum_{k=1}^{M}\left(c_k\cdot\left(\theta(n-k) - \theta(n+k)\right)\right) . \quad (8)$$

To use the formula (8) is necessary to know the position of the measured samples of both sides calculated speed – both earlier and later. This possibility exists in off-line calculation, as is typical, for example, for the class of repetitive control. In the case described in this paper only previous samples are available. One sided smooth numerical differentiation is special method for derivative estimation, which uses only past values of a function (causal calculation). Such differentiators can be written as:

$$\omega_{Hm}(n) = \frac{1}{2^{m-1}\cdot\tau}\cdot\sum_{k=0}^{m}\left(c_k\cdot\theta\left(n-k\right)\right) . \quad (9)$$

From signal processing point of view differentiators are digital filters with finite impulse response. In the work [11] it was pointed out that coefficients of best noise-robust differentiators are the elements of Catalan triangle (related to coefficients of Chebyshev's polynomials). For a given order of filter *M*, coefficients are defined as(10):

$$\text{for order } M\colon\; c_k = \begin{cases} 1 & \text{for} \quad k=0 \\ \binom{M-1}{k} - \binom{M-1}{k-1} & \text{for} \quad 0<k<M \,, \\ -1 & \text{for} \quad k=M \end{cases} \quad (10)$$

where binomial is defined as usually (11):

$$\binom{m}{k} = \frac{m!}{k!\cdot(m-k)!} \,. \quad (11)$$

Proposed in [11] smooth differentiators have efficient computational structure. Costly floating-point division can be completely avoided. Denominator is a power of 2 and division can be replaced by fast shift operation.

For example, in this section three smooth differentiators was tested, respectively, the order of 3, 6 and 10. They are described by equations (12) to (14).

$$\omega_{H3}(n) = \frac{1}{4\tau}\left(\theta(n) + \theta(n-1) - \theta(n-2) - \theta(n-3)\right), \quad (12)$$

$$\omega_{H6}(n) = \frac{1}{32\tau}\begin{pmatrix}\theta(n) + 4\cdot\theta(n-1) + 5\cdot\theta(n-2) + \\ -5\cdot\theta(n-4) - 4\cdot\theta(n-5) - \theta(n-6)\end{pmatrix}, \quad (13)$$

$$\omega_{H10}(n) = \frac{1}{512\cdot\tau}\begin{pmatrix}\theta(n) + 8\cdot\theta(n-1) + 27\cdot\theta(n-2) + \\ +48\cdot\theta(n-3) + 42\cdot\theta(n-4) + \\ -42\cdot\theta(n-6) - 48\cdot\theta(n-7) + \\ -27\cdot\theta(n-8) - 8\cdot\theta(n-9) - \theta(n-10)\end{pmatrix} . \quad (14)$$

The advantage of filter realized in the structure of the FIR is the constant group delay, depending on the degree of filter:

(15) $$\tau_{\text{fil}} = \tau \cdot \frac{M-1}{2}.$$

This allows for easy taking into account delays in the synthesis of the controller.

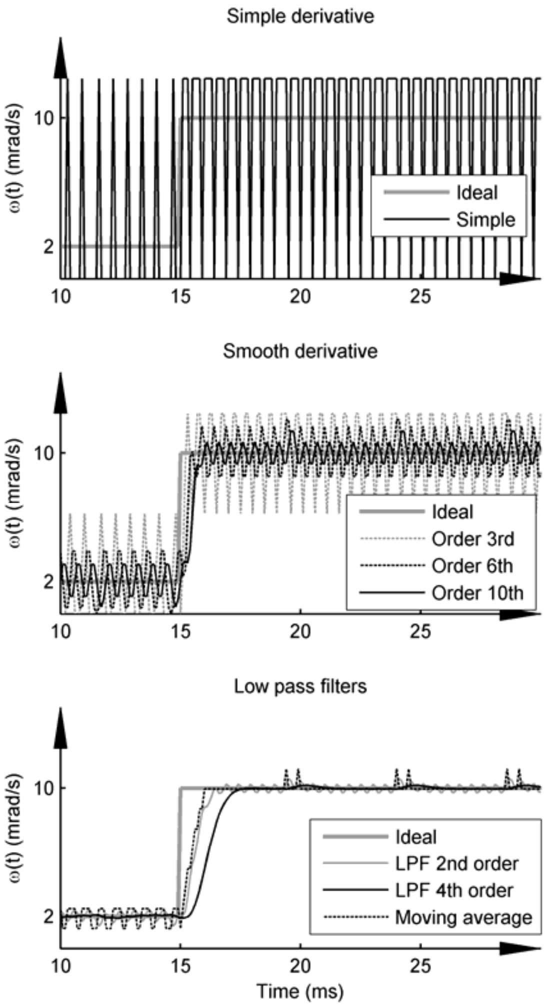

Fig.2 Step responses of the speed

Step response of proposed speed estimation methods

To verify proper operation and to evaluate the effectiveness of presented algorithms simulation tests was carried out. Simulation model of the drive consists of a programmable generator of model speed (ideal speed) and integrator block. The model includes a discrete position measurement and sampling with a fixed period. Numeric data for the model were taken from the papers [14, 15], which presents the laboratory direct drive with permanent magnet synchronous motor. Resolution of position measurement sensor is *N*=5,000,000 values per revolution, while the sampling period is $\tau = 100\ \mu\text{s}$. According to (1) the quantum of speed calculation is equal $|\Delta\omega| = 12.6$ mrad/s.

In test the step response was analyzed. The ideal speed is step changing from 2 to 10 mrad/s. The whole step change of speed is less than one quantum of speed computation. The test results were collectively shown in Figure 2. The second subplot from the top shows the response of smooth derivative speed. Responses are quick and without overshoot. Because smooth derivative is realized in the structure of the FIR filter, the delay depends on the order of the filter. Ripple in speed signal for 10th order filter are already very small. On the last subplot low pass filters responses and moving average filter response are compared. In the case of moving average, pulses occur periodically, resulting from the calculation for very low speeds. The use of LPF filters ensures very good smoothness of the signal, but there are long delays. Therefore, only the smooth derivative, described in equation (1) were selected for further analysis.

Parameter tuning of PI speed controllers

For the purpose of the speed controller synthesis, a simplified drive model described by equations (16)-(17) was adopted. Current close loop control is replaced by:

(16) $$i = \frac{e^{-s\cdot\tau_{CD}}}{1 + s\cdot\tau_C} \cdot i^{\text{ref}},$$

where τ_{CD} and τ_C are respectively an equivalent delay and time constant of the first order inertia. The electromechanical subsystem is described by the equation:

(17) $$\dot{\omega} = \frac{1}{J}\left(k_T \cdot i - T_L\right),$$

where J is the equivalent moment of inertia, k_T describes the relationship between the current and the generated electromagnetic moment, while T_L describes the total load torque in the system.

It is considered speed controller PI, described by equations:

(18) $$\begin{aligned}\upsilon(n) &= \upsilon(n-1) - K\cdot\omega_{\text{fil}}(n) + \\ &+ \frac{\tau}{T_i}\left(K\left(\omega^{\text{ref}}(n) - \omega_{\text{fil}}(n)\right) - z(n)\right)\end{aligned},$$

(19) $$i^{\text{ref}}(n) = \text{sat}\left(\left(\upsilon(n) - K\cdot\omega^{\text{fil}}\right), \pm I^{\max}\right),$$

(20) $$z(n) = \left(\left(\upsilon(n) - K\cdot\omega^{\text{fil}}\right) - i^{\text{ref}}(n)\right)\cdot\lambda,$$

where K, T_i, τ are the gain, time constant and sampling time of the controller, respectively. The block diagram of the controller is shown in Fig. 3. The internal output signal υ, calculated following equation (18) is limited according to formula (19), to the range [$-I^{\max}$, $I^{\max}$]. The saturation function $\text{sat}(\cdot)$ is used, which is linear within the preset limits. The output from the saturation block is the signal of the reference current i^{ref} component creating the electromagnetic torque. The difference between the internal output signal υ and the signal after limitation block i^{ref} is provided as an additional signal z, preventing integrator saturation (20). The weight factor of this anti-windup tracking system is selected as λ = 5.

Due to the stability criterion, the controller gain *K* is limited by total delay in the control loop:

(21) $$K < K_{\text{cr}} = \frac{\pi\cdot J}{2\cdot k_T\cdot\left(\tau_{\text{fil}} + \tau_{\text{del}}\right)},$$

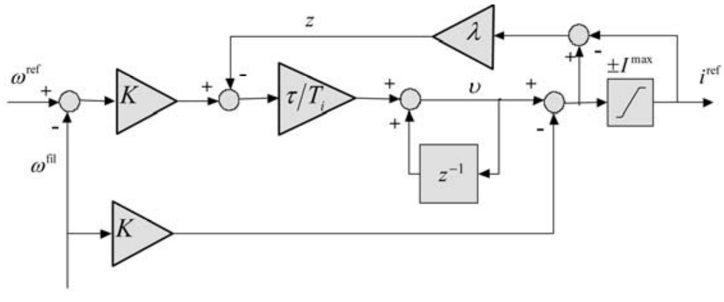

Fig.3. Block diagram of the controller

where J, k_T, τ_{del} are the inertia, torque constant and sum of other delays in control loop, respectively. The value K_{cr} corresponds to the critical gain for which the closed speed control system reaches the stability limit. Appropriate margin amplitude is provided where

(22) $$K = 0.5 \cdot K_{\mathrm{cr}} .$$

Ripple of measured feedback signal (12-14) on speed controller input cause ripple in the reference current:

(23) $$\left|\Delta i^{\mathrm{ref}}\right| = K \cdot \left|\Delta \omega_{\mathrm{fil}}\right| .$$

For assumed an acceptable level ρ of ripple current in relation to the maximum current $I^{\max}$ controller gain must be limited

(24) $$K < \frac{\rho \cdot I^{\max}}{\left|\Delta \omega_{\mathrm{fil}}\right|} .$$

From (22) and (24) the controller gain should be chosen so as to fulfill the condition

(25) $$K \le \min\left(0.5 \cdot K_{cr},\ \frac{\rho \cdot I^{\max}}{\left|\Delta \omega_{\mathrm{fil}}\right|}\right) .$$

For example, the following data was used: J=2.4 kgm^2, k_T=17.5 Nm/A, $\rho = 4\%$, $I^{\max}$=6 A, $\tau_{\mathrm{del}} = 6.5 \cdot \tau$ [6]. Results are shown on Fig. 4. The preferred damping in the loop will be provided when the integration time satisfies:

(26) $$T_i = \frac{2 \cdot J}{K \cdot k_T} .$$

Simulation results

Based on Fig. 3 27-degree filter has been selected. In this case the greatest controller gain can be use: K=52.8 As/rad, accordingly T_i = 5.2 ms was also chosen. The results of other tests have shown, that these settings ensure the best possible dynamic of control system. Figures 5 presents the results of tests on an accurate simulation model. Such an exact model has been built with a full, not simplified current control system. Test consists of two parts: step change of reference speed from 0 to 0.1 rad/s and step change od load torque from 0 to 30 Nm.

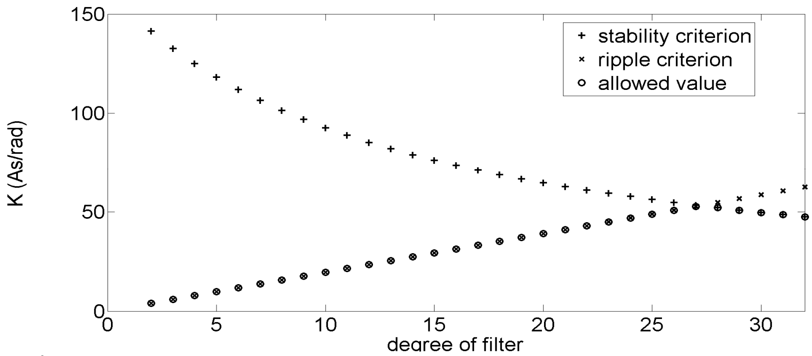

Fig. 4 Selection of controller gain

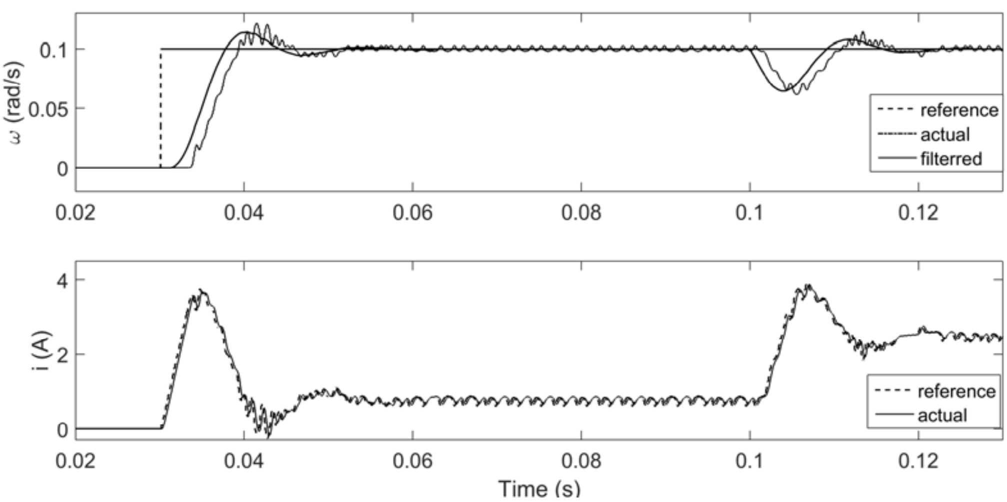

Fig. 5 Step response of speed and current

Conclusions

This paper presents the relationship between the degree of differentiation and controller setting. The controllers gain is limited by the permissible ripple in reference current and the stability conditions. These criteria define the optimal degree of differential filter. FIR filter structure ensures a constant group delay, which is included in the design. In the future, it is also advisable to extend the work for other methods of filtering the speed signal. Experimental verification on the direct-driven astronomical telescope mount is planned.

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