Global Stability of Backstepping Control with Robust Nonlinear Observer of Induction Motor in \((\alpha, \beta)\) Frame

**Abstract.** This paper deals with the design of an advanced control law by backstepping with an observer for a special class of nonlinear systems. We design an observer with a single adjustment gain as a function of speed. Our contribution is developed by demonstrating a nonlinear control law by backstepping using the global Lyapunov stability of the controller, the nonlinear observer and the induction motor. We study the behavior of the torque tracking and the rotor flux of the induction motor in the natural frame \((\alpha, \beta)\). The control algorithm obtained is studied through simulations and applied in many configurations (flux and speed and torque disturbances), and is shown to be very efficient.

**Streszczenie.** Niniejszy artykuł dotyczy projektowania zaawansowanego prawa kontroli poprzez odtwarzanie z obserwatorem specjalnej klasy systemów nieliniowych. Projektujemy obserwatora z jedynym wzmacnieniem regulacji w funkcji prędkości. Nasz układ jest rozwiązany przez Demonstrowanie nieliniowego prawa kontrolnego poprzez odtwarzanie za pomocą globalnej stabilności Lapunowa kontrolera, nieliniowego obserwatora i silnika indukcyjnego. Badamy zachowanie sterowania momentu i strumienia wirnika silnika indukcyjnego w naturalnej ramie \((\alpha, \beta)\).

Użyskany algorytm sterowania jest badany za pomocą symulacji i stosowany w wielu konfiguracjach (strumieni i zakończenia prędkości i momentu obrotowego) i jest bardzo wydajny. **Globalna stabilna sterowań silnikiem indukcyjnym z odpornym nieliniowym obserwatorem**

**Keywords:** nonlinear observer, backstepping control, global stability, Lyapunov stability.

**Słowa Kluczowe:** nieliniowy obserwator, kontrola cofania, stabilność globalna, stabilność Lapunowa

**Introduction**

Induction motors are nonlinear, coupled, multivariable process. Nevertheless, they become more and more appealing because of their reliability, robustness and low cost or maintenance [3]. We built a globally stable nonlinear control law with real effectiveness for the adopted strategies and we describe a speed dependent observer. We based the initial strategy on backstepping control. Here we design the observer based on a nonlinear control law in order to ensure the global stability of the process observer-controller system. The main contributions of the paper are the following: First, we propose a new observer modified for a special class of nonlinear systems applied to the induction motor [1]. Secondly, the model is nonlinear in the frame \((\alpha, \beta)\) without making the FOC. Third, the demonstration of the nonlinear observer, backstepping control, global stability, Lyapunov stability, boundedness and regulation of the induction motor model. In Section 3, we present a nonlinear observer for a special class of nonlinear systems applied to the induction motor. Lastly, we present a Backstepping control algorithm. The main contributions of the paper are the following: First, we propose a new observer modified for a special class of nonlinear systems applied to the induction motor \([1]\). Secondly, the model is nonlinear in the frame \((\alpha, \beta)\) without making the FOC. Third, the demonstration of the nonlinear observer, backstepping control, global stability, Lyapunov stability, boundedness and regulation of the induction motor model. In Section 3, we present a nonlinear observer for a special class of nonlinear systems applied to the induction motor.

**Modeling of the induction motor**

The model used is a classical induction model of Park in a frame \((\alpha, \beta)\) fixed to stator, given by \([1]\):

\[
\dot{x} = f(x) + g u
\]

with

\[
x = \begin{bmatrix} i_{qa} & i_{qb} & \varphi_{qa} & \varphi_{qb} & \Omega \end{bmatrix}^T, \quad u = \begin{bmatrix} u_{qa} & u_{qb} \end{bmatrix}^T
\]

Here \(x\) contains four electrical states (flux and currents components, respectively \(\varphi_{qa}, \varphi_{qb}\) and \(i_{qa}, i_{qb}\)) and one mechanical state \(\Omega\) governed by a mechanical equation. The motor is driven by two voltage components, \(u_{qa}\) and \(u_{qb}\). We define the control input matrix by

\[
g = \begin{bmatrix} \frac{1}{\sigma L_s} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sigma L_s} & 0 & 0 & 0 \end{bmatrix}
\]

with

\[
T_r = \frac{L_s}{R_r}, \quad \sigma = 1 - \frac{M^2}{L_s \omega_n}
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\]

with

\[
T_r = \frac{L_s}{R_r}, \quad \sigma = 1 - \frac{M^2}{L_s \omega_n}
\]
We assume that both \( f, g, m \) and \( \beta \) are known. This system can be viewed as a cascade connection of two components, as shown in Fig. (1), the first component is (5), which \( \xi \) as input, and the second component (6). Where for the system in (5), a desired feedback \( \alpha(x) \)

\[
y = h(x)
\]

**Application to asynchronous motor**

We are applied backstepping control (see Fig. 2).

**Step 1:**

We define \( e_1 \) and \( e_2 \) respectively represent the error between the actual speed \( \Omega \) and \( \Omega_{ref} \) the reference speed and the error between the module of the flux \( \varphi \) and \( \varphi_{ref} \) its reference.

\[
e_1 = \Omega_{ref} - \Omega
\]
\[
e_2 = \varphi^2_{ref} - \varphi^2
\]

where \( \varphi^2 = \varphi^2_{ra} + \varphi^2_{rb} \)

The first Lyapunov function is chosen as follows:

\[
V_1 = \frac{1}{2}(e_1^2 + e_2^2)
\]

**Step 2:**

We define other tracking errors concerning the components of the stator current and reference

\[
e_3 = (i_{sa})_{ref} - i_{sa}
\]
\[
e_4 = (i_{sb})_{ref} - i_{sb}
\]

The final Lyapunov function is given by:

\[
V_2 = V_1 + \frac{1}{2}(e_3^2 + e_4^2)
\]

**Nonlinear observer and application to the Induction Motor:**

In this section, based on extensions of the observer design strategy to the multi-output case [4] and the application to the induction motor, we propose a new observer with nonlinear terms. We are going to apply the result given in the preceding part to construct a full order observer for an induction motor written in the (\( \alpha, \beta \)) frame
The proposed observer uses the measurements of the stator voltage and current, and the rotor speed. More precisely, we design the observer up to an injection of the speed measurements so that only the electrical equations are considered. First, we define

$$
\begin{align*}
\dot{x}_e &= \begin{bmatrix}
-\gamma & 0 & K & 0 & \frac{p\Omega K}{T_r}
\end{bmatrix} \begin{bmatrix}
x_e \\
\dot{x}_e \\
M & 0 & M & -\frac{1}{T_r} & 0 & -\frac{p\Omega K}{T_r} & 0
\end{bmatrix} + f_{ia} [\frac{\dot{\varphi}_{ra}}{\varphi_{ra}}] + f_{ib} [\frac{\dot{\varphi}_{rb}}{\varphi_{rb}}]
\end{align*}
$$

\[\dot{x}_e = x_e - \hat{x}_e\]

Where \(x_e, \dot{x}_e\) and \(\hat{x}_e\) real states, estimated and observation error vectors respectively.

We show the diagram block of this observer (see Fig. 3).

We will define a Lyapunov function for proving stability.

$$
V(x) = V_1(x) + V_2(x)
$$

Therefore the Lyapunov function of all

For \(V(x)\) is then a Lyapunov function for the overall system. Consequently, the whole process is stable and the convergence is exponential. We ensure flux, speed and torque tracking.

Results and simulations

We used a first-order filter after each time derivative in order to eliminate the large amplitude of the pulses produced by the derivatives; we placed a filter of the first order of unity gain and of fast time constant so as not to influence it on the behavior and shape of the signal that is derived.

We design the general block diagram as shown (see Fig. 3). In addition to that; we perform a simulation with Matlab-Simulink by using the Benchmark (see Fig. 4) and the motor parameters in Table 1 [6].

<table>
<thead>
<tr>
<th>Designation</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor resistance</td>
<td>(R_r)</td>
<td>4.3047 (\Omega)</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>(R_s)</td>
<td>9.63 (\Omega)</td>
</tr>
<tr>
<td>Mutual inductance</td>
<td>(M)</td>
<td>0.4475 (H)</td>
</tr>
<tr>
<td>Stator inductance</td>
<td>(L_s)</td>
<td>0.4717 (H)</td>
</tr>
<tr>
<td>Rotor inductance</td>
<td>(L_r)</td>
<td>0.4718 (H)</td>
</tr>
<tr>
<td>Rotor inertia</td>
<td>(J_m)</td>
<td>0.0293 (kgm^2)</td>
</tr>
<tr>
<td>Pole pair</td>
<td>(p)</td>
<td>2</td>
</tr>
<tr>
<td>Viscous friction coefficient</td>
<td>(f_m)</td>
<td>0.0038 (N m sec^{-1})</td>
</tr>
<tr>
<td>Mechanical power</td>
<td>(P_{mech})</td>
<td>1.1 (KW)</td>
</tr>
<tr>
<td>Nominal voltage</td>
<td>(V_m)</td>
<td>220 (V)</td>
</tr>
<tr>
<td>Nominal current</td>
<td>(I_m)</td>
<td>2.6 (A)</td>
</tr>
<tr>
<td>Nominal speed</td>
<td>(\Omega_{in})</td>
<td>1410 (Nm)</td>
</tr>
</tbody>
</table>

This benchmark [1] (see Fig. 4) reveals the following profile: a rise in speed, a load, inversion speed and a load in recovery, and a return at a low speed.

Control Algorithm

We will define a Lyapunov function \(V_1(x)\) between the motor and the observer and other Lyapunov function \(V_2(x)\) between the motor and controller as:

\[
V_1(x) = \frac{1}{2} (\dot{\varphi}_{ra}^2 + \dot{\varphi}_{rab}^2 + \dot{\varphi}_{rb}^2)
\]

\[
V_2(x) = \frac{1}{2} (\dot{\varphi}_{ra}^2 + \dot{\varphi}_{rb}^2)
\]

Therefore the Lyapunov function of all

\[
\dot{V}(x) = \dot{V}_1(x) + \dot{V}_2(x)
\]

Its derivative is:

\[
\dot{V}(x) = -k_2 \dot{e}_2^2 - k_3 \dot{e}_3^2 - k_4 \dot{e}_4^2 - k_5 \dot{e}_5^2 - k_6 \dot{e}_6^2 - \frac{1}{2} \dot{\varphi}_{ra}^2 - \dot{\varphi}_{rb}^2
\]

For \(\dot{V}(x) < 0\)

\[
\begin{align*}
K_1 & = \gamma < 0 \\
K_2 &= (K + M + K_2) \frac{1}{T_r} \left(\dot{\varphi}_{ra}^2 + \dot{\varphi}_{rb}^2\right)
\end{align*}
\]

Hence

\[
K_1 < \gamma
\]

\[
\begin{align*}
\dot{V}(x) &= \dot{V}_1(x) + \dot{V}_2(x)
\end{align*}
\]

Fig. 3. Scheme of the backstepping with an observer.

It is placed after each impulse derivative in order to eliminate the large amplitudes of the pulses produced by the derivatives; we have placed a filter of the first order of unity gain and of fast time constant \(\tau = 0.008\) so as not to eliminate it. Influence on the behavior and shape of the derived signal.
We have used first-order filter (see Fig. 5).

We note that the drive torque follows the load torque when the speed is constant. During an increase or decrease in the speed, a difference of $\pm 4 \text{Nm}$ appears between the two torques, (see Fig. 6).

The three stator control voltages follow the profile of the norm current, except if speed varies linearly. Their amplitude varies in the same propositions, (see Fig. 8) and their shape for each time interval, has a normal physical relationship with a speed, motor torque, stator current and control voltage.

We note a good tracking by looking at the two errors of observation and regulation; (see Fig. 9, 10, 11, 12). During an increase or a decrease in the speed of $0.3 \text{ rad/s}$ is observed.

We show the plot of the norm stator current in (see Fig. 7). The norm of the current is equal 3.2 A, in the interval of $t = [0 \, 1]s$, $t = [3.4 \, 5]s$ and $t = [7.4 \, 9.3]s$ because the speed increase or decrease and the load torque are zero.

Between $t = [1.5 \, 3.4]s$, $t = [5 \, 5.4]s$, $t = [6.5 \, 7.4]s$ and $t = [8.3 \, 9]s$, the norm is minimal and equal to 2.3A. In this phase, the speed is constant and the load torque is zero. The amplitude of the current reaches a maximum value of 4.7A at $t = [1.5 \, 2.5]s$ because the load torque of 7 Nm appears at that point. Speed remains always constant. Therefore, the behavior of the norm of the stator current is normal.
We study sensitivity to rotor and stator resistance disturbances for three values of $R_s = 4.3047 \Omega$, $7.318 \Omega$, $8.6094 \Omega$ and $R_r = (9.65; 16.4050; 18.30) \Omega$, i.e. an increase by 20% and 70% and 100%. In the error curve tracking flux (see Fig. 10, 11, 12), we note the appearance of the waves at time $t = 4.4s$ and $t = 8s$ because the induction motor not observable in zero speed, moreover variation of $R_s$ and $R_r$. The greater the variation in the $R_s$ and $R_r$, increased the amplitude of waves. 

For load torque

We notice a good and logical behavior of the motor torque $T_m$.

\[
\begin{align*}
J \dot{\omega} &= T_m + I_i \\
T_m &= J \dot{\omega} - I_i
\end{align*}
\]

For $t \in [0 1]s$: the speed is a line of constant slope and no load torque.

For $t \in [1.5 2.5]s$: the speed is constant and its load torque and no variation in the behavior is normal.

For $t \in [2.5 3.5]s$: the speed decreases, its slope is negative, the load torque is zero $T_m = 0 N.m$, the motor torque changes the direction of rotation to become negative, which translates the normal behavior. At the point $t = 4.2s$ and around it we speak of low speed, it is a singular point are almost discontinuous, and continues the process of the system.

For $t \in [3.5 4.2]s$: the speed changes sign and start to become negative (a negative slope), the motor torque also changes direction and becomes negative, which is normal.

For $t \in [4.2 5]s$: the speed passes through $\omega = 0$ and changes sign (sense) to become negative (a negative slope), the motor torque also changes direction and becomes negative, which is normal.

Conclusion

We have presented two contributions in this article. The first is the use of the dynamic model in the frame (aE) of the asynchronous motor, model approximated. The second is the demonstration of the overall stability of the system composed of the asynchronous motor (nonlinear coupled system) and multivariable of the observer with a single adjustment parameter $\theta$ (which also nonlinear) and the controller which used the backstepping, to produce a nonlinear control law. We used the Lyapunov function, which involves the errors of the current, flux and their variable in square form to approach a form of energy.

We have disturbed the rotor and stator resistors in order to show the robustness of this control and the good estimate of the observer and the results were conclusive.

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REFERENCES


Marcin MORAWIEC, Arkadiusz LEWICKI, Zbigniew KRZEMIŃSKI, Obserwator prędkości kątowej wirnika maszyny indukcyjnej klatkowej oparty na metodzie backstepping ze ślizgowymi funkcjami przełączającymi, PRZEGLĄD ELEKTROTECHNICZNY