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doi:10.15199/48.2019.12.12

Analysis of interaction of forces of working elements in electromagnetic mill

***Abstract**. The paper is devoted to the analysis of the operating modes of an electromagnetic mill, a device designed for grinding or mixing various substances, which occurs due to the interaction of these substances with moving ferromagnetic working elements. The factors impacting this force interaction are revealed. One of the possible structural solutions of the inductor of such a mill is given. The dependences of the projections of the equatorial force vector applied to the center of the mass of the working element are calculated from the above factors, in particular, from the phase of the resulting magnetomotive force of the inductor, the angular and radial coordinates of the location of the working element, its dimensions and the angle of rotation around its own axis. The results and conclusions obtained during this study make it possible to choose the method of mathematical formalization of the description of the force interaction of mill elements in a rotating magnetic field.*

***Streszczenie.** W publikacji zaprezentowano analizę stanów pracy młyna elektromagnetycznego - urządzenia służącego do rozdrabniania i mieszania różnych substancji. Opisano czynniki, które wpływają na współdziałanie siłowe mieszanych substancji z ruchomymi ferromagnetycznymi mielnikami. Ponadto przedstawiono jedno z możliwych rozwiązań konstrukcyjnych induktora tego typu młyna. W artykule zawarto również zależności rzutów wektora wypadkowej siły, przyłożonej do środka ciężkości mielnika od fazy wypadkowej SMM induktora, promieniowych i kątowych współrzędnych położenia mielnika, jego wielkości i kąta obrotu wokół własnej osi. Wyniki i wnioski uzyskane dzięki tym badaniom umożliwiły wybranie sposobu sformalizowania matematycznego oddziaływania siłowego mielników z wirującym polem magnetycznym. (**Analiza współdziałania sił elementów roboczych młyna elektromagnetycznego**)*

Keywords: electromagnetic mill, grinding, mixing, rotating magnetic field inductor, FEM-analysis.
Słowa kluczowe: młyn elektromagnetyczny, rozdrabnianie, mieszanie, induktor wirującego pola magnetycznego, analiza MES.

Introduction

Electromagnetic mill (EMM) is a process equipment designed for grinding or mixing non-aggressive and aggressive mixtures, for example, solutions that do not interact with the *material of its working elements and chamber where grinding/mixing takes place. The electromagnetic mill is* also used to intensify the flow of various physical and chemical processes characterized by energy efficiency, reliability at work, environmental safety and ease of use.

The working elements of the electromagnetic mill will hereinafter be called grinding media, and the volume in which the process of grinding or mixing occurs is called a working chamber.

The development of issues related to the creation of such equipment, the implementation of the latest structural and technological decisions and methods of calculation is undoubtedly a challenging task. The solution of the problem is technically promising, it is also particularly attractive from an economic point of view, and can contain a significant innovative component.

Analysis of recent research

An overview of publications on this topic proves its relevance and highlights existing problems. Thus, the paper [1] emphasizes the efficiency of using the electromagnetic mill in comparison with a traditional ball mill, and [2] shows positive experience of using the electromagnetic mill for the disposal of bottom slag. The technical features of grinding brown coal for subsequent combustion in boilers with a boiling layer are considered in [3].

Existing approaches to the analysis of processes in the electromagnetic mill are used, in particular these include genetic algorithms and neural networks in order to take into account the hidden interconnections between energy consumption and the parameters of the process of grinding [4], which are based on experimental data obtained on existing prototypes of the electromagnetic mill. Also, statistical processing of the results of physical experiments [5] is used.

The authors [6-10] take into account the impact of the physical and technological features of the electromagnetic mill control system in relation to regulatory and control devices, such as air or fluid flow regulators, filters and/or separators based on a variety of operating principles, cooling systems, heat exchangers, recirculation flow of the substance for additional grinding, etc. Based on the results of mathematical modeling of such systems, conclusions on the effectiveness of the proposed electromagnetic mill design are presented.

A number of papers are devoted to the experimental research of the aforementioned equipment [1, 3, 11, 12]. They provide a valuable material for testing all sorts of theoretical approaches taken for the analysis of processes directly related to the grinding of substances. The problems of quality control of grinding [2], temperature monitoring [11] and others are investigated.

However, in the papers [4, 6, 7], which contain attempts to mathematically describe the processes of the electromagnetic mill, only the connection of the mill parameters with its efficiency is declared, but direct influence on the efficiency of the grinding process is not shown. The construction of the magnetic conductors of the electromagnetic mill inductors used in [3-11], in our opinion, is far from optimal in terms of their mass-dimensional indices. Moreover, lack of efficient cooling of the winding of the inductor in the proposed designs also significantly reduces the productivity of the mill.

The survey gives us the right to assert that to date no methods of the electromagnetic mill synthesis were found, which at the stage of making the design decisions would allow to predict its productivity depending on the design parameters, i.e. the dimensions of the inductor, working chamber and grinding media, the winding data, etc.

Statement of the task

The initial stage of the study requires the calculation of the field of forces of electromagnetic origin, acting on the ferromagnetic mill in the middle of the working chamber. The analysis of the obtained results will enable to find out the factors influencing the magnitude and direction of these forces, to select from among them the most weighty one and to choose the method of mathematical formalization of the description of force interaction in a rotating magnetic field of the mill.

The aim of the study. Detection of factors influencing the field of forces in the working chamber of an electromagnetic mill; determination of the dependencies of the projections of the equatorial force vector applied to the mill on the above-mentioned factors, in particular, on the size of the mill and its position in the working chamber; the choice of mathematical formalism for describing these dependencies.

The object of the study. Electromagnetic processes causing the force interaction of the grinding media in the electromagnetic mill working chamber.

The results and conclusions obtained during this study will be used in the future for the mathematical formulation of the problem of calculating the trajectories of the motion of a set of ferromagnetic grinding media in a rotating magnetic field of the electromagnetic mill. Creating such model, in our opinion, will allow to link the design parameters of this device with its performance.

Description of the design and operating principle

An electromagnetic mill will be called a device consisting of a magnetic field inductor 1, with its own cooling system 2 and working chamber 3 with ferromagnetic elements 4 (grinding media) (Fig.1). The electromagnetic mill inductor, in turn, consists of an inductor core 5, housing 6 and winding 7.

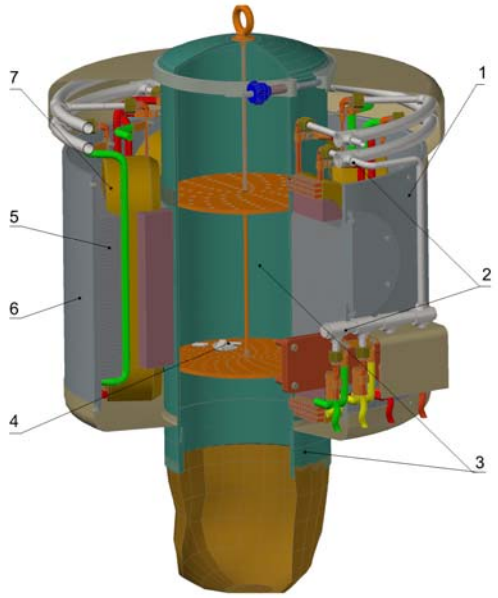

Fig.1. Design of the inductor together with the operating chamber of the electromagnetic mil

The inductor is connected to the industrial three-phase AC network with the frequency of 50 Hz and creates rotating magnetic field in the working chamber. This leads to the emergence of forces acting on the ferromagnetic mill and causing its movement.

The interaction of moving grinding media with the substance that is fed into the working chamber leads to the transformation of the electric energy consumed by the inductor from the network into the mechanical energy of fluctuations and blows of the grinding media.

The electromagnetic mill inductor prototype (Fig.1) is equipped with an autonomous cooling system, a capacitor battery to compensate for reactive power consumption and a control system that contains starting equipment and equipment for protection and control.

Research methods

The calculation of the forces of electromagnetic origin is based on the FEM analysis of the magnetic field, taking into account the most important factors influencing their magnitude, in particular, the saturation of the ferromagnetic elements of the magnetic core. The algorithm is based on the quasi-stationary approximation of the magnetic field of a mill in a two-dimensional domain.

The approach we have used is based on the method of calculating the force by means of the so-called Maxwell stress tensor [13, 14]. The force in it acting on any part of the volume V, limited by the surface S, can be obtained by summing the elementary forces $d\vec{F} = F_n d\vec{S}$, applied to the surface elements $d\vec{S}$ with S, respectively

(1) $$\vec{F} = \int_S F_n d\vec{S}$$

where F_n is the force of tension, acting from the outside on the elementary surface, the outer normal to which is directed along $\vec{n}$.

In a two-dimensional form, the vector of the resulting force is calculated as

(2) $$\vec{F} = \frac{1}{\mu_0}\oint_l \boldsymbol{T_m}\vec{n}dl$$

where $\boldsymbol{T_m} = \begin{Vmatrix} \frac{1}{2}(B_x^2 - B_y^2) & B_x B_y \\ B_x B_y & \frac{1}{2}(-B_x^2 + B_y^2) \end{Vmatrix}$ – is the Maxwell stress tensor for an isotropic medium; B_x, B_y – are projections of the magnetic induction vector on the surface S; $\overline{n}$ – is the normal to this surface.

The quantitative estimation of the calculation results the field of the electromagnetic force vector F_m acting on a separate grinding media in the working chamber of the electromagnetic mill is carried out using the mathematical model of the electromagnetic mill inductor with a working chamber of 6300 cm^3 in volume. Its main dimensions are as follows: the estimated length of the core is 200 mm, the diameter of the working chamber is 200 mm. The thickness of the wall of the working chamber is 4 mm, the number of turns in the phase is 8. The core is made of electrical steel M600-50A, and the grinding media is made of carbon steel A283/C. The average value of magnetic flux density in the working chamber is 0.153 T. The cross-section and the discretization mesh are shown in Fig.2.

One of the obvious factors impacting the force F_m is the presence of an ‘environment’ from other grinding media. In all the further studies described in this paper devoted to the grinding media for which the force F_m will be determined, there are 8 other types of grinding media of the same size as the investigated one (Fig.2). In our opinion, such method will increase the adequacy of the results.

We denote the angular position of the grinding media in relation to the center of symmetry of the cross section of the working chamber having a radius R_k is taken as γ_m; the radial position of the grinding media relative to this center is r_m; the angle of turn of the grinding media around its own axis is α_m, the size of the grinding media is d_m and l_m (Fig.3).

The vector of the resulting force acting on a real grinding media will obviously depend on the mutual orientation of the magnetic field density vectors in the vicinity of this grinding media. For the mathematical formalization of this dependence, the magnitude for the phase of the resulting magnetomotive force S_Σ of the three-phase winding of the inductor φ is introduced as the argument of the above-mentioned dependence. Let’s determine φ through the phase of current of the ’first’ phase winding ϕ. In the steady mode of operation of the mill inductor

(3) $$\varphi = \operatorname{atan}\left(\frac{Sy_{\Sigma}}{Sx_{\Sigma}}\right)$$

where

(4) $Sx_{\Sigma} = \sin(\omega t)\cos(\phi) + \cos(\omega t)\sin(\phi)$;

$Sy_{\Sigma} = \sin(\omega t)\sin(\phi) - \cos(\omega t)\cos(\phi)$ – projections of the vector of the resulting magnetomotive force vector.

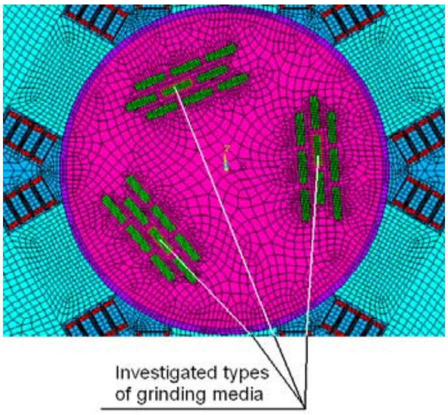

Fig.2. Location of the investigated grinding media and 'environment'

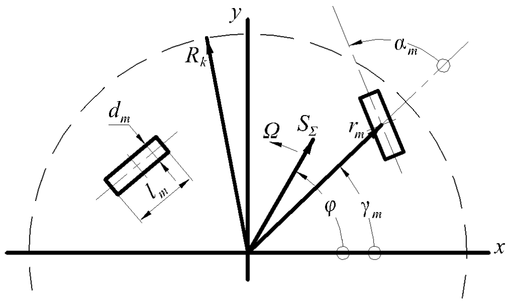

Fig.3. Coordinates of the location of the grinding media and their sizes

At this stage of our research, we have manufactured a model of an electromagnetic mill inductor with an autonomous forced winding cooling system (Fig.4). Its main dimensions and winding data are as follows: the length of the core is 200 mm, the diameter of the working chamber is 120 mm, the number of turns in the phase is 80, the number of pairs of poles is 2. The core is made of sheets of amorphous alloy VITROPERM 500F, and the grinding media is made of carbon steel A283/C.

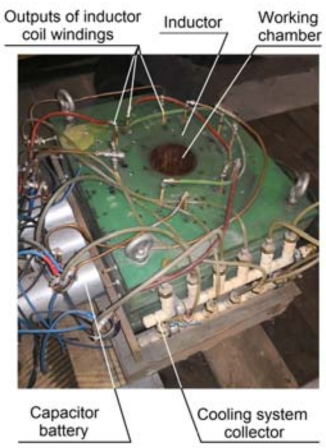

Fig.4. Electromagnetic mill inductor prototype (photo provided by MEGATECH Zbigniew Gałuszkiewicz)

Results

Using the aforementioned model, the hodographs of the resulting forces F_m acting on the grinding media are arranged in different positions within the working chamber $F_m(\varphi,\gamma_m)$. The coordinate γ_m takes values of 0, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156, 168 [º]; the radius r_m for all grinding media is 83 mm and the angle of rotation is α_m=90º. Dimensions of the grinding media are d_m=2.5 mm and l_m=10 mm. Hodographs of some of the forces F_m are depicted in Fig.5 in the polar coordinate system. Its horizontal axis is responsible for the magnitude of force in [N], and the circular axis is the angular position of the grinding media γ_m. The magnetomotive force phase φ varies in the range of 0 ÷ 180 °.

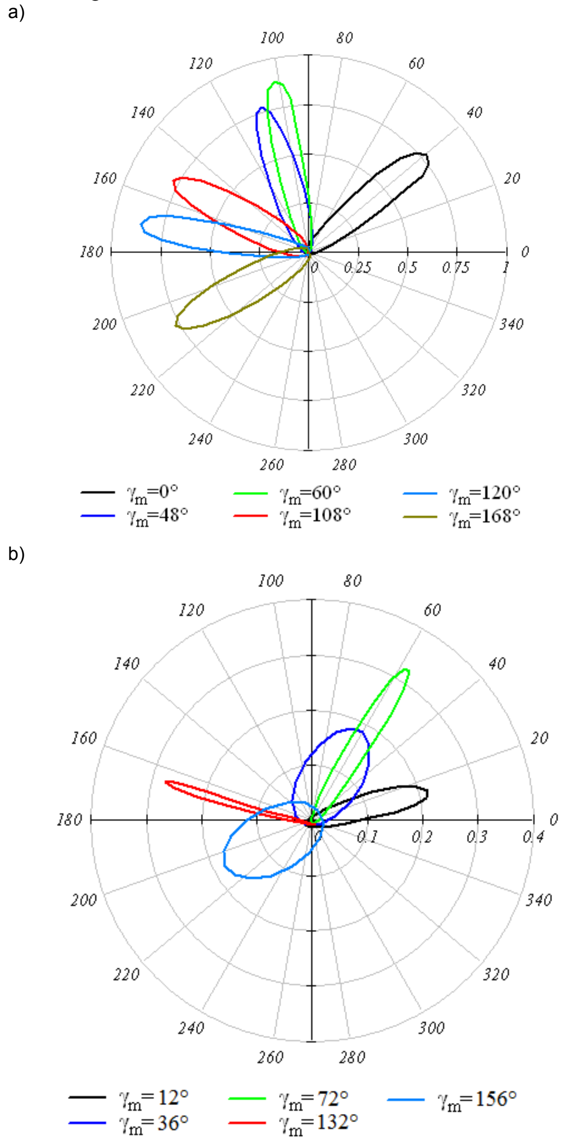

Fig.5. Hodographs of force vectors F_m at different angular positions of grinding media γ_m

In order to align the directions of F_m with the location of their application, a vector diagram of forces is constructed (Fig.6). Please note that it only shows the largest vectors F_m in the module.

Analysis of the calculation data confirms the theoretical conclusions on the periodic nature of the dependencies $F_m(\varphi)$ and $F_m(\gamma_m)$. The period of the first one is 180° (as an electric angle), and the period of the second one is 360°/p (p is the number of pole pairs of the inductor). It is also established that the radial component of force, regardless of the angle of the grinding media position, exceeds the tangential value by 0.8 ÷ 4.0 times. This will obviously result in the accumulation of grinding media in the vicinity of the inner surface of the working chamber.

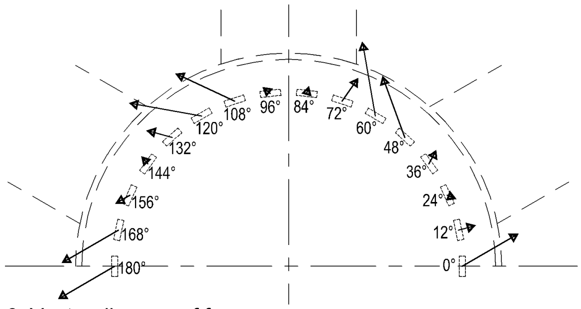

Fig.6. Vector diagram of forces F_m

The field of forces of electromagnetic origin, where there is grinding media, causes its movement, on the one hand, around the center of the working chamber, and, on the other hand, around its own center of mass, that is, in addition, the rotating moment acts on each grinding media relative to its center. As our studies have shown, its size is relatively small, and the direction is always such that it tries to orient the longitudinal axis of the grinding media along the lines of magnetic field.

a)

b)

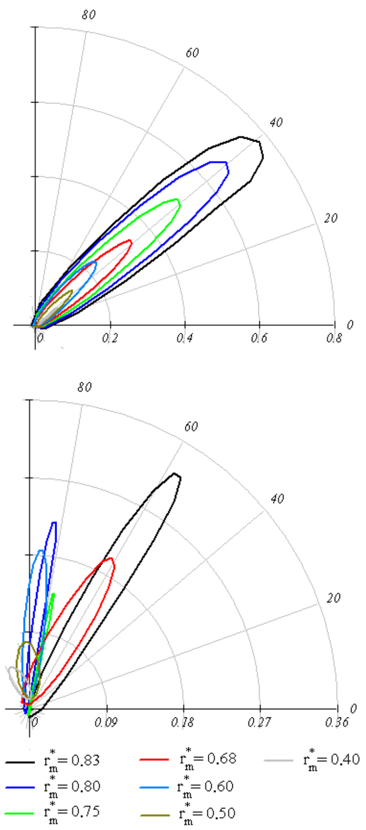

Fig.7. Hodographs of the force vectors F_m depending on the coordinates r_m: a) γ_m = 0°; b) γ_m = 108°

The next factor that definitely affects the value of F_m is the radial coordinate of the grinding media r_m. Calculations of F_m for changes of r_m in the range from 0.4R_k to 0.83R_k were carried out. Figure 7 shows the homographs of these forces for grinding media with angular coordinates γ_m = 0°, 108°, respectively. Their radial coordinates r_m change discretely, acquiring values of 0.40; 0.50; 0.60; 0.68; 0.75; 0.80; 0.83 relative unit. Note that $r^*_m = r_m / R_k$.

The parameter α_m in all cases is 90°. The graphs show that for the grinding media located near the magnetic concentrators, the change of F_m vs. r^*_m occurs almost linearly, with the direction of force almost unchanged.

However, in front of the teeth, as well as for increased distance from the wall of the working chamber the linear connection is violated. This kind of dependence $F_m(r_m)$ allows to argue that the analytical methods of its description, for the purpose of further use in solving the problems of the dynamics of movement of the grinding media in a rotating magnetic field are practically not realized. Instead, its solution is possible by methods of the theory of invariant approximation of functions.

Next we consider the impact of the angle of rotation of the grinding media α_m on its own axis. Figure 8 shows that not only the magnitude, but also the direction of force F_m depends on this angle. Dependence is investigated for a steady radial position r^*_m = 0.75. So, for grinding media γ_m= 0° and 108° the change of F_m direction is about ± 30° for α_m = -90° ÷ +90°. For other positions, for example, γ_m = 36° and 72°, the direction change takes place even in a wider range, but for lower amplitude values of F_m (<0.3F_{mmax}).

a)

b)

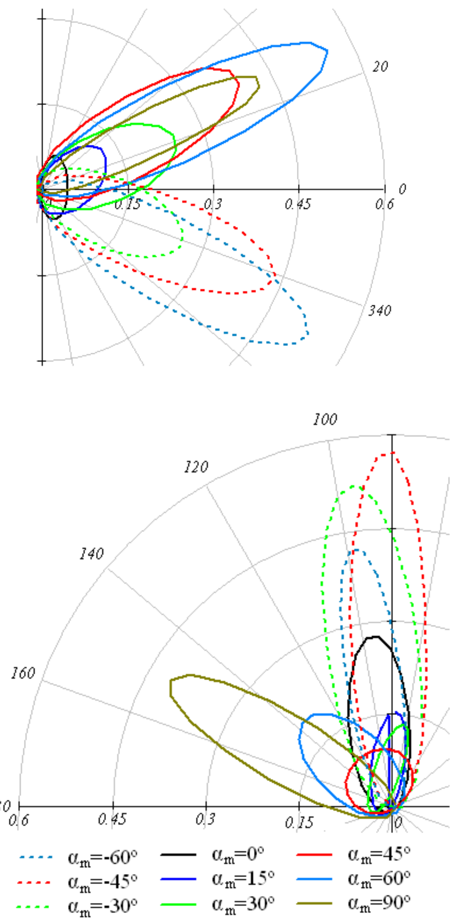

Fig.8. Hodographs of the force vectors F_m depending on the coordinates α_m: a) γ_m = 0°; b) γ_m = 108°

It is also noticeable that the change in amplitude of $F_m(\alpha_m)$ is subordinated to the harmonic law with a period of 180°.

This type of dependence $F_m(\alpha_m)$ is explained by the change in the magnetic conductivity of local zones in the working chamber due to the redistribution of the magnetic flux closing through the grinding media. In these cases, they also play the role of magnetic field concentrators.

The dependence $F_m(\alpha_m)$ is investigated for the constant size of grinding media. The ratio l_m/d_m is equal to 4. Clearly, reduction of this ratio will reduce the effect of α_m on the force F_m.

Finally, let us consider the direct impact of the size of the grinding media. Increasing the size while simultaneously maintaining a constant value l_m/d_m leads to narrowing the range of change in the direction of force α_{Fm}, depending on the phase of the resulting magnetomotive force of the inductor $F_m(\varphi)$ (Fig.9). Also, for certain values of γ_m, with an increase in d_m, the direction of force (its average value) shifts somewhat vs. the direction of the rotational field (Fig.9b).

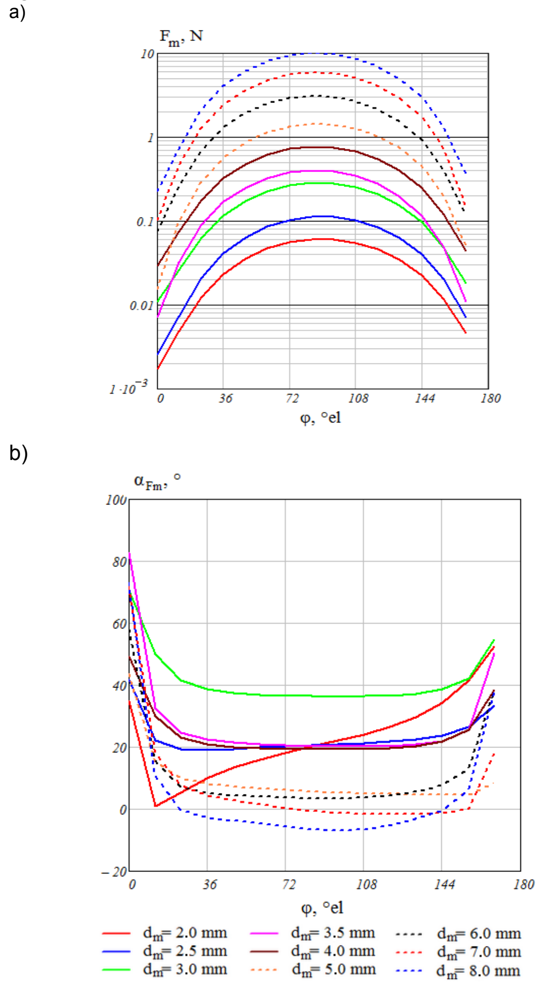

Fig.9. Dependence of the module F_m (a) and the direction α_{Fm} (b) on the phase of the magnetomotive force φ for different diameters of grinding media d_m, (ratio l_m/d_m=4 and γ_m = 12°)

In addition, not only the amplitude of force, but also the relation between its averaged value and gravity of the grinding media increases.

So for a grinding media with a diameter of 2 mm and l_m/d_m=4 for γ_m= 12°, α_m= 82° and r_m^* = 0.6, this ratio is ≈32, and for d_m=8 mm (all other indicators remain the same), it grows to ≈83.

Increasing the length of the grinding media with a constant diameter also leads to an increase in the amplitude of the force, but, unlike the effect of the change in the diameter, this dependence is characterized by strong nonlinearity (the increase in the amplitude of the force as the length l_m increases) (Fig.10). The change in the direction of force, in certain positions occurs stiffly, which is explained by the local re-magnetization of the toothed structure of the inductor.

Comparison of dependencies in Fig.10 (d_m=2 mm and d_m=4 mm) indicates a practically quadratic dependence of the force module F_m on the diameter of the grinding media. This type of connection is observed in the range of change in d_m from 1.0 till 6 mm and is explained based on the basic provisions on the connection between the force of electro-magnetic origin, the magnetic flux and the area of the axial section of the grinding media.

The superficial analysis of dependence of the force F_m on the size of grinding media indicates the absence of extremums in these functions. This means that the formulation of optimization problems to find the optimal ratio of dimensions l_m/d_m, taking into account only the field F_m and not taking into account the mechanical interaction of the grinding media with each other and with the material to be ground (touch, impact, slip), does not make sense.

Summarizing the conducted analysis of the field of electromagnetic forces acting on ferromagnetic elements in a rotating magnetic field of the working chamber of the electromagnetic mill, it can be argued that the analytical description of the impact of all factors considered here in their interconnection is impossible, owing to the non-linear nature of this interaction. The formalization of consideration of all the above-mentioned impacts on the dynamic behavior of the set of grinding medias is possible, subject to the use of the mathematical apparatus of the theory of interpolation for vector functions of the vector argument. Also, in our opinion, the formulation of such a task will have to be simplified by reducing the number of independent variables of the function F_m.

Conclusions

1. To date, the dependence between the main dimensions of the electromagnetic mill with electromagnetic loads and performance is not established. At the initial stage of designing, we propose to implement it on the basis of the analysis of the factors of impact on the field of electromagnetic forces in the working chamber of the electromagnetic mill.
2. The tangential component of the force acting on the grinding media, on average, is 0.3÷0.4 from the radial component. This fact indirectly indicates that an essential part of the forces acting against the movement of the grinding media will be the friction forces with the walls of the working chamber and the grinding media with each other. The thickness of the material to be ground is less than the density of the grinding material, which will lead to the accumulation of this substance in the center of the working chamber and the grinding media on its periphery.
3. Quantitative and qualitative analysis of the dependencies of the force acting on the grinding media from the phase of the resulting magnetomotive force of the inductor, the angular and radial coordinates of the location of the grinding media in the working chamber, its angle of rotation around its own axis and the size of the grinding media allow to establish the following:
- the time phase of the resulting magnetomotive force of the inductor changes the magnitude and direction of the aforementioned force, according to the periodic law with a period of 180º;
- the dependence of force on the angular position of the grinding media is periodic with the period 360°/p. The tooth harmonics of the magnetic field substantially (quantitatively) affect this dependence;
- the range of variation of the force module from the radial position of the grinding media is 5÷10, depending on its angular positions. This forces to take into account this factor as one of the main factors, in mathematical formalization of the distribution of the mentioned force;
- the dependence of force on the turning angle of the grinding media on its own axis is of periodic nature with a period of 180º. By reducing the value of the ratio l_m/d_m, this effect becomes relatively insignificant;

- the dependence of force on the size of the grinding media (length and diameter) is not smooth functions and may have ordinary discontinuities. In addition, there is a significant influence of the presence and the relative position of the 'environment' grinding media. This forces to remove these two design parameters from the list of independent variables of the task and assume that the size of the grinding media is constant.

4. The complex nature of the dependence of the electromagnetic force on the factors listed in item 3 practically excludes the possibility of analytical approaches in solving the problem of describing the dynamic behavior of a set of grinding media in a rotating magnetic field. Instead, the need to apply the theory of interpolation becomes apparent.

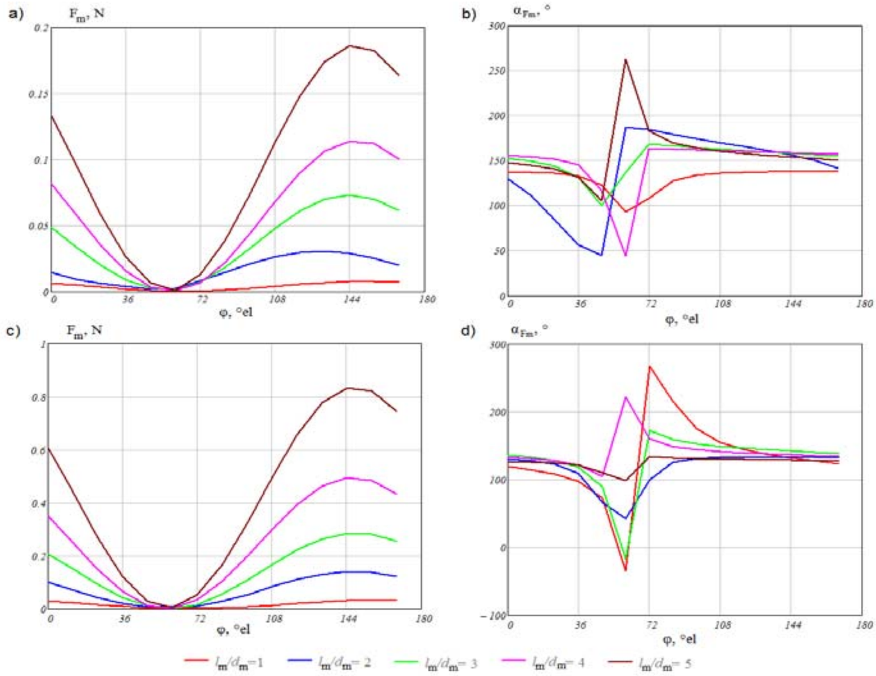

Fig.10. Dependence of F_m (a, c) and the direction α_{Fm} (b,d) on the phase of magnetomotive force φ, for a stable position γ_m = 120°, their diameters d_m = 2 mm (a, b) and d_m =4 mm (c, d) and different ratios of sizes of grinding media l_m/d_m

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