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Mathematical modelling of transient electromagnetic processes in a power grid

***Abstract**. A mathematical model of an open electric power grid including the key elements of three distributed parameter transmission supply lines is developed on the basis of an interdisciplinary method of mathematical modeling. Such an approach allows for application of a highly adequate theory of ordinary and partial derivative differential equations that addresses electromagnetic processes in the grid under analysis. Transient electromagnetic processes in the short-circuited state are analysed.*

Streszczenie.** W artykule na podstawie interdyscyplinarnej metody modelowania matematycznego opracowano model matematyczny otwartej sieci elektrycznej, kluczowymi elementami której są trzy długie linie zasilania o parametrach rozłożonych. Takie podejście daje możliwość zastosowania teorii równań różniczkowych o pochodnych zwyczajnych i cząstkowych z wysokim poziomem adekwatności uwzględniającą procesy elektromagnetyczne w analizowanej sieci. Przeanalizowano nieustalone procesy elektromagnetyczne w stanie zwarcia. (**Modelowanie matematyczne nieustalonych procesów elektromagnetycznych w sieci elektroenergetycznej).

Keywords: mathematical modelling, transmission supply line equations, Hamilton-Ostrogradski principle, electrical grid, distributed parameters, boundary conditions.
Słowa kluczowe: modelowanie matematyczne, równania linii zasilania, zasada Hamiltona-Ostrogradskiego, sieć elektroenergetyczna, parametry rozłożone, warunki brzegowe.

Introduction

Connection of local power subsystems into a single power system is the key objective of a high-voltage supply line. Design of new and operation of existing lines are impossible without comprehensive analysis of steady and transient operating conditions of a system [1] as ignoring complex processes in grids may lead to unstable operation of a power system.

Considering that supply lines are designed for specific lengths which are commensurate with electromagnetic wavelengths, supply lines are treated as distributed parameter systems, which allows for application of field approaches to development of power grid models. Ordinary and partial derivative differential equations are employed as a result. The fact not only initial but also boundary conditions must be determined is the most difficult part of solving such equations, a highly complicated task from both the power engineering and applied mathematics viewpoint. Addressing boundary conditions using the apparatus of applied mathematics, including second and third-type Neuman and Poincare conditions, is proposed here.

Analysis of recent research

A great number of research papers are devoted to analysis of steady and transient processes in power systems with transmission supply lines as their chief elements. There is also a broad range of complex software like *MatLab, EMTP-RV,* and others [2 – 6] which serve to analyse a variety of power system operating states. Each software has its advantages and drawbacks. Application of the principle of least action, on the other hand, is a fully physical approach, which does not require any formal assumptions.

The following **study objective** can be defined in light of the foregoing analysis: create a new method for modelling of complicated power grids consisting of high-voltage transmission supply lines while addressing additional electric apparatus present in a system.

System description

A calculated circuit of an open electricity grid under analysis that consists of four power sub-systems is shown in Figure 1. These subsystems comprise electromotive forces SEM, effective resistances, and inductances that are connected with one another with transmission supply lines. Air-core coils are used in the grids to compensate for reactive power, presented in the circuit as branches of effective resistance and inductances in series. A three-phase open electrical grid analysed here will be a single line, since only symmetrical operating states of the power system are assumed.

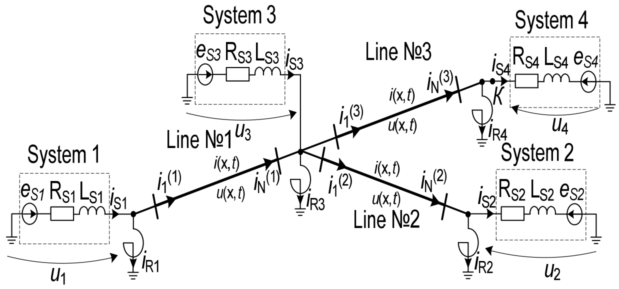

Fig.1. A calculated equivalent circuit of the studied part of an open electrical grid

Our team will traditionally apply variational approaches [7 – 11] to build a mathematical model of the electric grid shown in Figure 1, namely: an interdisciplinary variational method based on a modification to the famous Hamilton-Ostrogradski integral variational principle, described in [7].

The extended functionality of the mathematical operation for the studied system according to Hamilton-Ostrogradski will be similar to that presented in [7]:

$$S = \int_0^{t_1} L^* dt + \int_0^{t_1}\int_l L_l \, dl \, dt, \quad I = \int_l L_l \, dl \,, \tag{1}$$

where: S – action according to Hamilton-Ostrogradski, L^* – extended Lagrange function, L_l – linear density of the modified Lagrange function, I – energy functional.

The extended functionality of the action for the studied system according to Hamilton-Ostrogradski will be as follows [7]:

(2) $$L^* = \tilde{T}^* - P^* + \Phi^* - D^*, \qquad L_l = \tilde{T}_l - P_l + \Phi_l - D_l ,$$

where: $\tilde{T}^*$ – kinetic coenergy, P^* – potential energy, Φ^* – energy dissipation, D^* – energy of outside nonpotential forces, with index l for the corresponding linear densities of energies.

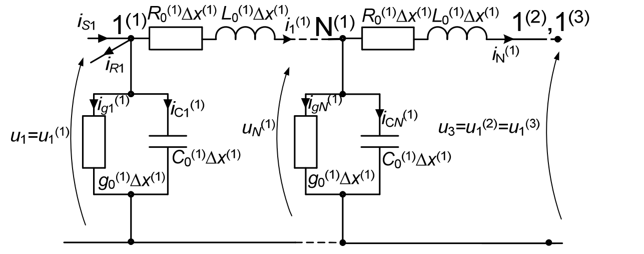

Fig.2. Equivalent circuit of elements of line No. 1 in the electric grid analysed

Let us write down elements of the extended Lagrange function [7], [9]

(3) $$T^* = \sum_{m=1}^{4}\left(\frac{L_{S,m}i_{S,m}^2}{2}+\frac{L_{R,m}i_{R,m}^2}{2}\right);$$

(4) $$\Phi^* = \frac{1}{2}\int_0^t \sum_{m=1}^{4}\left(R_{S,m}i_{S,m}^2 + R_{R,m}i_{R,m}^2\right)d\tau , \; D^* = \int_0^t \sum_{m=1}^{4}\left(e_{S,m}i_{S,m}\right)d\tau ;$$

(5) $$\frac{\partial T^{\langle k\rangle}}{\partial x} \equiv T_l^{\langle k\rangle} = \frac{L_0^{\langle k\rangle}Q_t^{2\langle k\rangle}(x,t)}{2}, \; \frac{\partial P^{\langle k\rangle}}{\partial x} \equiv P_l^{\langle k\rangle} = \frac{1}{2C_0^{\langle k\rangle}}Q_x^{2\langle k\rangle}(x,t);$$

(6) $$\frac{\partial \Phi^{\langle k\rangle}}{\partial x} \equiv \Phi_l^{\langle k\rangle} = \Phi_{l3}^{\langle k\rangle} - \Phi_{lB}^{\langle k\rangle} =$$

$$= \int_0^t \left(\frac{R_0^{\langle k\rangle}}{2}Q_t^{2\langle k\rangle}(x,t) - \frac{g_0^{\langle k\rangle}}{2C_0^{2\langle k\rangle}}Q_x^{2\langle k\rangle}(x,t)\right)\Bigg|_{t=\tau} d\tau , \; k = 1,\,2,\,3 ,$$

where: m = 1, 2, 3, 4 – numbers of the electric grid and the air-core coil; $L_{S,m}$ – inductance of the m^{th} power system; $L_{R,m}$ – inductance of the m^{th} air-core coil; $R_{S,m}$ – effective resistance of the m^{th} electric grid; $R_{R,m}$ – effective resistance of the m^{th} air-core coil; $e_{S,m}$ – electromotive force of the m^{th} electric grid; $i_{S,m}$ – current of the m^{th} electric grid; $i_{R,m}$ – current of the m^{th} air-core coil; $i(x, t)$ – current in the line; R_0, g_0, C_0, L_0 – distributed parameters of the line; Φ_{l3} – linear density of external energy dissipation; Φ_{lB} – linear density of internal energy dissipation; $Q(x, t)$ – the line charge; k – the line number.

The method of similar equations is presented in [9], [10], [11], for example. Therefore, for space considerations, we only suggest finished equations of the electromagnetic state of the object, Fig. 1.

(7) $$\frac{\partial v^{\langle k\rangle}}{\partial t} = \left(C_0^{\langle k\rangle}L_0^{\langle k\rangle}\right)^{-1}\left(\frac{\partial^2 u^{\langle k\rangle}}{\partial x^2} - \left(g_0^{\langle k\rangle}L_0^{\langle k\rangle} + \right.\right.$$

$$\left.\left. + C_0^{\langle k\rangle}R_0^{\langle k\rangle}\right)v^{\langle k\rangle} - g_0^{\langle k\rangle}R_0^{\langle k\rangle}u^{\langle k\rangle}\right), \quad \frac{\partial u^{\langle k\rangle}}{\partial t} = v^{\langle k\rangle};$$

(8) $$\frac{di_{S1}}{dt} = \frac{1}{L_{S1}}\left(e_{S1} - R_{S1}i_{S1} - u_1\right), \; \frac{di_{S2}}{dt} = \frac{1}{L_{S2}}\left(u_2 - R_{S2}i_{S2} - e_{S2}\right);$$

(9) $$\frac{di_{S3}}{dt} = \frac{1}{L_{S3}}\left(e_{S3} - R_{S3}i_{S3} - u_3\right), \; \frac{di_{S4}}{dt} = \frac{1}{L_{S4}}\left(u_4 - R_{S4}i_{S4} - e_{S4}\right);$$

(10) $$\frac{di_{R,m}}{dt} = \frac{1}{L_{R,m}}\left(u_m - R_{R,m}i_{R,m}\right), \quad m = 1, 2, 3, 4,$$

where: u_1, u_2, u_3, u_4 – substation busbar voltage of the power systems 1, 2, 3 and 4, respectively.

Kirchhoff's Second Law for electrical circuits with distributed parameters is used as boundary conditions for (7) [9]:

(11) $$-\frac{\partial u^{\langle k\rangle}}{\partial x} = R_0^{\langle k\rangle}i^{\langle k\rangle} + L_0^{\langle k\rangle}\frac{\partial i^{\langle k\rangle}}{\partial t}.$$

We will write down (7) and (11) in discrete space for the j-node of the line (using the concept of the central derivative):

(12) $$\frac{dv_j^{\langle k\rangle}}{dt} = \left(C_0^{\langle k\rangle}L_0^{\langle k\rangle}\right)^{-1}\left(\frac{u_{j-1}^{\langle k\rangle} - 2u_j^{\langle k\rangle} + u_{j+1}^{\langle k\rangle}}{\left(\Delta x^{\langle k\rangle}\right)^2} - \left(g_0^{\langle k\rangle}L_0^{\langle k\rangle} + \right.\right.$$

$$\left.\left. + C_0^{\langle k\rangle}R_0^{\langle k\rangle}\right)v_j^{\langle k\rangle} - g_0^{\langle k\rangle}R_0^{\langle k\rangle}u_j^{\langle k\rangle}\right), \; u_1^{\langle k\rangle} = u(x,t)\big|_{x=0} ,$$

$$u_N^{\langle k\rangle} = u(x,t)\big|_{x=l} , \; u_1^{\langle 1\rangle} \equiv u_1 , \; u_1^{\langle 2\rangle} \equiv u_1^{\langle 3\rangle} \equiv u_3 , \; u_N^{\langle 1\rangle} \neq u_3 ,$$

$$u_N^{\langle 2\rangle} \neq u_2 , \; u_N^{\langle 3\rangle} \neq u_4 ;$$

(13) $$\frac{di_j^{\langle k\rangle}}{dt} = \frac{1}{L_0^{\langle k\rangle}}\left(\frac{u_{j-1}^{\langle k\rangle} - u_{j+1}^{\langle k\rangle}}{2\Delta x^{\langle k\rangle}} - \frac{R_0^{\langle k\rangle}}{L_0^{\langle k\rangle}}i_j^{\langle k\rangle}\right);$$

(14) $$\frac{du_j^{\langle k\rangle}}{dt} = v_j^{\langle k\rangle}, \quad j = 1,\ldots,N , \; k = 1,\,2,\,3 .$$

Having analysed (12) and (13), it can be seen that in order to find voltage and current on the first and the last nodes of discretization, it is necessary to find the unknown voltages in the fictitious nodes $u_0^{\langle k\rangle}$ and $u_{N+1}^{\langle k\rangle}$. Let's search for the voltage.

Let's note the equation of scleronomic constraint (see Fig. 2):

(15) $$i_{S1} - i_1^{\langle 1\rangle} - i_{g1}^{\langle 1\rangle} - i_{C1}^{\langle 1\rangle} - i_{R1} = 0 , \quad i_{g1}^{\langle 1\rangle} = \Delta x^{\langle 1\rangle}g_0^{\langle 1\rangle}u_1^{\langle 1\rangle};$$

(16) $$i_{C1}^{\langle 1\rangle} = \Delta x^{\langle 1\rangle}C_0^{\langle 1\rangle}\frac{du_1^{\langle 1\rangle}}{dt} = \Delta x^{\langle 1\rangle}C_0^{\langle 1\rangle}v_1^{\langle 1\rangle}.$$

The next step is to differentiate (15) and (16) with regard to time:

(17) $$\frac{di_{S1}}{dt} - \frac{di_1^{\langle 1\rangle}}{dt} - \frac{di_{g1}^{\langle 1\rangle}}{dt} - \frac{di_{C1}^{\langle 1\rangle}}{dt} - \frac{di_{R1}}{dt} = 0 , \; \frac{di_{g1}^{\langle 1\rangle}}{dt} = \Delta x^{\langle 1\rangle}g_0^{\langle 1\rangle}v_1^{\langle 1\rangle};$$

(18) $$\frac{di_{C1}^{\langle 1\rangle}}{dt} = \Delta x^{\langle 1\rangle}C_0^{\langle 1\rangle}\frac{dv_1^{\langle 1\rangle}}{dt}.$$

Now, based on (13) and Fig. 2, we will write:

(19) $$\frac{di_1^{\langle 1\rangle}}{dt} = \frac{1}{L_0^{\langle 1\rangle}}\left(\frac{u_0^{\langle 1\rangle} - u_2^{\langle 1\rangle}}{2\Delta x^{\langle 1\rangle}} - R_0^{\langle 1\rangle}i_1^{\langle 1\rangle}\right);$$

(20) $$\frac{di_N^{\langle 1\rangle}}{dt} = \frac{1}{L_0^{\langle 1\rangle}}\left(\frac{u_{N-1}^{\langle 1\rangle} - u_{N+1}^{\langle 1\rangle}}{2\Delta x^{\langle 1\rangle}} - R_0^{\langle 1\rangle}i_N^{\langle 1\rangle}\right);$$

(21) $$\frac{di_N^{\langle 1\rangle}}{dt} = \frac{1}{L_0^{\langle 1\rangle}\Delta x^{\langle 1\rangle}}\left(u_N^{\langle 1\rangle} - R_0^{\langle 1\rangle}\Delta x^{\langle 1\rangle}i_N^{\langle 1\rangle} - u_1^{\langle 3\rangle}\right).$$

Substituting the first equation of (8), the equation (19), the second equation of (17), the equations (18) and (10) to the first equation of (17), for m = 1, will eventually produce the final voltage of a fictitious discretisation node of the first line, $u_0^{\langle 1\rangle}$:

$$(22)\quad u_0^{\langle 1\rangle} = \frac{2\Delta x^{\langle 1\rangle} L_0^{\langle 1\rangle}}{3}\left[\frac{1}{L_{S1}}\left(e_{S1} - R_{S1} i_{S1} - u_1^{\langle 1\rangle}\right) - \frac{1}{L_{R1}}\left(u_1 - R_{R1} i_{R1}\right) + \left(\frac{\Delta x^{\langle 1\rangle} g_0^{\langle 1\rangle} R_0^{\langle 1\rangle}}{L_0^{\langle 1\rangle}} + \frac{2}{L_0^{\langle 1\rangle}\Delta x^{\langle 1\rangle}}\right)u_1^{\langle 1\rangle} - \frac{1}{2L_0^{\langle 1\rangle}\Delta x^{\langle 1\rangle}}u_2^{\langle 1\rangle} - \left(\Delta x^{\langle 1\rangle} g_0^{\langle 1\rangle} - \frac{\Delta x^{\langle 1\rangle}\left(g_0^{\langle 1\rangle}L_0^{\langle 1\rangle} + C_0^{\langle 1\rangle}R_0^{\langle 1\rangle}\right)}{L_0^{\langle 1\rangle}}\right)v_1^{\langle 1\rangle} + \frac{R_0^{\langle 1\rangle}}{L_0^{\langle 1\rangle}} i_1^{\langle 1\rangle}\right].$$

By equating (20) and (21), we proceed to find voltage functions of the second fictitious discretisation node of the first line, $u_{N+1}^{\langle 1\rangle}$:

$$(23)\quad u_{N+1}^{\langle 1\rangle} = -2\left(u_N^{\langle 1\rangle} - u_1^{\langle 3\rangle}\right) + u_{N-1}^{\langle 1\rangle}.$$

Voltages across fictitious discretisation nodes of another two lines (Fig. 1), $u_0^{\langle 2\rangle}$, $u_{N+1}^{\langle 2\rangle}$ and $u_0^{\langle 3\rangle}$, $u_{N+1}^{\langle 3\rangle}$), are found like for the line No. 1. Due to space considerations, sample voltage functions for the line No. 2 $u_0^{\langle 2\rangle}$, $u_{N+1}^{\langle 2\rangle}$ are only presented:

$$(24)\quad u_0^{\langle 2\rangle} = \frac{2\Delta x^{\langle 2\rangle} L_0^{\langle 2\rangle}}{3}\left[\frac{\left(u_N^{\langle 1\rangle} - R_0^{\langle 1\rangle}\Delta x^{\langle 1\rangle} i_N^{\langle 1\rangle} - u_1^{\langle 2\rangle}\right)}{L_0^{\langle 1\rangle}\Delta x^{\langle 1\rangle}} + \frac{1}{L_{S3}}\left(e_{S3} - R_{S3} i_{S3} - u_1^{\langle 2\rangle}\right) - \frac{1}{L_{R3}}\left(u_3 - R_{R3} i_{R3}\right) - \frac{3}{2\Delta x^{\langle 3\rangle} L_0^{\langle 3\rangle}} - \sum_{n=2}^{3}\left\{\frac{\Delta x^{\langle n\rangle}}{L_0^{\langle n\rangle}}\left(\frac{u_2^{\langle n\rangle} - 2u_1^{\langle n\rangle}}{\left(\Delta x^{\langle n\rangle}\right)^2} - \left(g_0^{\langle n\rangle}L_0^{\langle n\rangle} + C_0^{\langle n\rangle}R_0^{\langle n\rangle}\right)v_1^{\langle n\rangle} - g_0^{\langle n\rangle}R_0^{\langle n\rangle}u_1^{\langle n\rangle}\right) + \Delta x^{\langle n\rangle} g_0^{\langle n\rangle} v_1^{\langle n\rangle} + \frac{1}{L_0^{\langle n\rangle}}\left(\frac{1}{2\Delta x^{\langle n\rangle}}u_2^{\langle n\rangle} + R_0^{\langle n\rangle} i_1^{\langle n\rangle}\right)\right\}\right],\ n = 2, 3;$$

$$(25)\quad u_{N+1}^{\langle 2\rangle} = \frac{2L_0^{\langle 2\rangle}\Delta x^{\langle 2\rangle} L_{R2} L_{S2}}{L_{R2}L_{S2} + L_0^{\langle 2\rangle}\Delta x^{\langle 2\rangle}\left(L_{S2} + L_{R2}\right)}\left[\frac{u_N^{\langle 2\rangle}}{L_0^{\langle 2\rangle}\Delta x^{\langle 2\rangle}} - \frac{R_0^{\langle 2\rangle}}{L_0^{\langle 2\rangle}} i_N^{\langle 2\rangle} + \frac{R_{R2}}{L_{R2}} i_{R2} + \frac{\left(R_{S2} i_{S2} + e_{S2}\right)}{L_{S2}}\right] + u_{N-1}^{\langle 2\rangle} - 2u_N^{\langle 2\rangle}.$$

The following system of differential equations is subject to joint integration: (8) – (10), (12) – (14) including (22) – (25).

Computer simulation findings

Analysis of transient electromagnetic processes in the electric grid in Figure 1 is computer simulated. A steady state is the initial condition, then a grid short-circuiting is modelled.

Analysis of transient processes begins at time t = 0 s. SEM values are: e_{S1} = 605 $\sin(\omega t + 5.8°)$ kV, e_{S2} = 592 $\sin(\omega t - 2.1°)$ kV, e_{S3} = 607 $\sin(\omega t + 6°)$ kV, e_{S4} = 635 $\sin(\omega t + 8.1°)$ kV. After a steady state is produced at t = 0.12 s, a short-circuiting at point K is produced – Figure 1.

Parameters that fully correspond to real parameters of a 750 kV grid are introduced as part of modeling of the power system's operation. For instance, line No. 1 connects the nuclear power plant in Rivnensk and Albertirsa switching station. Line No. 1 is 282 km long, line No. 2 – 476 km, and line No. 3 – 360.5 km long. Parameters of the calculated grid circuit in Figure 1: R_{S1} = 2.05 Ω, R_{S2} = 2.55 Ω, R_{S3} = 2.15 Ω, R_{S4} = 2.35 Ω, L_{S1} = 0.205 H, L_{S2} = 0.155 H, L_{S3} = 0.175 H, L_{S4} = 0.2 H, $R_{R1} = R_{R2} = R_{R3} = R_{R4}$ = 3.41 Ω, $L_{R1} = L_{R2} = L_{R3} = L_{R4}$ = 5.97 H. The parameters of three transmission supply lines with distributed parameters are as follows: $R_0 = 1.9\cdot10^{-5}$ Ω/m, $L_0 = 9.24\cdot10^{-7}$ H/m, $C_0 = 1.3166\cdot10^{-11}$ F/m, $g_0 = 3.25\cdot10^{-11}$ Sm/m.

The simulation results are shown in Figures 3 – 7.

Figures 3 and 4 illustrate spatial voltage (1) and current (2) waveforms across the supply line No. 2 at the instant t = 0.008 s and of the line No. 3 at t = 0.005 s. The Figures demonstrate the voltage and current waveforms are similar and their only differences arise from the different lengths of both the lines: line No. 2 is 115 km longer than line No. 3.

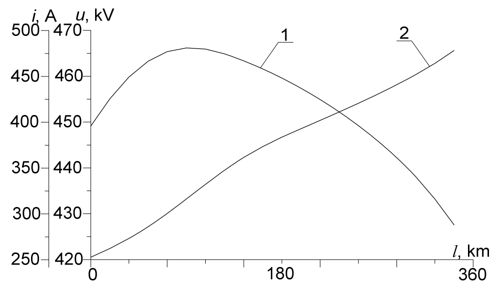

Fig.3. The spatial waveforms of voltage (1) and current (2) in the line No. 2 at the time t = 0.008 s.

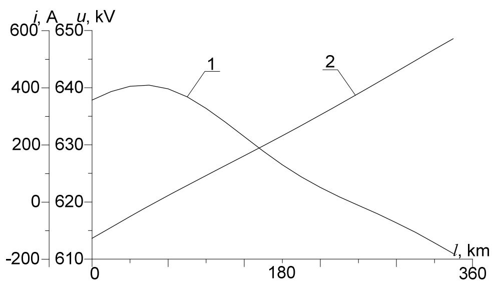

Fig.4. The spatial waveforms of voltage (1) and current (2) in the line No. 3 at the time t = 0.005 s.

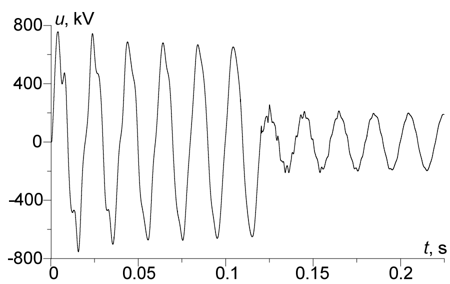

Fig.5. Voltage at the midpoint of line No. 3

Figures 5 and 6 present a voltage transient process at the midpoint of line No. 3 and a current transient process at the end of line No. 2. Both the Figures imply once the system is in a steady state, a temporary voltage amplitude is 652 kV (Fig. 5) and temporary current amplitude is 750 A (Fig. 6). Following a short-circuiting at point K, voltage in the midpoint of line No. 3 reduced to 200 kV (Fig. 5) while the surge current at the end of line No.2 reached 2 kA, whereas its amplitude in the steady state remained unchanged (Fig. 6).

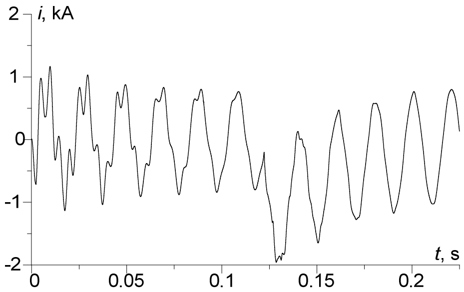

Fig.6. Current at the end of line No. 2

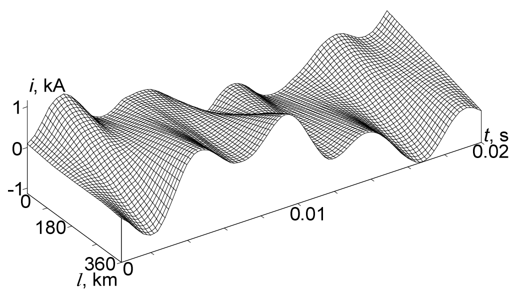

Fig.7. Temporal-spatial waveforms of current functions in the line No. 3 at the time t [0; 0,02] s

Figure 7 shows a temporal-spatial waveform of current in the line No. 3 as the power system is switched on until it reaches a steady state. We can see maximum current fluctuations at the start and end of the line and minimum in its midpoint. Physics of these processes is highly complicated and depends on wave processes across the line. Vanishing of current oscillation results from damping of the wave motion by the line cables.

Conclusion

1. Application of the interdisciplinary variation method based on a modification to the famous integral Hamilton-Ostrogradski principle to development of mathematical models of complicated power systems helps to avoid decomposition of a unified dynamic system. Transient electromagnetic processes in power grids are only analysed by means of the sole energetic approach.
2. Spatial waveforms of basic line parameters affirm physical principles of electrodynamics with regard to wave processes in transmission supply lines. Three-dimensional temporal-spatial current waveforms present information on wave processes in supply lines with high adequacy.
3. Computer simulation results in graphic format are of great use to designing and operation of complicated dynamic power systems including distributed parameter transmission supply lines.

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