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# The universal Mayr-Pentegov model of the electric arc

***Abstract.*** *The article presents a universal mathematical model of electric arc. Mayr and Pentegov's assumptions were used to create it. An analytical form of static voltage-current characteristics was set, which allows taking into account the residual conductance of the column in the low-current range and the effect of voltage rise in the high-current range. A column state equation was obtained which contains a non-linear damping function depending on the state current and on constant parameters. Families of static voltage-current characteristics with changes in parameters are presented. Simulations of circuit processes with a universal arc model and with sinusoidal excitation were performed.*

***Streszczenie.*** *W artykule przedstawiono uniwersalny model matematyczny łuku elektrycznego. Do jego utworzenia wykorzystano założenia Mayra i założenia Pentegowa. Zadano postać analityczną charakterystyki napięciowo-prądowej statycznej, która umożliwia uwzględnienie konduktancji resztkowej kolumny w zakresie słaboprądowym i efektu narastania napięcia w zakresie silnoprądowym. Uzyskano równanie stanu kolumny, które zawiera nieliniową funkcję tłumienia zależną od prądu stanu i od stałych parametrów. Przedstawiono rodziny charakterystyk napięciowo-prądowych statycznych z uwzględnieniem zmian parametrów. Wykonano symulacje procesów w obwodzie z uniwersalnym modelem łuku i z działającym wymuszeniem sinusoidalnym.* (***Uniwersalny model Mayra-Pentegowa łuku elektrycznego***).

**Keywords**: electric arc, mathematical model, Mayr model, Pentegov model.
**Słowa kluczowe**: łuk elektryczny, model matematyczny, model Mayra, model Pentegowa.

## Introduction

Changes in the arc current are accompanied by changes in the volume and distribution of temperature, plasma pressure and velocity in the arc column, as well as changes in the geometric size and temperature of the active areas on the electrode surfaces. The basic effect of the transformation of electricity is heat, although it coexists with other processes (mostly secondary), such as optical, acoustic, mechanical and chemical. Thermal characteristics of materials of a certain mass (e.g. coefficient of thermal conductivity, specific enthalpy, specific density [1]) clearly indicate the inertia of heating processes (changes in plasma temperature) in relation to changes in instantaneous current values. Usually conducted experimental studies [2] relate to determining the response (voltage) to the abruptly increasing current change. Depending on the operating methods of external factors (gas, liquid-water flows, electrode movements), heating processes may be slightly different from cooling processes. In addition, a momentary power decay may be due to a residual plasma conductance that facilitates the re-development of an arc.

In the Pentegov model [3], the condition of strict dependence of plasma enthalpy on the shape of the voltage-current static curve characteristic is assumed. Enthalpy and voltage integral are bound by a time constant. This assumption limits the effectiveness of the approximation of electrical processes used by the Pentegov model with possible different variants of physical changes in arc plasma parameters. The abandonment of this condition with the simultaneous use of the Mayr assumption may give results of simulations more convergent with real ones coming from experimental studies.

## Assumptions of the Mayr-Pentegov model of the electric arc

The proposed Mayr - Pentegov model uses the assumptions introduced by these two pioneers of arc modeling. According to Mayr, the change in thermal plasma enthalpy $Q$ affects the changes in conductance $g$ as follows

$$\exp\left(\frac{Q}{Q_p}\right) = \frac{g}{G_p} \tag{1}$$

In this relationship, the constant $Q_p$ - means the subtangent of the graph of the function $g(Q)$, and $G_p$ - the intersection of the graph with the ordinate axis $g(0) = G_p$ (Fig. 1). Hence, after differentiation, a relationship can be obtained

$$\frac{dQ}{Q_p} = \frac{dg}{g} \tag{2}$$

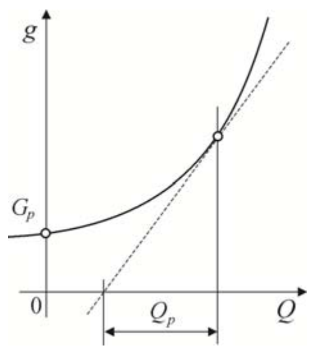


Fig. 1. Graph of dependence $g(Q)$

The Pentegov model can be used to mathematically map the dynamic characteristics of the electric arc column [3]. In this model, instead of the actual arc, a hypothetical arc is considered, in which the conductivity of the arc column is defined as a function of the virtual state current $i_\theta(t)$, changing with a specific time constant $\theta$ = const. The Pentegov model reproduces a non-linear double clamp electric element, which is: energetically balanced, thermally inertial, first order, linear, stationary and electrically non-inertial. According to the assumption, the currents and voltages of the model meet the condition

$$\frac{i}{u} = \frac{i_\theta}{U} = g \tag{3}$$

where $U(I)$ - static voltage-current arc characteristic. Coming out of the power balance equation in the column

$$\frac{dQ}{dt} + Ui_\theta = ui \tag{4}$$

(where: $dQ/dt$ - derivative of the internal plasma energy change, $ui$ - electric power supplied; $Ui_\theta$ - electric power dispersed from the column), the first order differential equation is obtained

(5) $$\theta(i_\theta, p)\frac{di_\theta^2}{dt} + i_\theta^2 = i^2$$

where the designation of the damping function was introduced

(6) $$\theta(i_\theta, p) = Q_p \frac{dg}{di_\theta^2}$$

which, as can be seen, may depend on the parameter vector $p$. Experimental studies of the arc corresponds to the search for the parameters of the selected formula for approximation of the static characteristic $U(I,\ p)$ and the constant $Q_p$.

The Pentegov assumption imposes limits on the parameters of the mathematical model through dependence $Q(i_\theta, p) = 2\theta\int_0^{i_\theta} U(i_\theta, p)di_\theta = const$. Depending on the type of current excitation and the selected version of the Pentegov model [4, 5], the operation of determining its parameters can be one or two stages. In the second case, it is possible to separately determine the family of static characteristics of the arc $U(I)$, and then the time constant $\theta$. Static characteristics can have any shape and can cover a wide range of current changes. In addition, they may take into account the impact of various external factors on the plasma column (e.g. column length, gas pressure) [6, 7].

**Properties of the Mayr-Pentegov model with static *g*(*I*) characteristics having saturation**

Let us assume that the shape of the static characteristic of the arc $g(I)$ is as shown in Figure 2. It can be described by a rational function in the form [8]

(7) $$\frac{g - G_1}{G_2} = \frac{1}{1 + \frac{P_1 G_2}{I^2}}$$

where $\mathrm{tg}\alpha \approx P_1$ - constant equivalent to Mayr power, $G_1$, $G_2$ - constant parameters of static characteristics. If external factors influence the arc column and they change quasi-statically, they can be taken into account using the functions $G_1(p)$ and $G_2(p)$. These characteristics can also be saved in a different form

(8) $$I^2 = \frac{P_1(g - G_1)}{1 - \frac{g - G_1}{G_2}}$$

or as a voltage-current characteristic

(9) $$G_2 U = \frac{I}{G_1 + \frac{I^2}{P_1 + \frac{I^2}{G_2}}}$$

Depending on the values of the $G_1$ and $G_2$ parameters, the static voltage-current characteristic may have a different shape (Fig. 3).

The inflection point in Figure 3a corresponds to the abscissa $I_p = \sqrt{P_1 G_2 / 5}$. In contrast, coordinates point to local extremes

(10) $$I_M = \frac{P_1 G_1 G_2}{G_1 + G_2}\frac{4}{1 + \sqrt{1 - 8\frac{G_1}{G_2}}}$$

(11) $$U_M = \frac{I_M}{G_1 + G_2}\left(1 + \frac{2}{1 - \sqrt{1 - 8\frac{G_1}{G_2}}}\right)$$

(12) $$I_m = \frac{P_1 G_1 G_2}{G_1 + G_2}\frac{4}{1 - \sqrt{1 - 8\frac{G_1}{G_2}}}$$

(13) $$U_m = \frac{I_M}{G_1 + G_2}\left(1 + \frac{2}{1 + \sqrt{1 - 8\frac{G_1}{G_2}}}\right)$$

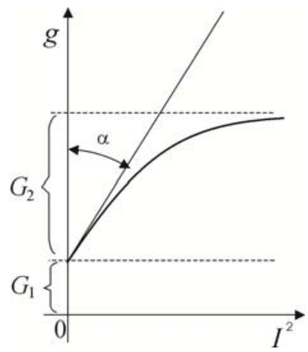


Fig. 2. Static characteristics $g(I^2)$ of the arc column

a) b)

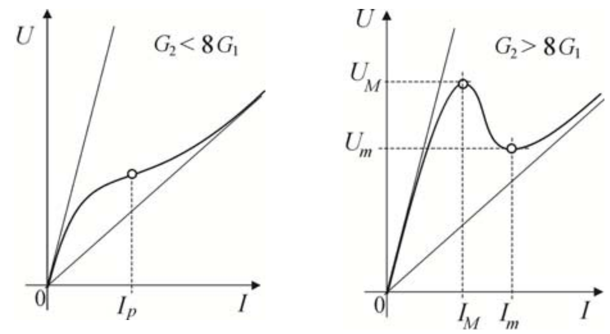


Fig. 3. Static voltage-current $U(I)$ characteristics: a) characteristic with inflection point; b) characteristics with two local extremes

After differentiation of the static characteristics (7) and using the dependencies (6), the expression for the suppression function is obtained

(14) $$\theta(I) = Q_p \frac{dg}{dI^2} = \frac{Q_p}{P_1}\frac{1}{\left(1 + \frac{I^2}{P_1 G_2}\right)^2}$$

After substituting this function for equation (5), the equation of arc column state is obtained [8]

(15) $$\frac{\theta_M}{\left(1 + \frac{i_\theta^2}{P_1 G_2}\right)^2}\frac{di_\theta^2}{dt} + i_\theta^2 = i^2$$

If parameter $G_2 \to \infty$, then equation (15) takes the form of the Mayr model. The impact of changes in current and static characteristics on the damping function (14) of the model is shown in Figure 4.

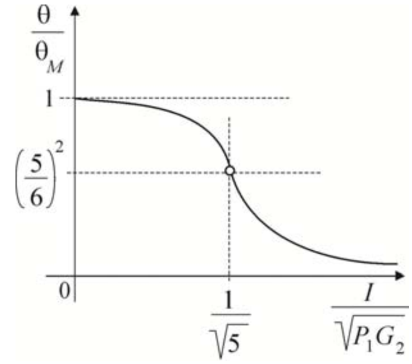

Fig. 4. Relation of the relative value of the modified model suppression function to the relative value of the state current

### The results of computer simulations

The possibilities of using the proposed function (7) to approximate the voltage-current static curve characteristics are shown in Figures 5, 6 and 7. This function allows the residual conductance of the column to be taken into account in the low-current range. A non-zero conductance value can improve the stability of simulation programs and be a factor that facilitates better mapping of dynamic characteristics [7, 9]. It can be seen that the increase in the $G_1$ parameter value causes a decrease in the arc ignition voltage value. A similar effect is obtained by increasing the $G_2$ parameter. However, increasing the value of $P_1$ causes an increase in ignition voltage.

a) b)

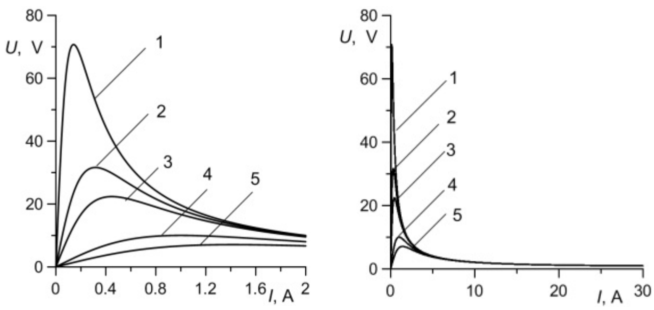

Fig. 5. The family of voltage-current static characteristics of the arc model in the case of the variable $G_1$ parameter: a) in the low current range; b) in a wide range of current changes ($P_1$ = 200 W, $G_2$ = 10 S, 1 – $G_1$ = 0.0001 S, 2 – $G_1$ = 0.0005 S, 3 – $G_1$ = 0.001 S, 4 – $G_1$ = 0.005 S, 5 – $G_1$ = 0.01 S)

a) b)

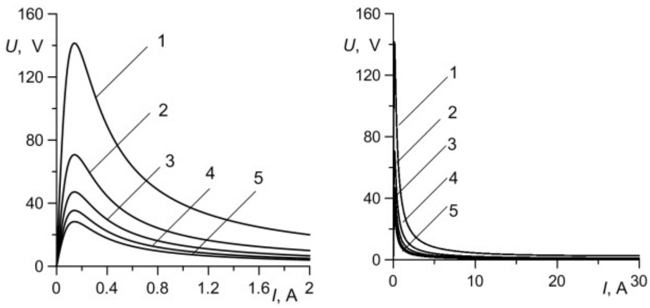

Fig. 6. The family of voltage-current static characteristics of the arc model in the case of the variable $G_2$ parameter: a) in the low current range; b) in a wide range of current changes ($P_1$ = 200 W, $G_1$ = 0.0001 S, 1 – $G_2$ = 5 S, 2 – $G_2$ = 10 S, 3 – $G_2$ = 15 S, 4 – $G_2$ = 20 S, 5 – $G_2$ = 25 S)

a) b)

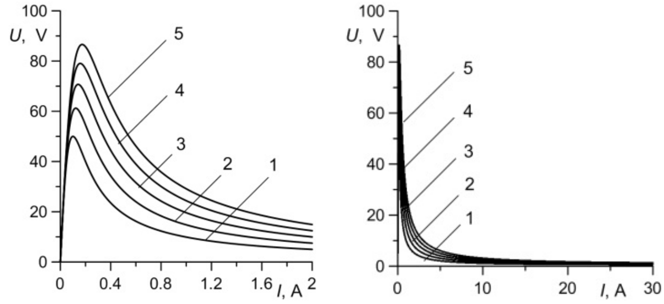

Fig. 7. The family of voltage-current static characteristics of the arc model in the case of the variable $P_1$ parameter: a) in the low current range; b) in a wide range of current changes ($G_1$ = 0.0001 S, $G_2$ = 10 S, 1 – $P_1$ = 100 W, 2 – $P_1$ = 150 W, 3 – $P_1$ = 200 W, 4 – $P_1$ = 250 W, 5 – $P_1$ = 300 W)

Then a circuit was created with the macromodel of the electric arc, which contained a current source with an amplitude of 50 A and a frequency of 50 Hz. In addition, the value of the sum of electrode electrode voltage drops of 18 V was imposed. The results of the simulations performed are shown in Figure 8. Despite the wide changes in the value of the value parameter $\theta_M$, stable operation of the numerical integration program was found. The macromodel parameters were selected in such a way that the dynamic characteristics obtained correspond to the low-voltage arcs used in welding.

a) b) c) d)

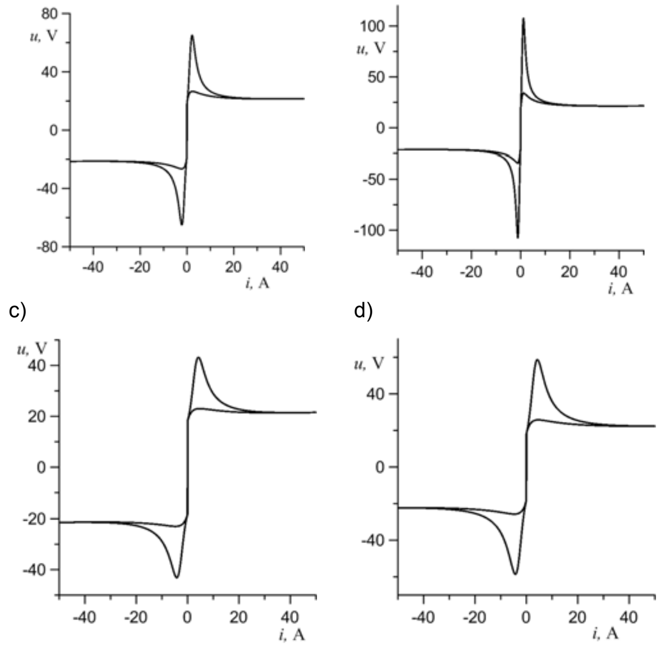

Fig. 8. Dynamic voltage-current arc characteristics ($G_1$ = 0.0001 S, $G_2$ = 5 S): a) with the following parameters: $P_1$ = 300 W, $\theta_M$ = $1{\cdot}10^{-4}$ s; b) with the following parameters: $P_1$ = 300 W, $\theta_M$ = $5{\cdot}10^{-5}$ s; c) with the following parameters: $P_1$ = 300 W, $\theta_M$ = $2{\cdot}10^{-4}$ s; d) with the parameters: $P_1$ = 500 W, $\theta_M$ = $2{\cdot}10^{-4}$ s

### Conclusions:

1. The use of Mayr and Pentegov assumptions enables obtaining a mathematical model reflecting arc burning in a wide range of current excitation.
2. The described mathematical model does not require the use of the assumption for a relationship of time constant

with plasma enthalpy and state current, which may limit the usefulness of approximation of experimental data.

3. The approximating static characteristic function allows to take into account the effects of residual conductance in the low-current range and voltage rise in the high-current range.

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## REFERENCES

[1] Krouchinin A.M., Sawicki A., A theory of electrical arc heating, The Publishing Office of Technical University of Częstochowa, Częstochowa 2003.
[2] Kalasek V., Measurements of time constants on cascade d.c. arc in nitrogen, TH-Report 71-E18, Eindhoven 1971.
[3] Пентегов И.В., Сидорец В.Н., Сравнительный анализ моделей динамической сварочной дуги, Автомат. Сварка, 1989, № 2 (431), 33-36. (in Russian) Pentegov I.V. and V.N. Sydorets: Comparative analysis of models of dynamic welding arc. The Paton Welding Journal (2015), no. 12, 45-48
[4] Sawicki A., Haltof M., Wyznaczanie parametrów modeli matematycznych łuku elektrycznego w wybranych stanach pracy urządzeń TIG, Przegląd Elektrotechniczny, R. 91 (2015), nr 12, 212-217 (DOI: 10.15199/48.2015.12.55)
[5] Sawicki A., Two-point method of determining the time constant of selected variants of the Pentegov model of electric arc, XXIX Sympozjum Środowiskowe PTZE Zastosowania Elektromagnetyzmu we Współczesnej Inżynierii i Medycynie. Janów Podlaski 2019, 9-12.06.2019. (IEEE Xplore Digital Library, DOI: 10.23919/PTZE.2019.8781718 )
[6] Sawicki A., Haltof M., Nieliniowe modele matematyczne łuku elektrycznego, Przegląd Elektrotechniczny, R. 92 (2016), nr 11, 257-261 (doi: 10.15199/48.2016.11.62)
[7] Marciniak L., Implementacje modeli łuku ziemnozwarciowego w programach PSCAD i Matlab/Simulink, Przegląd Elektrotechniczny, R. 88 (2012), nr 9a, 126-129.
[8] Wąsowicz S., Zmodyfikowany model Mayra-Pentegowa z parametrami. (Częstochowa 2019. – unpublished work)
[9] Sawicki A., Klasyczne i zmodyfikowane modele matematyczne łuku elektrycznego, Biuletyn Instytutu Spawalnictwa (2019), nr 4, 73-76.