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Comparative study to assess reliability in the presence of two geometric defect shapes for non-destructive testing

***Abstract**. This study aims at evaluating the reliability for non-destructive eddy currents control. Two geometrical forms of the defect are generated whose dimensions and the electrical conductivity of the studied material are random variables of Gaussian type. The model SSFEM constructs allows the treatment and the post-treatment of the problem posed in a single step. the results of the impedance variation are compared with those of the Monte Carlo simulation and the experimental measurements. The state of reliability is quantified for the two forms of the defect.*

Streszczenie.** Analizowana jest wiarygodność detekcji metodą prądów wirowych. Badana jest obecność dwóch różnych defektów któeych kształt jest przypadkowy. Wykorzystywano model SSFEM. Metodę porównano z rezultatami przy wykorzystaniu metody |Monte Carlo oraz z eksperymentem.* ***Studium porównawcze wiarygodności detekcji dwóch różnych defektów przy defektoskopii wiroprądowej

Keywords: Spectral Stochastic Finite Element - Gaussian random variable- Random geometry- Probability of failure.
Słowakluczowe:: defektoskopia wiroprądowa, metody statystyczne

Introduction

The study of electromagnetic systems requires knowing the input data to obtain the output information. The difficulty encountered in solving a practical problem is the ability to identify physical properties such as electrical conductivity, magnetic permeability, which must be known before the problem is addressed. In this case, most case studies, input data are often searched for using an experiment or using them with some uncertainty [4] - [9].

Today, researchers are focusing more and more on modeling in uncertain environments using stochastic calculations. Deterministic calculus requires inverse computation. Often, for post-processing, such as evaluation of the reliability of the electromagnetic system, the hazard on the physical properties of the system are not taken into account. Knowing the forms of defects is also necessary in the inspection process. In this situation, input data is often used with some uncertainty [6].

The spectral stochastic finite element intrusive method (SSFEM) used takes into account the random variables of the input variables, they are considered random variables of Gaussian type. The method (SSFEM) developed also has a rather important advantage, in particular; the study and analysis of the sensitivity of the system in one step [3] - [5].

The present work concerns the study of the reliability of the Non - destructive control device when the fault concerns the uncertainty on the electrical conductivity [2] and the shape of the defect (rectangle and triangle).Considered as a random variable of Gaussian type, with an average value of the associated variable and a standard deviation set for this study at 0.9, they are developed in series of Hermit polynomials.

A finite element code is developed under Matlab considering a non-destructive system. The post-processing allows us to evaluate the variation of the impedance in the defect, for the two geometrical shapes, the data of the impedance for a rectangular defect of shape are confronted with those obtained by the Monte Carlo simulation and the experimental measurements[1]. The reliability study is represented by the evaluation of the reliability index and the probability of system failure. The study is carried out in 2D axisymmetric hypothesis.

A flowchart presenting the different stages of calculation is presented in Fig. 7.

Formulation of the intrusive method of spectral stochastic finite elements SSFEM

Let consider the random variable Y, having as input parameters the vector ξ of *M* independent stochastic variables It can be shown that if Y has a finite variance then *Y* can be written as a linear combination of multivariate polynomials on the basis of the Hermit polynomials [2]-[4].

(1) $$Y = \sum_{i=1}^{p-1} \alpha_i H_i(\xi_1, ..., \xi_n)$$

where : p , represents the degree of polynomial chaos.
$\{H_i(Y), i=0,\dots,\infty\}$ are Hermits polynomials,$\{\alpha_i, i = 0,\dots,\infty\}$ are coefficients of Hermits polynomials where ξ is a reduced centred Gaussian random variable (r.c.g.r.v) with the following density of probability [14, 7].

(2) $$\Phi(G) = \frac{1}{\sqrt{2\pi}} e^{\frac{-\sigma^2}{2}}$$

In (1) α_i are unknown coefficients to be determined. We notice that all Hermit polynomials are orthogonal with regard to the Gaussian measure according to the orthogonally property

(3) $$\mathrm{E}[\mathrm{H_n}(\xi(\theta))\,\mathrm{H_m}(\xi(\theta))] = 0 \quad \text{if } n \neq m$$

E [.] denotes the mathematical expectation.

For the computation of the coefficients of Hermits polynomials, the work develops the method of projection. Thus, the coefficients of Hermits polynomials are given by the following expression [10,5].

(4) $$\alpha_i = \frac{E[GH_i(\xi)]}{i!}$$

The identification of the Hermit polynomial coefficients in the case of is probabilistic transformation is performed using the next formula:

(5) $$\alpha_i = \int_R F_G^{-1}(\Phi(t)) H_i(t) \phi(t) dt$$

when G follows a Gaussian law of average μ_t and of standard deviation σ_t, the calculation of the coefficients of the development (4) is as follows:

(6) $$\alpha_0 = \mu_t$$

(7) $$\alpha_1 = \sigma_t$$

(8) $$\alpha_i = 0 \qquad if\ i \neq 1$$

Random geometry definition

In the case of random geometry, the two random variables G_h and G_w of chaos polynomial of order 2 is expressed as following:

(9) $$\begin{bmatrix} G_h \\ G_w \end{bmatrix} = \begin{bmatrix} \alpha_{01} & \alpha_{11} & 0 & \alpha_{12} & 0 & 0 \\ \alpha_{02} & 0 & \alpha_{12} & 0 & 0 & \alpha_{22} \end{bmatrix} \begin{bmatrix} 1 \\ \xi_1 \\ \xi_2 \\ \xi_1^2 \\ \xi_1\xi_2 \\ \xi_2^2 \end{bmatrix}$$

$\alpha_{01}, \alpha_{11}, \alpha_{12}$ and coefficients $\alpha_{02}, \alpha_{12}, \alpha_{22}$ are obtained from (6), (7) and (8) for each random variable G_h and G_w.

In our development the random variables G_h an G_w are the height and the width of the defect respectively.

The projection of the defects dimensions, the height and the width, G_h and G_w respectively in the basis 0of Hermit polynomials leads to [2].

(10) $$G_h = \sum_{i=1}^{p-1} h_i H_i(\xi_1, ..., \xi_n)$$

(11) $$G_w = \sum_{i=1}^{p-1} w_i H_i(\xi_1, ..., \xi_n)$$

h_i and w_i are the random coefficients of defect geometry to be searched.

Deterministic electromagnetic model

The electromagnetic equation is obtained from Maxwell's equations associated with those of the middle relations and the law of ohms. In case of harmonic hypothesis, the electromagnetic formulation for determinist problem is given as follows [2, 7].

(12) $$\nabla \wedge \left(\upsilon \nabla \wedge \vec{A}_z\right) + j\sigma_s \omega \vec{A}_z = \vec{J}_{sz}$$

Where : $\vec{A}_z$: Magnetic vector potential magnetic along z direction; $\omega = 2\pi f$; ν : Absolute magnetic reluctivity [H/m] $^{-1}$; f : Frequency [Hz]; σ_s : Electrical conductivity [S.m^{-1}]; J_{sz} : Source current density next z [A / m^2]

The finite element formulation leads, considering homogeneous Dirichlet boundary conditions, to the matrix system, and whose elements are as follows

(13) $$[M][A] = [F]$$

(14) $$M_{ij} = \iint_{\Omega} \nu \left(\frac{\partial \alpha i}{\partial x} . \frac{\partial \alpha j}{\partial x} + \frac{\partial \alpha i}{\partial y} . \frac{\partial \alpha j}{\partial y} \right) dxdy$$

(15) $$F_i = \iint_{\Omega} J_{sz} \alpha_i dxdy$$

(16) $$[A] = [A_1, A_2, ..., A_n]^T$$

Stochastic linear system construction

The stochastic finite element linear system is constructed after the projection into the Hermit polynomial basis of the random variables considered below. The projection of the unknown A in the basis of Hermits polynomials leads to [4,3].

(17) $$A = \sum_{i=1}^{p-1} A_j H_j(\xi_1, ..., \xi_M)$$

(18) $$\sigma = \sum_{i=1}^{p-1} \sigma_i H_i(\xi_1, ..., \xi_M)$$

The Non-destructive testing of interest problem is built with the consideration that the source current density and the electrical conductivity of the medium are given.

The minimization when using Galerkin method means requiring the orthogonality of the residue with the base of projection functions $\{\psi_j, j = 0 ... p\}$. By taking as test function α_i the chaos polynomial ψ_j. This leads to [9,8].

(19) $$\sum_{j=0}^{p-1} M_0 A_j E[\psi_j \psi_k] + \sum_{i=0}^{p-1}\sum_{j=0}^{p-1} N_i A_j E[\psi_i \psi_j \psi_k]$$

$$k = 0, ..., p-1$$

The final form of the stochastic finite element and harmonic linear system is:

(20) $$F_k^s = \sum_{j=0}^{p-1} \left(M_{jk}^s + j\omega N_{jk}^s \right) A_j$$

(21) $$M_{jk}^s = C_{0jk} M_0$$

(22) $$N_{jk}^s = \sum_{i=0}^{p-1} C_{ijk} N_i^s$$

(23) $$C_{ijk} = \begin{cases} \dfrac{i!\,j!\,k!}{\left(\frac{i+j-k}{2}\right)!\left(\frac{j+k-i}{2}\right)!\left(\frac{k+i-j}{2}\right)!} & if \begin{Bmatrix} (i+j+k) \quad even \\ k \in [|i-j|, i+j] \end{Bmatrix} \\ 0 & otherwise \end{cases}$$

M^s, N^S, F^s are the random linear matrixes and source vector respectively related to solving problem.The stochastic system obtained from (23) is:

(24) $$\begin{bmatrix} R_{00}^s & R_{10}^s & R_{20}^s \\ R_{01}^s & R_{11}^s & R_{21}^s \\ R_{02}^s & R_{12}^s & R_{22}^s \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} F_0 \\ F_1 \\ F_2 \end{bmatrix}$$

The expression of the resulting matrix R is obtained by equations (21), (22) as follows:

(25) $$R_{jk}^s = M_{jk}^s + j\omega N_{jk}^s$$

where A_0, A_1, A_2 are the solutions of the stochastic complex algebraic system and F_0, F_1, $F_2$are the source vector components.

Post treatments

After solving the stochastic matrix system, we obtain three random solutions of the potential vector A, which are A0, A1, A2, the number of solutions is equal to p, which represents the degree of polynomial chaos.

The post treatment, in the first step, consists of calculating the impedance Z, in the suspected fault zone, the expression of the impedance is as follows:

(26) $$\mathrm{Re}(Z) = -\frac{N^2}{J_{sz}.S^2}\omega.\iint_S 2.\pi.r.\mathrm{Im}(A).ds$$

(27) $$\mathrm{Im}(Z) = \frac{N^2}{J_{sz}.S^2}\omega.\iint_S 2.\pi.r.\mathrm{Re}(A).ds$$

where: J_{sz}: Current density [A / m²]; S: Conductors Section [m²]; N: Number of turns

The limit state function g_{SFEM} is obtained with the threshold value of the impedance and the stochastic solutions of the impedance.

(28) $$g_{SFEM} = Z_s - Z^{i0}$$

where: Z_s: reference value of the impedance; Z^{i0}:Stochastic impedances.

Reliability analysis

The reliability of the structures seeks to calculate the probability of failure of a structure taking into account the variability of parameters (geometry, material and source). The parameters are represented by random variables. Let X be the random vector grouping them, which is given the joint probability density $f_X(x)$. Let S(X) be the electromagnetic response of the system. For each failure mode of the structure, a limit state function g (X, S (X)) is defined in the parameter space. By convention, the safety domain is defined by [10]:

(29) $$D_s = \{X \setminus \mathrm{g}(\mathrm{X},\mathrm{S}(\mathrm{X})) > 0\}$$

And the failure domain by:

(30) $$D_f = \{X \setminus \mathrm{g}(\mathrm{X},\mathrm{S}(\mathrm{X})) \leq 0\}$$

The boundary $\{g(\mathrm{X}, \mathrm{S}(\mathrm{X})) = 0\}$ is the boundary state surface. The probability of failure is given by:

(31) $$P_f = \mathrm{Pro}b[\mathrm{g}(\underline{X},S(\underline{X})) \leq 0] = \int_{g(\underline{X},S(\underline{X}))\leq 0} f_{\underline{X}}(\underline{x})d\underline{x}$$

The computation of this integral is not possible analytically because the domain of failure is defined implicitly starting from S(X) which is given by the computation to the finite elements. When a stochastic finite element analysis of the problem has been carried out beforehand, the development of the response on the polynomial chaos is available. Any limit state function is therefore approximated in the form of:

(32) $$g(S(\underline{X})) \approx g\left(\sum_{j=0}^{p-1} S_j\psi_j\right)$$

Considering a service limit state related to the exceeding of a threshold us by the displacement at the node i_0, the limit state function can be put in the form:

(33) $$g(S(\underline{X})) = u_s - \sum_{j=0}^{p-1} u_j^{i0}\psi_j$$

where: u_s : Threshold value

The reliability index β is given directly by the limit state function g_{SSFEM}, generated by the stochastic finite element calculation code [4].

(34) $$\beta = Min\sqrt{g_{SSFEM}}$$

From the values of the reliability index a numerical calculation is carried out on the basis of the Lagrange polynomial, this one allows us to obtain the formulation of the probability of failure as well as its evolution according to the index of reliability.

Description of the application

The application chosen is a non-destructive testing device, which represents a steam generator tube in nuclear power plants subjected to eddy currents inspection [2,13].

The obtained axisymmetric stochastic model is applied for numerical simulations of a cylindrical device behavior. A reliability analysis and an estimate of the probability of failure are made. Thus, the study was conducted under the 2D axisymmetric hypothesis.

The device for NDT consists of a 2D approximation of a conducting cylinder as shown inFigure1[4], whose conductivity is equal to 0.76 MS/m. The defect is characterized by a length of 10 mm, 0.22 mm width and a depth of 0.75 mm.

The inductor, representing the sensor, has 140 turns, it is powered by an alternating current of amplitude 0.008A and a frequency of 150 kHz [11].

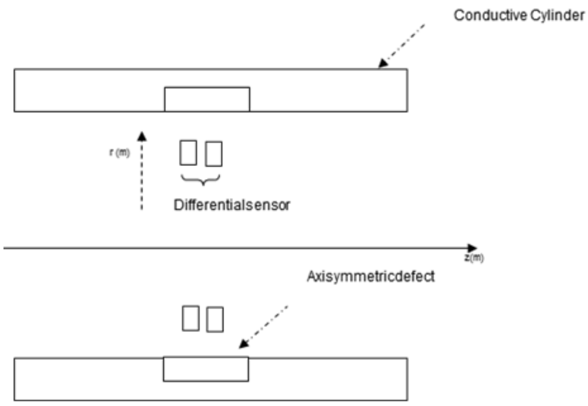

Fig.1. 2D axisymmetric NDT device

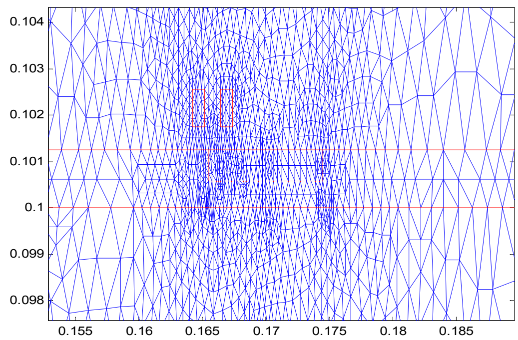

Fig.2. Mesh of the studied domain for rectangular geometry

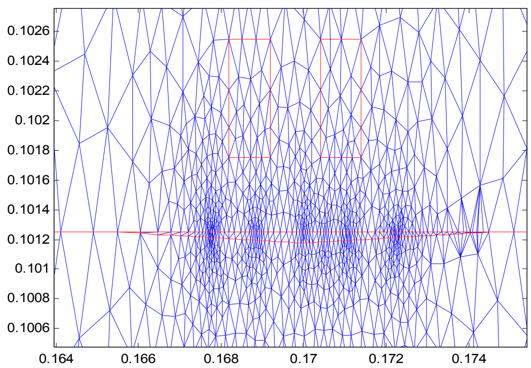

Fig. 3. Mesh of the studied domain for triangular geometry

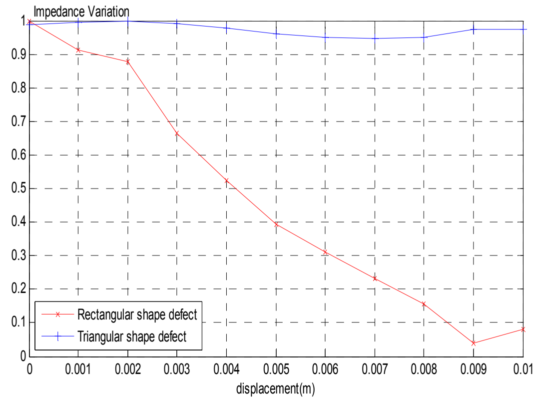

Fig.4. Impedance variation for triangle and rectangle defects shapes.

Interpretation of Results

The results obtained are shows by Fig.2, Fig.3, Fig.4, Fig.5, Fig.6 and in Table 2.

Figure.2 and 3 show the mesh of the domain of study of the defect with rectangular and triangular geometry respectively. This mesh is used for the resolution of the axisymmetric 2D electromagnetic system.

The figure 4 shows the impedance variation for both rectangle and triangle defects shapes.

Experimental setup

The experiment consists of a GW - INSTEK 8101G LCR meter which produces the excitation voltage, a pancake coil and a conductive plate (with and without fault). This experiment is characterized by the parameters indicated in

Table 1. Parameters of the test experiments

Coil	Plate	Defect
Inner diameter10 mm	Thickness2 mm	A/- Crack Length 10 mm Width 2 mm Depth 1 mm
Outer diameter12 mm	Electric conductivity 58 MS/m	
Number of turns200	Lift-off 1 mm	

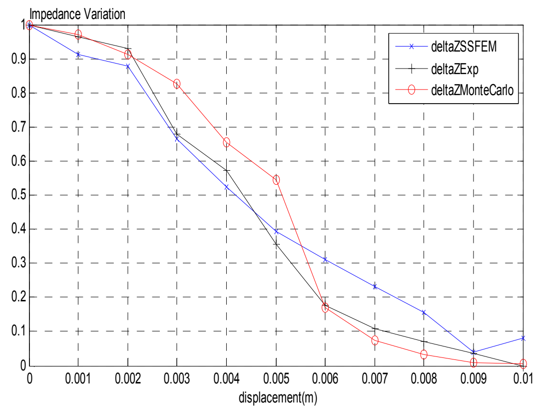

Fig.5. Comparison of impedance variation between SSFEM, Monte Carlo and experimental measurements

Table 1. shows the impedance signal obtained from the ECT probe is recovered and represented as a function of the position of the sensor [1].

A comparison of the variation of the impedance in the rectangular shape defect is illustrated in Figure 5, which shows a fairly marked agreement between the experimental values obtained from the conditions mentioned in Table 1, and those obtained by the simulation. of the spectral stochastic finite element method SSFEM, in parallel, we note the paces obtained by the two simulation methods respectively SSFEM and Monte Carlo are very close; which allows us to argue that the experimental curve gives us a good validation of the SSFEM model and the Monte Carlo simulation

Table 2 presents the values of the probability of failure evaluated by the numerical method of interpolation of the Lagrange for the two studied geometries in the zone of supposed defect. We note through these results that the probability of failure for the triangular defect is very small compared to the rectangular shape defect.

Table.2. Probability of failure in the zone of supposed defect for the two studied geometries

Displacementsx(m)	Probability of failure Rectangular geometry	Probability of failure (Pr) %	Probability of failure Triangular geometry	Probability of failure (Pr) %
0.001	09.67		3.00e-04	
0.002	13.57		3.53e-04	
0.003	0.62		3.81e-04	
0.004	0.19	Pr<14	3.23e-04	Pr<0.1
0.005	0.032		2.46e-04	
0.006	0.095		2.86e-04	
0.007	02.88		3.58e-04	
0.008	09.67		3.50e-04	

The figure 6 shows the appearance of the probability of failure. The development of the Lagrange polynomial based on reference data[6] of the reliability index and the probability associated with the values of the reliability index obtained by the stochastic computation, allowed us to evaluate the probability of failure in the defect.

The two curves are practically superimposed, which allows us to argue that the exploitation of the Lagrange polynomial is a rather appreciable technique in the evaluation of the probability in failure from values of the reliability index generated by the SSFEM model.

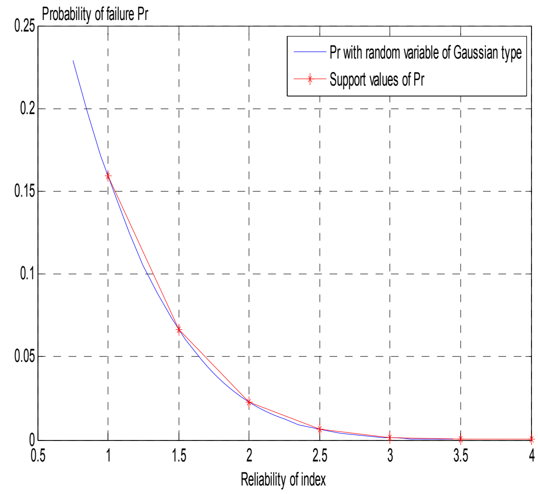

Fig.6. Trend of the probability of failure according to the reliability index

Conclusion

The present work presents the development of the intrusive method of spectral stochastic finite elements to, on the one hand generate the random geometric form of a defect and on the other hand to solve the 2D electromagnetic equation in axisymmetric hypothesis by considering the electrical conductivity a Gaussian type 0.9 standard deviation random variable [2].

The application represents a non-destructive eddy current device with a defect. Two geometric shapes are considered, the exploitation of the SSFEM model, allowed us to calculate the impedance variations in the two forms of defects, a comparison on the gaits is carried out, a first observation appeared, when to the importance of the variation of the impedance depends directly on the geometrical shape of the defect, on the margin of this comparison, we realized a comparison of the impedance variation behavior between experimental values [1],

The SSFEM model and Monte Carlo simulation, we can say that the results obtained for the three methods are very satisfactory, we draw the attention when the relevance of using this or that method in this type of control,

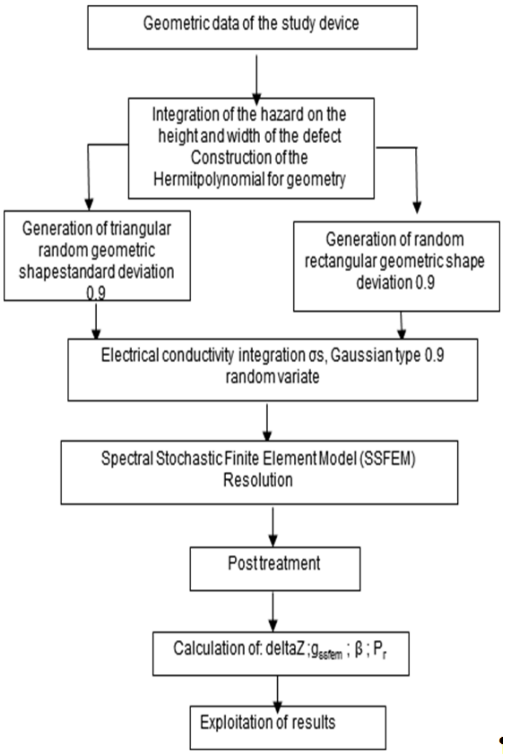

Fig.7. Synthesis flowchart under Matlab

The experimental is not often available and requires standardized test platforms, and accredited what is not very accessible, Monte Carlo requires large draws and remains very expensive in industrial settings Nevertheless, its place remains privileged despite the need to go to the opposite problem for exploitation; when the spectral stochastic finite element method has the advantage of doing the study and the operation in a single step, which is not negligible in terms of time of realization and control of the simulations, and especially this method allows to consider a hazard from the design which is not negligible compared to the opportunities for evaluation of reliability and the probability of failure which remains a fundamental theme in the industry.

This work was completed by the evaluation of the probability of failure and comparisons were made for both geometric shapes, which confirmed that the latter is largely weak when the defect is triangular while keeping the height and width of the defect on the surface considered the same for both geometric shapes. At the end of this study we can confirm that the surface of the defect, see its volume, has a direct impact on the rate of probability of failure.

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