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Mathematical models of dynamics of friable media and analytical methods of their research

***Abstract**. The method of studying the process of vibroseparation or adhesion of friable media has been developed. The analytical solution of this mathematical model of the researched system for resonance and nonresonance cases was constructed. It is established that the relative amount of friable medium motion greatly affects the quantitative and qualitative parameters of the system oscillations, in particular, the increase in the amount of motion of a friable media causes a decrease in the amplitude of the system in the transition through the resonance.*

***Streszczenie.** Opracowano metodę badania procesu separacji wibracyjnej oraz przyczepności materiałów sypkich. Zbudowano model matematyczny badanego systemu dla przypadków występowania rezonansu i jego braku. Ustalono, że wielkość przesunięcia względnego materiału sypkiego znacznie wpływa na parametry ilościowe i jakościowe oscylacji systemu, w szczególności zwiekszenie przesunięcia materiału sypkiego powoduje zmniejszenie amplitudy systemu podczas przejścia przez rezonans. **(Modele matematyczne dynamiki materiałów sypkich oraz metody analityczne ich badań)**.*

Keywords: friable media, adhesion, separation, sieve, transverse perturbation of motion.
Słowa kluczowe: materiał sypki, przyczepność, separacja, sito, poprzeczne zaburzenie ruchu.

Introduction

Methods of vibration treatment, vibration transporting, separation, adhesion, and hence the corresponding machines and equipment have been widely used in various industries [1-4]. Their implementation contributes to the intensification of various production processes, increases the level of mechanization of labor-intensive work, increases economic efficiency and productivity. With an increase in the intensity of production, the scope of application of vibrating machines expands, and new ones are created for various technological operations [1,4,5]. This is due to the simplicity of their designs, and in many cases, higher performance than that of conventional machines. Especially effective application of vibration equipment is in the chain of technological processes, which are associated with the processing of various friable mixtures, their dosage, mixing, separation, sealing, etc. [6,7]. In this case, along with the intensification of technological processes during the vibrational impact on the material, the quality of the final product improves: due to constant vibration, a high degree of separation of the mixture by its physical, mechanical or other properties is achieved.

Despite the significant progress in the creation of a new and modernization of existing vibration equipment, theoretical studies related to the vibrational processes of friable media - sieve have not been properly developed due to the lack of satisfactory methods for analyzing these complex dynamic processes [8,9]. Above mentioned, on the one hand, led to the intensive introduction of vibration processes into production, on the other hand - it requires to change the basic dynamic characteristics of the working containers, and on this - the dynamics of the friable media, that is, using vibration machines of a controlled type [1,5,8,10-13]. For them, as a rule, there is no transition via the resonances of individual units and the machine as a whole, which ultimately reduces the power of driving engines compared to uncontrolled vibration machines. Therefore, it is important to study the influence of external factors on the amplitude and frequency of oscillations of friable media. The latter characteristics are also determinative for the calculation of the efficiency of the separation process [1].

As a rule, the friable media has a complex structure and its characteristics are determined by the physical and mechanical properties of individual parts, their size, interaction, etc. [7,10,11] However, the study of the dynamics of the friable media on the basis of the dynamics of some of its parts does not yield the desired results, because such an approach requires an analysis of a large number of ordinary differential equations, in addition, the mechanism of interaction between individual parts of the friable media is difficult. A relatively simpler method for studying the dynamics of friable media is the so-called integral approach. Its essence is as follows: the friable media is modeled as an continuous medium (layering of flat homogeneous or heterogeneous beams) with some integral characteristics [2,7,10]. Such an approach in studying the dynamics of the media, its effect on the process of vibration separation is seen from the mathematical side easier, and from the practical - more acceptable. However, it is designed to be sufficient (for practical calculations) only for the lengthwise oscillations in a friable media. At the same time, the case of transverse oscillations of the friable media is more important from the practical side. It corresponds to a qualitatively new mathematical model, which has not been properly researched due to purely mathematical problems.

The study of the process of the dynamics of friable media during the process of vibration separation or adhesion within the framework of the physical model [2,7] under the condition of transverse perturbation of the motion of the friable media is the subject of this work. To do this: an appropriate mathematical model is constructed under the condition of transverse perturbation of its motion; dependencies have been obtained describing the parameters of the dynamics of friable media, and hence the separation process itself. Thus, we consider the dynamic process of an integrate friable media, which moves continuously along the inclined horizon under some angle to the sieve. The latter is exposed to external periodic perturbation, and is considered one-dimensional nonlinearly-elastic body. Moving continuous flow of friable medium is considered to have zero bending stiffness. The purpose of this work is to investigate the influence of the basic physical and mechanical characteristics of the sieve and the friable media, the parameters of the external periodic perturbation, the relative velocity of the friable media movement on the amplitude - frequency characteristic of the oscillations of the friable medium - the sieve.

To solve the research problem it is necessary to construct a mathematical model of the process within the framework of the accepted physical model of the object being studied; to get analytical solution to it, to investigate the influence of the main external and internal parameters of the investigated system on the determining parameters of the process of vibration separation or adhesion (amplitude and frequency).

Mathematical model of dynamics of the system „Friable media – sieve"

As was noted above, the physical model of the process of the separation or adhesion of the friable media is applied to the layering of flat "beams" of zero stiffness, which inactivate contact with the sieve. In this case, the position of the system continuous flow of friable media - a sieve is uniquely determined by the sieve bend function $u = u(x,t)$. The given value is calculated in the direction that is perpendicular to the longitudinal axis of the undeformed sieve and x - the running coordinate of the cross section of the sieve model, deducted from the upper part of it along the middle line, t - time. Let: m - weight of unit of length of elastic sieve; $m_1(x)$ - the mass of the unit of length is conditionally separated from the friable media, which moves with relative speed V to the same function $m_1(x)$ is slowly variable; E - the modulus of the elasticity of the sieve material, and I - the moment of inertia of the cross-section of the system relative to the axis, which coincides with the neutral axis in the undeformed position (the axis is perpendicular to the plane of oscillations). To obtain a mathematical model of dynamics of the investigated system, consider the "dynamic equilibrium" of arbitrary conditionally selected element. The forces acting on it and the forces of inertia of this element are determined as following (fig. 1):

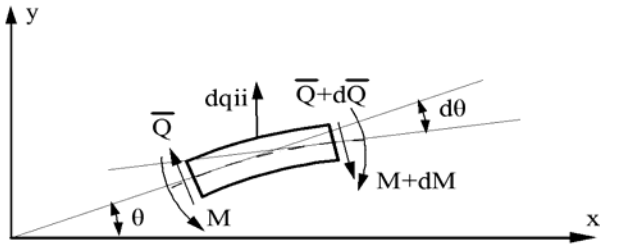

Fig. 1. The distribution of forces acting on the conditionally selected element of the system is friable media - the sieve

M - the bending moment of the cross section of the sieve with the coordinate x, and - $M + \frac{\partial M}{\partial x}dx$ in the section with the coordinate $x + dx$; Q - cross-sectional forces in the section with coordinate x and $Q + \frac{\partial Q}{\partial x}dx$, respectively, in the section with the coordinate $x + dx$; θ_1 - the angle which forms from the axis OX tangent to the neutral line of the normal section with the coordinate x; θ_2 - angle of inclination, which forms an axis OX tangent to a neutral line of normal section with coordinate $x + dx$; $q(x, u, u_t, \vartheta)dx$ - equivalent "small" external forces acting on a conditionally allocated element of the system, $q(x,u,u_t,\vartheta)$ - a function that describes their distribution along the length of the sieve (including resistance forces) $\vartheta(t) = \mu t$, μ the frequency of external periodic perturbation.

Taking into account that for small oscillations of the considered element of the system $\frac{\partial u}{\partial x}$ and θ are also small quantities and there is a connection between them $\sin\theta_1 = \frac{\partial u}{\partial x}$, $\sin\theta_2 = \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2}dx$. In addition, taking into account the connection between the cross-sectional force Q and the bending moment M in the form $Q = \frac{dM}{dx}$, the equation of the "dynamic equilibrium" of the selected element of the elastic body along which the continuous fluid flow of the medium takes the form

(1) $$dq_{in} + \frac{\partial Q}{\partial x}dx + q(x,t)dx = 0 .$$

In it, the forces of inertia dq_{ii} are determined by the ratio

(2) $$dq_{in} = m_1 \frac{d^2 u(x,t)}{dt^2}dx + m\frac{\partial^2 u(x,t)}{\partial t^2}dx .$$

In the last expression, a symbol $\frac{d^2}{dt^2}$ means a complete derivative of the corresponding function of time, therefore, for a friable media in the case of a constant velocity, its displacement takes place

(3) $$\frac{d^2 u(x,t)}{dt^2} = \frac{\partial^2 u(x,t)}{\partial t^2} + \frac{\partial^2 u(x,t)}{\partial x^2}V^2 + 2\frac{\partial^2 u(x,t)}{\partial t \partial x}V.$$

The last dependence on the physical side can be interpreted as accelerating the friable media in a complex movement. Indeed, if we take transverse oscillations of a sieve in a portable motion, then the first term of this dependence is nothing more than an acceleration of the friable media in the portable motion, and the other two, respectively, in relative (at constant velocity of the friable media along the sieve) motion and coriolis or rotary acceleration.

If we take into account that the elastic properties of the material of the sieve satisfy the nonlinear technical law of elasticity [9] $\sigma = E\left(\varepsilon_1 + \varepsilon\varepsilon_1^3\right)$ ($\varepsilon_1 = \frac{\partial u(x,t)}{\partial x}$ - the relative elongation of the sieve, and the parameter ε characterizes the deviation of its elastic properties from the linear law, and below it is considered small in comparison with the elastic modulus E), the equation (2) takes the form

(4) $$\begin{aligned}&(m+m_1)\frac{\partial^2 u(x,t)}{\partial t^2} + 2m_1 V\frac{\partial^2 u(x,t)}{\partial t \partial x} + m_1 V^2\frac{\partial^2 u(x,t)}{\partial x^2} + \\ &+\frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}\left(EI\left(\frac{\partial^2 u(x,t)}{\partial x^2} + \mu\left(\frac{\partial^2 u(x,t)}{\partial x^2}\right)^3\right)\right)\right] = \\ &= q\left(x,u,u_t,\vartheta(t)\right)\end{aligned}$$

The following restrictions on force factors allow the presented differential equation to be presented as

(5) $$\begin{aligned}&\frac{\partial^2 u}{\partial t^2} + \beta^2\frac{\partial^2 u}{\partial x^2} + \alpha^2\frac{\partial^4 u}{\partial x^4} = -\varepsilon\frac{EI}{m+m_1}\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u}{\partial x^2}\right)^3 + \\ &+\frac{1}{m+m_1}\left(q\left(x,u,u_t,\vartheta(t)\right) - 2Vm_1\frac{\partial^2 u}{\partial x \partial t}\right)\end{aligned}$$

where $\alpha^2 = \frac{EI}{m+m_1}, \beta^2 = \frac{V^2 m_1}{m+m_1}$ and the maximum value of its right part is small in comparison with the maximum value of the terms of its left part (l – the length of the sieve). To analyze the dynamic process on the basis of the obtained equation, we add boundary conditions that are consistent with the hinge connection of the sieve and the container, i.e.

(6) $$u(x,t)_{|x=0,j} = \frac{\partial^2 u}{\partial x^2}_{|x=0,j},\ j=0;l\ .$$

The task is to determine the influence of the basic physical and mechanical characteristics of the system of the moving flow of friable media - the sieve, as well as external factors on the determinant parameters of the oscillations of the friable media, and thus on the process of separation itself.

Obtained dependences can be based on the approximate analytical solution of the nonlinear differential equation (4) under the boundary conditions (5), since the exact solution of this problem cannot be found. The conditions imposed on the right side of equation (4) allow us to use the general ideas of perturbation methods for constructing the solution of the later. The later are most effectively used under the condition of the existence of a closed solution of an unperturbed analogue of it, that is, the equation

(7) $$u(x,t)_{|x=0,j} = \frac{\partial^2 u}{\partial x^2}_{|x=0,j},\ j=0;l\ ,$$

under boundary conditions similar to (5). We shall show that the dynamic process for the unperturbed movement can be interpreted as an overlay of two waves, that is

(8) $$u_0(x,t) = C_1\cos(\kappa x + \omega t + \phi) + C_2\cos(\kappa x - \omega t + \psi)\ ,$$

where $C_{1,}C_{2,}\kappa,\phi,\psi$ - the content and meaning of which will be set below.

The representation of the solution of the equation in the form (7) will satisfy the boundary conditions (5) if the wave number k and the frequency of the wave process ω are bound by the dispersion relation

(9) $$\omega^2 + \beta^2\kappa^2 - \alpha^2\kappa^4 = 0\ .$$

The obtained dispersion relation determines the frequency of the process as a function of a wave number in the form $\omega = \sqrt{\alpha^2\kappa^4 - \beta^2\kappa^2}$. To determine the constant values C_1 and C_2, as well as the relation between the initial phases of the direct and reflected waves, that is, the parameters φ and ψ boundary conditions allow to set: $\kappa = \frac{k\pi}{l}$, $\phi = -\psi$ and $C_1 = -C_2 = a$. Thus, the single-frequency wave process of the boundary value problem, which is described by the unperturbed equation (4), can be represented as a dependence

(10) $$u_0(x,t) = a\left[\cos\left(\frac{k\pi}{l}\left(x + \sqrt{\alpha^2\left(\frac{k\pi}{l}\right)^2 - \beta^2}\,t + \psi\right)\right) - \cos\left(\frac{k\pi}{l}\left(x - \sqrt{\alpha^2\left(\frac{k\pi}{l}\right)^2 - \beta^2}\,t - \psi\right)\right)\right].$$

The obtained result is at the same time a is a mandatory condition for describing the dynamics of perturbed motion of a friable media - a sieve. According to them, the first asymptotic approximation of the single-frequency process of the studied system is described by the dependence

(11) $$u(x,t) = a(t)\times\left[\cos\left(\frac{k\pi}{l}\left(x + \sqrt{\alpha^2\left(\frac{k\pi}{l}\right)^2 - \beta^2}\,t + \psi(t)\right)\right) - \cos\left(\frac{k\pi}{l}\left(x - \sqrt{\alpha^2\left(\frac{k\pi}{l}\right)^2 - \beta^2}\,t - \psi(t)\right)\right)\right] + \varepsilon U(a,x,\psi,\vartheta).$$

That is, for perturbed motion parameters a and ψ will already be functions of time, in addition, the function $\mathrm{U}(a,x,\psi,\vartheta)$ must be periodic for arguments ψ and ϑ with the period 2π and not contain the first harmonics of the phase of the own oscillations. The specified will be executed on condition

(12) $$\int_0^{2\pi} U(a,x,\psi,\vartheta)\begin{Bmatrix}\sin\psi \\ \cos\psi\end{Bmatrix} d\psi = 0\ .$$

The physical content of the superimposed condition is as following: the amplitude of the dynamic process coincides with the amplitude of its first mode, and the function $\mathrm{U}(a,x,\psi,\vartheta)$ only affects the form of oscillations and is determined by the right-hand side of equation (10). In the case of unknown functions of time $\mathrm{a}(t)$ and $\psi(t)$, then they are determined in such a way that the asymptotic representation of the solution with the accuracy to the second order of smallness satisfies the initial equation and boundary conditions. The basis for finding it is the differential equation

(13) $$\frac{\partial^2 U}{\partial\psi^2}\omega^2 + \frac{\partial^2 U}{\partial\vartheta^2}\mu^2 + 2\frac{\partial^2 U}{\partial\psi\partial\vartheta}\omega V + \alpha^2\frac{\partial^4 U}{\partial x^4} = 4a\omega\frac{da}{dt}\sin\frac{\pi x}{l}\sin\psi + 4a\frac{d\psi}{dt}\omega\times \times s\mathrm{in}\frac{\pi x}{l}\cos\psi - V^2 a\left(\frac{\pi}{l}\right)^2\sin\frac{\pi x}{l}\cos\psi + +2Va\omega\frac{\pi}{l}\cos\frac{\pi x}{l}\sin\psi + F(a,x,\psi,\vartheta),$$

where: $\mathrm{F}(a,x,\psi,\vartheta)$ - known function.

Practical implementation of the proposed method of studying the process of vibroseparation or adhesion of friable media

Finding the laws of change of the defining parameters of the oscillations of a friable media - a sieve solves the problem. Therefore, the following will be the main focus it is concentrated precisely on determining the laws of changing these oscillations of the specified system. It must be emphasized that it is exposed to external periodic perturbations frequency μ. The result of this action essentially depends on the ratio between μ that ω, that is, between the frequency of external perturbations and the frequency of its own oscillations. If there is a connection between the given frequencies $p\mu \approx q\omega$ (p and q are mutually prime numbers), this case of oscillations is called resonance, and it is characterized by a significant increase in the amplitude of oscillations. A simpler case of oscillations is nonresonance, i.e. $p\mu \neq q\omega$. Without going into the mathematical procedure of finding these functions, we give only the resulting dependencies for the case when $q(u,x,\vartheta(t) = \delta\left(\frac{\partial u}{\partial t}\right)^s + h\cos\mu t$, h, S, δ - constants. Thus, for the nonresonance case, the amplitude-frequency response of the system oscillations is determined by the system of ordinary differential equations

(14) $$\frac{da}{dt} = -\frac{\delta}{m+m_1}(\omega)^{s-1}a^s,\quad \frac{d\psi}{dt} = \omega - \frac{3\varepsilon}{32}\frac{\pi^2}{l^2}\frac{a^2}{\omega(m+m_1)} - -\left(\frac{\pi}{l}\right)^2\frac{m_1}{m+m_1}\frac{V^2}{8\omega}.$$

Figures 2 and 3 show the laws of the change in the amplitude and frequency of oscillations according to the different kinematic characteristics of the friable media, in accordance with the dependences (13), at $E=2{,}06\cdot10^9 N/m^2$, $l=2m$, $m_1=25kg/m$, $m=7kg/m$.

The calculations are made for different relative velocities: 1) $V=0$ m/s; 2) $V=2{,}5$ m/s; 3) $V=3{,}5$ m/s; 4) $V=7{,}5$ m/s;

(15) $$\omega_1=\omega-\frac{3\varepsilon}{32}\frac{\pi^2}{l^2}\frac{a^2}{\omega(m+m_1)}-\left(\frac{\pi}{l}\right)^2\frac{m_1}{m+m_1}\frac{V^2}{8\omega},$$

The obtained dependencies show that for a nonresonance case, the influence of the small-size external periodic perturbation on the transverse vibrations of a system friable media - a sieve is insignificant. To a greater extent, the influence of the velocity is manifested, more precisely, of the relative amount of motion on both the amplitude and the frequency of damped oscillations, namely:

- for friable media with more relative motion along the elastic sieve, the rate of attenuation of their own oscillations is greater;
- the own frequency of the system of a moving flow of friable media - a sieve is smaller at higher values of the relative amount of motion of the friable media.

The obtained theoretical results have practical significance not only for the evaluation of the separation process, vibration processing or adhesion, but also serve as a basis for avoiding such undesired processes as resonance.

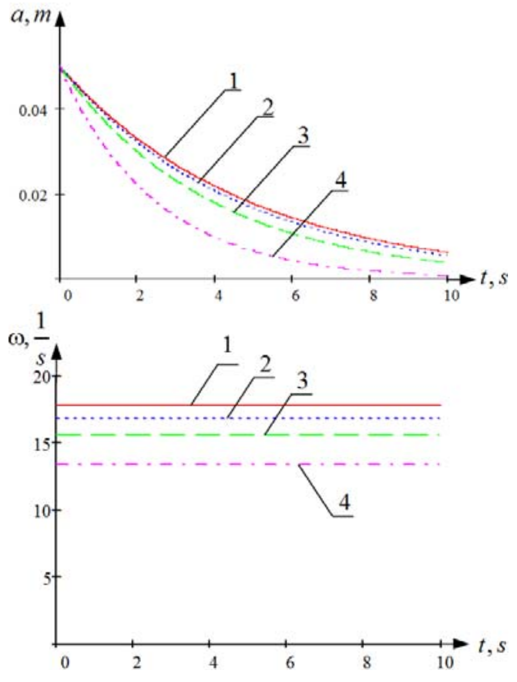

Fig. 2. Time dependence of the amplitude and frequency of a system of a friable media - a sieve with different kinematic characteristics of friable media

More complicated is the resonance case of the studied system. Below we will stop only on the case of the main resonance, i.e. $\mu\approx\omega$. As shown in [11,12,14-16], resonant amplitude value greatly depends on the phase difference of its own and forced oscillations. In addition, the amplitude of the passage of the resonance significantly depends on the rate of change in the frequency of the force in the resonance zone. In the same way as for a nonresonance case, we will not go into mathematical calculations, but let us give only the dependencies that describe the amplitude of the system's friable media - the sieve. Thus, the basic parameters describing the resonance oscillations of the system under study in the case of the main resonance are of the form

(16) $$\begin{aligned}\frac{da}{dt}&=-\frac{\bar{\delta}}{m+m_1}(\omega)^{s-1}a^s-\\&-\frac{2\delta h}{\pi(\omega+\nu(t))(m+m_1)}\cos\gamma,\\ \frac{d\gamma}{dt}&=\omega-\mu(t)-\left(\frac{\pi}{l}\right)^2\frac{m_1V^2}{8\omega(m+m_1)}+\\&+\frac{2h}{\pi(\omega+\mu(t))(m+m_1)a}\sin\gamma-\frac{3\varepsilon}{32}\frac{\pi^2}{l^2}\frac{a^2}{\omega(m+m_1)},\end{aligned}$$

where γ - the phase difference between the own and forced oscillations, and the frequency $\mu(t)$ in (15) takes close to the frequency of its own oscillations ω.

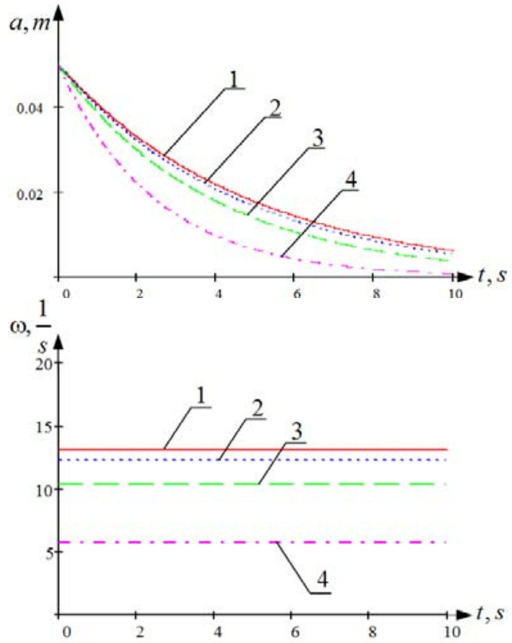

Fig. 3. Time dependence of the amplitude and frequency of a system of a friable media - a sieve, on condition that m_1=50 kg/m

In fig. 4 and 5 corespondently to differential equations (15), are represented the law of variation of the amplitude of oscillations for the different values of the friable media $(k=m_1V)$.

The presented dependencies show that the resonance phenomenon of the moving along the sieve of a friable media is determined not only by the frequency of the external periodic perturbation but also by the number of its relative motion along the last. For friable media with linear mass, which differ in a small amount at identical velocities along the sieve, the amplitudes of the transition through the resonance are almost identical. For the passage of the resonant area of friable media with a greater number of relative motion, the amplitude of passage of the resonance is less. The above mentioned can serve as a basis to certain extent for the creation of means for suppressing the oscillations of certain mechanical systems.

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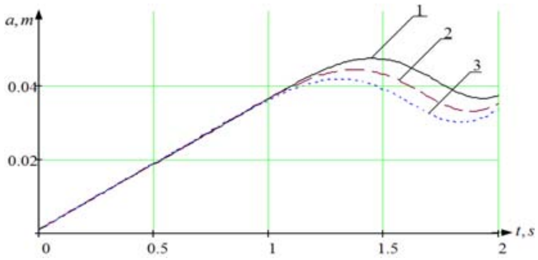

Fig. 4. The law of variation of the amplitude of oscillations transition through the main resonance for the big values of relative r move of the media: 1-k=60 kg·m/s; 2-k=120 kg·m/s; 3-k=180 kg·m/s

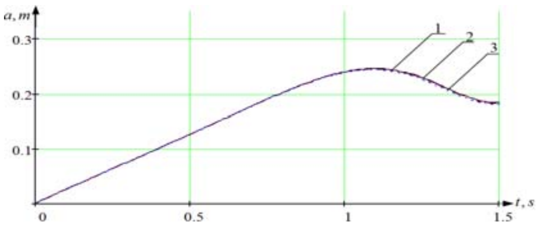

Fig. 5. The law of variation of the amplitude of oscillations in the transition through the main resonance at: 1-k=10 kg·m/s; 2-k=20 kg·m/s; 3-k=30 kg·m/s

The basic idea of the above work can be used for the case of nonlinear longitudinal oscillations of moving friable media along the sieve. A mathematical model of dynamics of a friable medium - a sieve will be a differential equation for the corresponding boundary conditions.

(17)
$$\left(m+m_1\right)\frac{\partial^2 u\left(x,t\right)}{\partial t^2}+2m_1 V\frac{\partial^2 u\left(x,t\right)}{\partial t\partial x}- -\left(ES-m_1V^2\right)\frac{\partial^2 u\left(x,t\right)}{\partial x^2}=q\left(x,u,u_t,\vartheta\left(t\right)\right).$$

In equation (16) all symbols are saved, the same as for the case of transverse oscillations, only $u(x,t)$ longitudinal movement cross section of the sieve with a coordinate x at any moment time t, and S - cross-sectional area of the sieve.

Conclusions

The obtained theoretical and constructed graphic dependencies show:

- with increasing in the relative amount of motion the friable media along the sieve, own frequency of oscillations to the systems "a friable medium - a sieve" decreases, which negatively affects the process of vibration processing or separation;
- or larger values of the relative amount of motion of a friable medium, velocity attenuation amplitude increases;
- amplitude of oscillations when passing through the main resonance takes less value for a more rapid change in the frequency of the force in the resonance region;
- the amplitude of passage of the resonance is less for larger values of the relative amount movement of a friable medium.

At the same time, the basic results of the work can be used to study the dynamics of the system “a friable media – a sieve”, as well as more general properties of the friable media and slowly variable its speed along the sieve.

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