A model of destructive processes based on interval fuzzy rough soft sets

Abstract. This work presents a spatial model of destructive processes for the real-time GIS-based decision support systems. A dynamic fuzzy rough soft topology represents a structure of a geoeotechnogenetic system that contains a multitude of interacting processes, which evolve in space and time. In disaster conditions, some of the interacting processes can be destructive. Their dynamics are modeled using the spread model. The area of interest is represented as an approximation by a grid of cubic cells. This allows taking into account the peculiarities of the initial information obtained from drones using remote sensing techniques and having a significant uncertainty. The proposed model reduces the computational complexity and provides the acceptable performance of real-time DSS.


Model procesów destrukcyjnych opartych na interwałach rozmytych zbiorach przybliżonych.

Keywords: destructive processes, spatial model, fuzzy rough soft set, grid of cells, blurred boundaries

Słowa kluczowe: procesy destrukcyjne, model przestrzenny, rozmieć, przybliżone zbiorzy miękkość, siatka komórek, niewyraźne granice.

Introduction

Complex systems containing territories with natural and artificial objects as well as a multitude of interacting processes, which evolve in space and time, can be considered as geoeotechnogenetic systems (GETS). Some processes arising within GETS are destructive because they give rise to a danger and risk to the certain valuable objects causing their destructions and can often lead to critical situations or emergencies.

Solving decision support problems in disaster situation requires real-time geographic information systems (GIS) containing a spatial model of confined space (area of interest, AOI), where the destructive processes take place, as well as adequate models of the destructive processes exposed onto the spatial model. However, the most of the destructive processes are poorly observed and their spreading over the AOI is weakly modeled, so developing decision support systems (DSS) is a complex and non-trivial task, which becomes more complicated due to uncertainty of information, a wide geographically distribution of events and, as usual, a lack of time [1]. The efficiency of decision-making strongly depends on the availability of online disaster monitoring tools aimed at the real-time computation of the most important parameters related to the spreading of the destructive processes.

Today, a suite of the most modern methods and techniques, such as remote sensing, GIS, geospatial analysis, unmanned aerial vehicles (UAV), can be synergistically used to build GIS-based DSS. Remote sensing techniques allow generating a full range of data for disaster monitoring [2], which have a form of streams of great volumes that come from sensors on a continuous basis at a high rate and should be analyzed in a real-time [3]. UAVs can effectively perform long-time missions to obtain remote sensing data [4]. However, due to the instrumental inaccuracy and distortions caused by vibrations, remote sensing information obtained from UAVs is incomplete, imprecise, vague, and often blurred [5]. The dynamics of the spreading destructive process depends on the accuracy of determining the boundaries of its dynamic contour. However, the uncertainty of observations significantly reduces the accuracy of determining the boundaries of such contours [6]. Obtained remote sensing data should be correctly transferred to the disaster spread model, geolocated and mapped to the AOI.

Using the well-established approaches for the spatial modeling do not provide the required performance and acceptable efficiency of GIS-based real-time DSS in disaster conditions [7]. A key aspect to achieve the desired performance is to build an approximate spatial model of spreading destructive processes, taking into account their partial observability and uncertainty of observations. Thus, we need to soften the requirements for the accuracy of remote sensing data representation, which will give an opportunity to improve DSS performance. In this case, boundaries of the dynamic contours of the spreading processes can be vague and blurred.

There are several well-known approaches to deal with the uncertainty and vagueness in the spatial models, such as fuzzy set theory [8], rough set theory [9] and soft set theory [10]. Each of these approaches has its inherent difficulties as pointed out in [10]. It should be noted that due to the absence of some important information a priori, such as membership functions for fuzzy sets, equivalence relations for rough sets, or parameterizations for soft sets, these approaches cannot ensure the adequacy of the spatial model of the destructive process independently. Therefore, many researchers combine some of these approaches. Some authors proposed to use for spatial modeling the combinations of rough and fuzzy sets [1], rough and soft sets [11], fuzzy and rough sets [12]. In [13], the authors proposed the concepts of rough fuzzy soft sets and fuzzy rough soft sets, which have a number of advantages to build a blurred spatial model. Based on this, we can use soft topological spaces to build a spatial model of the destructive process, as well as the fuzzy rough method for its blurring [21,22].

The aim of this work is to develop the approximate spatial model of the destructive process within GETS in disaster conditions [14]. To overcome the computational complexity problem, we build a topological spatial model and soften the effects of discretization using the fuzzy rough sets. The
Modeling dynamics of gets in disaster conditions

Let us consider the AOI as an open connected subspace $X$ of two-dimensional Euclidean space endowed with the topological properties [15]. To build a topological space on $X$ we use an equivalence relation $\mathcal{R}_X \subseteq X \times X$ (reflexive, symmetric, and transitive) [1]. Then the pair $\text{apr}_X = (X, \mathcal{R}_X)$ is called the approximation space. The family of all composite sets is denoted by $\text{Def}(\text{apr}_X)$ and uniquely determines the topological space:

$$T = (X, \text{Def}(\text{apr}_X)).$$

Suppose that each point $x \in X$ has a non-empty finite set of attributes $A$, $V_x$ is a domain of $a \in A$ and $f$ is a function such that $f : X \times A \to V$. Let’s impose a metrical grid of coordinate lines with $\delta = \Delta \alpha_1 = \Delta \alpha_2 = \Delta \alpha_3$ within $X$, which form a set $C$ of square cells with the size being $\delta \times \delta \times \delta$. Thus, space $X$ is discretized by a grid $C$ of isometric square cells $c \in C$. Assume that a cell $c \in C$ is a spatial homogeneous object of a minimal size. Each cell $c \in C$ is associated with a set of attribute values, which is called the cell state, via the value function $f(c, A)$. The proposed discretization assigns equal values of the attributes to each point belonging to a certain cell $c$, therefore each cell $c \in C$ represents a homogeneous area of the AOI in terms of attribute values $A$, so it can be reduced to a point of $X$. It’s suggested to model disaster dynamics in GETS by means of a change of states of the cells covered by the disaster.

Suppose the set of attributes $A$ can be divided into subsets: not changing over time (static) attributes $A_0$, time-varying (dynamic) attributes $A_{x_1}$, slowly changing (environmental) attributes $A_{x_2}$. $A = A_0 \cup A_{x_1} \cup A_{x_2}$. Suppose $W = \{w_0, w_1, \ldots, w_k\}$ is an ordered set of the cell state categories (phases), where $w_0$ is the initial phase, $w_1$ is the final phase, and each $w_i$ is the transitional phase. We consider each significant change of the cell attribute’s value, which forces the cell to change its state, as an event. Assume, during the destructive process, the cell moves through a sequence of qualitatively different categories of states, which should be evaluated during continuous remote sensing. It is clear that the model of the destructive process can be represented as a model of dynamic change of states of a subset of cells covered by the process within the spatial model. Thus, the GETS structure can be represented as a topology space, which includes subspaces of cells of the same phase and makes it possible to assess the position and boundaries of the dynamic contour of the process. Since the belonging of each cell to a certain phase is determined approximately due to the uncertainty of remote sensing, the topological space describing the structure of GETS as well as the boundaries of the contour of the spreading process are blurred [26,27,28].

Representing structure system using soft sets

GETS structure at each time moment $t$ can be represented as a set of subsets of cells with the states being in one or another phase.

It’s suggested to represent GETS structure at each moment $t$ in the form of a soft set

$$Y_{w_0}(t) = \{(w, Y_{w_0}(w, t)) : w \in 2^{\mathcal{R}_0}, Y_{w_0}(w, t) \in 2^C\},$$

where $Y_{w_0}(w, t)$ is $w$-element of the soft set, namely a set of cells with the states belonging to the phase $w \in W_0$ at time moment $t$. Fig.1 shows a structure of GETS in the form of the soft set, which breaks down the set of cells into three subsets such as $w_1^+$, $w_1^-$, and $w_2$-elements.

### Fig. 1. GETS structure in the form of the soft set

Let $\text{Def} (Y_{w_0})$ be a set of all compositional sets of the soft set $Y_{w_0}(t)$. Then GETS structure can be represented in the form of topological space $T_{t=\infty} = (C, \text{Def} (Y_{w_0}))$.

Representing gets structure using fuzzy soft sets

By applying different ways of blurring to the soft set, we can obtain blurred GETS structures, which can be represented in the form of blurred topological spaces. The most general way of blurring is using fuzzy sets. Let $L$ be the interval $[0,1]$, $2^C$ be a family of all strict subsets of the set of cells $C$, and $\mu^C$ be a family of all fuzzy subsets of the set of cells $C$. Each fuzzy set is a mapping $\mu^C(t) : C \to L$. Then at each time moment, the blurred GETS structure can be represented as fuzzy soft set [20].

### Fig. 2. Blurring soft set using fuzzy set

It subdivides the set of cells $C$ into the phases (Fig.2):

$$Y_{w_0}(t) = \{(w, \tilde{Y}_{w_0}(w, t)) : w \in 2^{\mathcal{R}_0}, \tilde{Y}_{w_0}(w, t) \in \mu^C\}$$

where $\tilde{Y}_{w_0}(w, t) = \{(c, \tilde{Y}_{w_0}(w, t)(c)) : c \in C\} = \tilde{C}^+ (t)$, $\tilde{C}^+ (t) = \{(c, \tilde{C}^+(c)(t)) : c \in C\}$ is a fuzzy set of cells with the state (phase) $w \in W_0$ at the time moment $t$, $\tilde{Y}_{w_0}(w, t)(c) = \tilde{C}^+(c)(t)$ is a degree of membership of the cell $c$ to the fuzzy set $\tilde{C}^+$ of cells with the states...
belonging the phase \( w \in W_d \) (\( w \)-element of the soft set \( Y_{w_d} \)) at the time moment \( t \).

**Definition.** \( \forall t \in T \) a set \( \tau(t) = \text{Def}(\tilde{Y}_{w_d}(t)) \subseteq L_r \) is a fuzzy topology \( \tau(t) \) in the set \( C \), which meets the following conditions [16]:

a) \( \forall t \in T, \left\{ (\tilde{0}, \tilde{1} \in \tau(t)) \right\} \) or \( \left\{ (\tilde{1}, \tilde{C} \in \tau(t)) \right\} \);

b) \( \forall t \in T, \left\{ (C^+ (t), C^- (t) \in \tau(t)) \Rightarrow (C^+ (t) \wedge C^- (t) \in \tau(t)) \right\} \);

c) \( \forall t \in T, \left\{ (C^+ (t), j \in J \subseteq \tau(t)) \Rightarrow (\cup_{j \in J} C^+ (t) \in \tau(t)) \right\} \).

A pair \( T^{w_d}_C (t) = (C, \tilde{\tau}(t)) \) is called Chang topological space.

**Representing the gets structure using rough soft sets**

The fuzzy soft set is the most detailed description of blurred GETS structure, but in practice, determining the degrees of membership of cells to the phases can be impossible. If such degrees are unknown, the simplest way of blurring soft topology is using rough sets [9], which are built on the basis of determining the lower approximation (subset of cells belonging to the rough set unambiguously), upper set (subset of cells, which possibly belong to the rough set), and boundary set (subset of cells with the degrees of membership in the rough set being unknown) [29,30,31].

Let \( R^{w_d}_C \) be indiscernibility relation in the set of cells \( C \) such as \( R^{w_d}_C = \left\{ (c, c') \in C \times C \left| f(c, w_d) = f(c', w_d) \right. \right\} \), where \( f(c, w_d) \) is the phase, which corresponds to the state cell \( c \). Then \( \text{appr}^{w_d}_C = (C, R^{w_d}_C) \) is Pawlak approximation space [9].

**Definition.** Let \( Y_{w_d} = (Y, W_d) \) be a soft set in the set of cells \( C \). Then \( Y_{w_d} = \left( \overline{Y}, \overline{W_d} \right) \) is a lower rough approximation, and \( \overline{Y}_{w_d} = \left( \overline{Y}, \overline{W_d} \right) \) is an upper rough approximation of the soft set \( Y_{w_d} \). They constitute soft sets [18], such that

\[
\overline{Y}_{w_d}(w) = \left\{ c \in C \left| R^{w_d}_C(c) \subseteq \overline{Y}_{w_d}(w) \right. \right\},
\]

and

\[
\overline{Y}_{w_d}(w) = \left\{ c \in C \left| R^{w_d}_C(c) \cap \overline{Y}_{w_d}(w) \neq \emptyset \right. \right\}, \quad \text{for all } w \in W_d.
\]

If \( Y_{w_d}(w) = \overline{Y}_{w_d}(w) \), \( w \)-element of the soft set \( Y_{w_d} \) is a strict set, in other cases it is a rough set (Fig.3).

**Definition.** A rough set of cells with the states being in the phase \( w \) at the time moment \( t \) is determined by two approximations:

\( \overline{Y}_{w_d}(w,t) \) is a lower approximation containing the cells, which definitely belong to the set \( \tilde{Y}_{w_d}(w,t) \);

\( \overline{Y}_{w_d}(w,t) \) is an upper approximation containing the cells, which possibly belong to the set \( \tilde{Y}_{w_d}(w,t) \).

**Definition.** Negative area of the rough set \( \tilde{Y}_{w_d}(w,t) \) is a subset of cells \( C \), which definitely don’t belong to the set of cells with the states being in the phase \( w \): \( \text{NEG}(\tilde{Y}_{w_d}(w,t)) = C - \tilde{Y}_{w_d}(w,t) \).

**Definition.** Boundary area of the soft set \( \tilde{Y}_{w_d}(w,t) \) is a subset of the set of cells \( C \), which belong to the upper approximation \( \overline{Y}_{w_d}(w,t) \), but don’t belong to the lower approximation \( \tilde{Y}_{w_d}(w,t) \).

Let \( \text{Def}(\tilde{Y}_{w_d}(w,t)) \) be a family of all fuzzy rough sets, which represent the sets of cells belonging to one or another phase.

**Definition.** A set \( \tilde{\tau}(t) = \text{Def}(\tilde{Y}_{w_d}(t)) \) is a rough topology in \( C \). At each time moment, the pair \( \tilde{Y}^{w_d}_C (t) = (\overline{Y}, \tilde{\tau}(t)) \) is the rough topological space.

**Representing the gets structure using fuzzy rough soft sets**

Using remote sensing techniques during monitoring often allows assessing gradations of belonging the cells from rough sets boundary area to a certain rough set of cells. It is possible by means of visual information describing different gradations of gray or brightness. Such information allows representing GETS structure in the form of the fuzzy rough soft set [19].

The fuzzy rough soft set of cells, which subdivides the set of cells into phases at each time moment \( t \) is represented as the set of three elements consisting of upper and lower approximation of the rough set (Fig.4), as well as the boundary area of the rough set in the form of the fuzzy set:

\( \tilde{Y}_{w_d}(t) = \left\{ \tilde{Y}_{w_d}(t), \tilde{Y}_{w_d}(t), \text{BND}(\tilde{Y}_{w_d}(t)) \right\} \).

The fuzzy rough soft set subdivides the set of cells into \( w \)-elements, each of which is a rough set of cells belonging to a certain phase \( w \in W_d \):

\( \tilde{Y}_{w_d}(w,t) = \left\{ \tilde{Y}_{w_d}(w,t), \tilde{Y}_{w_d}(w,t), \text{BND}(\tilde{Y}_{w_d}(w,t)) \right\} \),

where \( \text{BND} \left( \tilde{Y}(w,c,t) \right) \) is a degree of belonging the cell \( c \), which is included in the boundary area \( c \in \text{BND} \left( \tilde{Y}_{w_d}(w,t) \right) \) of the rough set \( \tilde{Y}_{w_d}(t) \), to the fuzzy set \( \text{BND}(\tilde{Y}_{w_d}(w,t)) \) at time moment \( t \).
Let us consider a set of intervals \([20, 32]\).

The interval fuzzy set on the set \(W\) is the interval fuzzy rough topological space.

**Definition.** The GETS structure at time moment \(t\) can be represented as a fuzzy rough soft set of cells:

\[
\hat{Y}_{w_o}(t) = \left\{ \left( w, \hat{Y}_{w_o}(w, t) \right) : w \in W_o \right\},
\]

where each rough set of cells \(\hat{Y}(w, t)\) is \(w\)-element of the fuzzy rough set \(\hat{Y}_{w_o}(t)\).

Let \(Def(\hat{Y}_{w_o})\) be a family of all fuzzy rough sets, which represent the sets of cells belonging to one or another phase.

**Definition.** The set \(\hat{r}(t) = Def(\hat{Y}_{w_o}(t))\) is fuzzy rough topology in \(C\). At time moment \(t\), the pair \(\hat{T}_{w_o}^\tau(t) = (C, \hat{r}(t))\) is the fuzzy-rough topological space.

**Representing the gets structure using interval fuzzy rough soft sets**

Each \(w\)-element of the fuzzy-rough set contains a fuzzy set of cells belonging to the boundary area of a rough set and setting the degree of membership of each cell of the boundary region to the phase \(w\). In practice, setting the exact value of membership degree is impossible, but it is possible to assess the intervals to which such values can belong. In this regard, the authors have been suggested to model boundary area in the form of an interval fuzzy set [20, 32].

Let us consider a set of intervals

\[
L' = \left\{ [a, b] : 0 \leq a \leq b \leq 1 \right\}
\]

and a partial order relation \(\leq_{PL}\), such that

\[
[a, b] \leq_{PL} [a', b'] \iff a \leq a', b \leq b', \forall [a, b], [a', b'] \in L'.
\]

The interval fuzzy set on the set \(C\) is a mapping [20]

\[
\hat{C}^i : C \rightarrow L', \quad \text{where} \quad \hat{C}^i(c) = \left[ \hat{C}_c^i(c), \hat{C}^{ui}_c(c) \right], \forall c \in C,
\]

\(\hat{C}_c^i(c)\), and \(\hat{C}^{ui}_c(c)\) are lower and upper degrees of membership of the cell \(c\) to the fuzzy set \(\hat{C}^i(c)\). The interval fuzzy set of cells belonging to the boundary area of the soft set \(\hat{Y}_{w_o}(w, t)\) corresponding to the phase \(w\) at the time moment \(t\), can be defined in the following way:

\[
\hat{BND}^i(\hat{Y}_{w_o}(w, t)) = \left\{ \left( w, \hat{BND}^i(\hat{Y}_{w_o}(w, c, t)) \right) : w \in W_o \right\},
\]

\(c \in BND(\hat{Y}_{w_o}(w, t))\), where \(\hat{BND}^i(\hat{Y}(w, c, t))\) and \(\hat{BND}^{ui}(\hat{Y}(w, c, t))\) are lower and upper degrees of membership of the cell \(c\) to the fuzzy set \(\hat{BND}(\hat{Y}(w, t))\), which represents the boundary area of the soft set \(\hat{Y}(w, t)\) of cells whose states belong to the phase \(w\) at time moment \(t\).

The interval fuzzy rough soft set of cells, which subdivides the set of cells into the phases, at each time moment \(t\) is represented as the set (Fig. 5):

\[
\hat{Y}_{w_o}^t(w, t) = \left\{ \left( w, \hat{Y}_{w_o}(w, t), BND'(\hat{Y}_{w_o}(w, t)) \right) \right\}
\]
The results of the experiment shows that the proposed model provides acceptable performance in terms of accuracy and speed for all kind of topology. The fastest performance is demonstrated by the rough soft topology; however, it does not provide acceptable accuracy. The fuzzy topology significantly loses in terms of speed. The interval fuzzy rough topology shows sufficient results on the speed with enough accuracy.

**Conclusions**

The approximate spatial model of destructive processes for the real-time GIS-based DSS based on the interval fuzzy-rough soft sets is proposed. The model of the destructive process is represented as the model of dynamic change of states of the subset of cells covered by the process within the spatial model. As a result, the structure of GETS is represented as a topology space, which includes subspace of cells that are in the same phase. The soft topological spaces are used to build a spatial model, as well as the fuzzy-rough method is used for its blurring. Since the belonging of each cell to the certain phase is approximately determined due to the uncertainty of remote sensing, the topological space describing the structure of GETS is blurred, and the boundaries of the dynamic contour are also blurred. The proposed spatial model representing uncertain information about the disaster reduces the computational complexity and provides flexible and timely decision-making in real time.

**Authors:** prof. Dr. Techn. Sc., Volodymyr G. Sherstjuk, Kherson National Technical University, Berislavskiy shosse, 24, 73040, Kherson, Ukraine, E-mail: v.gsherstjuk@gmail.com, Ph.D., Associate Professor, Maryna V. Zharikova, Kherson National Technical University, Berislavskiy shosse, 24, 73040, Kherson, Ukraine, E-mail: marina.zharikova@gmail.com; Prof. Waldemar Wójcik, Lublin University of Technology, Institute of Electronics and Information Technology, Nadbystrzyska 38A, 20-618 Lublin, Poland, e-mail: Waldemar.Wojcik@pollub.pl; M.Sc. Aigul Syzdykpayeva, Sarsen Amanzholyev East Kazakhstan State University, email: aigul.uk@list.ru; M.Sc. Kuanysh Muslimov, Kazakh National Research Technical University named after K.I.Satpayev, email: k-u-a@mail.ru.

**REFERENCES**


[28] Analysis of microcirculatory disorders in inflammatory processes in the maxillofacial region on based of optoelectronic


