

## Development of a multi frequency phase method for measuring ranges

**Abstract.** The results of the development of a multi frequency phase method for measuring distance are presented. Mathematical transformation of a system of equations connecting complex amplitudes of harmonic summation signals and complex amplitudes of signals reflected from each object, the result of which is the obtaining of a polynomial equation. The solution of the polynomial equation gives the values of the unit vectors of signals reflected from each target.

**Streszczenie.** W pracy przedstawiono wyniki opracowania metody wieloczęstotliwościowej dla pomiaru odległości. Rozwiązanie równania wielomianowego daje wartości wektorów jednostkowych sygnałów odbijanych od każdego celu. (Opracowanie metody fazowej wieloczęstotliwościowej dla zakresów pomiarowych).

**Keywords:** phase method of measurement; range; radar; complex amplitude

**Słowa kluczowe:** fazowa metoda pomiaru, zakres, złożona amplituda

### Introduction

In the field of radar sounding, impulse methods for measuring ranges are traditionally used [1,2]. These methods are clear and easy to measure. But the finite duration of the probe pulse limits the accuracy and resolution of the methods. The use of frequency methods makes it possible to improve the accuracy of measuring distances [1,2]. Phase methods for measuring range are known [1,3]. These methods allow measuring the range of only one object, but with high accuracy. Also, phase methods, however, like frequency ones, do not have a "dead zone", which allows them to have a clear advantage over pulsed methods when measuring short ranges. But in the case of frequency methods, probing signals have a wider bandwidth than in the case of phase methods. The range of sounding frequencies is strictly limited to the minimum required quantity that allows to obtain the given accuracy, which has a distinct advantage over other methods.

When performing range measurements, the main task is to separate the signals reflected from each object. The developed multi-frequency phase methods allow me to separate the signals reflected from objects by analyzing the amplitude-frequency and phase-frequency characteristics of the total reflected signal, which represents the sum of the harmonic signals reflected from each object in a given frequency range [4-6]. Let us cite short results of the studies which made it possible to obtain a mathematical model on the basis of which it became possible to develop a multi frequency phase method for measuring distances to many objects [7].

Unlike time measurements, when signals reflected from each object are separated in time. In phase measurements, all harmonic signals reflected from all objects act simultaneously, and therefore, as the medium in which radio signals propagate can be considered linear systems, they are algebraically summed at each point.

### Method

Considering a system of N objects that are sounded by a harmonious signal, the condition for choosing the wavelength of the probing frequency is the correspondence of half the wavelength of the maximum measurable range in order to avoid the phenomenon of phase ambiguity.

As a result of probing objects with a harmonious signal with a frequency that satisfies the condition of absence of

phase ambiguity [1-2,6], it will be reflected from each object with a certain phase shift. The phase shift of the received signal can be determined by the formula:

$$(1) \quad \phi_i = \frac{4\pi l_i}{L_{max}} = \frac{8\pi l_i}{c} f_1,$$

where:  $l_i$  – is the range of the i-th object,  $L_{max}$  – the maximum measurable range equal to half the wavelength of the sounding harmonic signal,  $f_1$  – frequency of the sounding signal,  $c$  – velocity of propagation of an electromagnetic wave (speed of light).

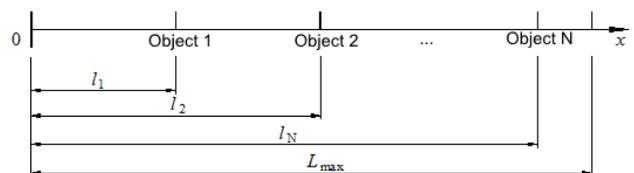


Fig. 1. Location of N radar sounding objects

The amplitudes of the reflected signals will be determined by the reflection coefficients of the objects. Then:

$$(2) \quad a_i = k_i \cdot a,$$

where:  $k_i$  – coefficient of reflection from the i-th object,  $a$  – amplitude of the sounding harmonic signal.

Thus, the resulting signal is a harmonic signal that carries information about each of the objects. In general, such a signal is a function of the range of each object and the reflectance of each object:

$$(3) \quad s_2(t) = f(l_1, l_2, \dots, l_n, k_1, k_2, \dots, k_n).$$

However, any signal is affected by a number of factors due to the passage of radio signals through physical media. These factors include the effect of attenuation in the material  $\alpha$ , the dispersion of signals during the passage of signals through the medium  $D$ , the partial transmission of signals through the object (the effect of transmission factors  $k_{ifn}$ ), and the partial absorption of signals by means of (absorption coefficient  $k_{aci}$ ). With all these factors, the signal will look like this:

$$(4) \quad s_2(t) = f \left( l_1, l_2, \dots, l_n, k_1, k_2, \dots, k_n, \alpha, D, k_{if1}, k_{if2}, \dots, k_{ifn}, k_{ac1}, k_{ac2}, \dots, k_{acn} \right).$$

A large number of parameters that affect the result of passing signals on Wednesdays do not allow us to establish

the basic regularities and relationships. Therefore, to develop the theoretical foundations of multi frequency phase measurements of ranges, it is proposed to use the generally accepted approach - from simple to complex. To identify the main regularities and relationships of multi frequency phase measurements, it is proposed to develop a number of simplified physical models for the passage of radio signals over a physical medium with several objects located at different distances on the same line. The simplest model should consider only the factors that affect the passage of signals. Each next model should take into account more factors. On the basis of the proposed physical models, it is necessary to develop mathematical models of signal propagation through physical media with several objects.

For a simplified consideration of the passage of signals over a physical medium, the following physical model can be proposed. This physical model is based on the following principles: All signals pass through the medium without damping; The propagation velocities of the radio wave in all media are the same; When the signal reaches the location of each object when it propagates in the forward direction, the signal is reflected according to the reflection coefficient of this object taking into account its electro physical parameters; When propagating signals in the forward and reverse directions, signals on objects are not absorbed and pass without distortion.

The latter principle contradicts the previous point, but this is the destiny of idealization for isolating the general patterns of signal transmission on media with several objects. According to the proposed principles for constructing a physical model of an idealized physical medium with several objects, let us consider the passage of harmonic signals. When carrying out measurements by the phase method, sounding signals have harmonic signals. And, with radar measurement, there are two-frequency or amplitude-modulated signals. In the most general case, harmonic sounding signals are used. To increase the accuracy increase the frequency, which leads to a proportional increase in accuracy. [1, 3, -8] In the presence of two or more objects, the sounding signal is reflected from each object in proportion to the effective scattering area. Therefore, all reflected signals return to the receiving point. All these signals are added together and a single total signal with a certain phase shift and amplitude  $a_{\Sigma}$  can be found from the following expressions:

$$(5) \quad a_{\Sigma}(j) = \sqrt{\left(\sum_{i=1}^N a \cdot k_i \cdot \cos(j\phi_i)\right)^2 + \left(\sum_{i=1}^N a \cdot k_i \cdot \sin(j\phi_i)\right)^2},$$

$$(6) \quad \phi_{\Sigma}(j) = \arctg \frac{\sum_{i=1}^N a \cdot k_i \cdot \sin(j\phi_i)}{\sum_{i=1}^N a \cdot k_i \cdot \cos(j\phi_i)},$$

where:  $a$  - the amplitude of the sounding signal;  $k_i$  - coefficient of reflection from the  $i$ -th object;  $\phi_i$  - phase shift of the signal reflected from the  $i$ -th object;  $N$  - number of objects;  $j$  - number of the sounding signal frequency [9-11]. Consider the case of two objects. With a vector description of the signals, the total signal will be the result of the vector sum of the signals reflected from both objects. Vector diagrams are shown in figure 2.

And if, as the frequency of the signal vector increases, the reflected signals from each object rotate in proportion to the frequency change, then the total signal changes its value nonlinearly.

As a result of sounding objects with a harmonious signal with a frequency, agreed with the condition of unambiguous range measurement, there will be a reflection from each

object with a certain phase shift, which can be determined from the expression:

$$(7) \quad \phi_i = \frac{2\pi l_i}{L_{\max}} = \frac{4\pi l_i}{c} f_1,$$

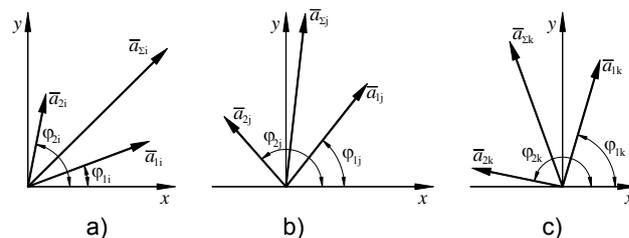


Fig. 2 Vector diagrams of signals reflected from two objects and the total signal on the  $i$ -th (a),  $j$ -th (b) and  $k$ -th (in) frequencies

$$(8) \quad \phi_i = \frac{4\pi l_i}{L_{\max}} = \frac{8\pi l_i}{c} f_1.$$

The amplitudes of the reflected signals will be determined by the reflection coefficients of the objects then:

$$(9) \quad a_i = k_i \cdot a.$$

The total signal is the sum of the signals reflected from each object:

$$(10) \quad s_{\text{ref}}(t) = \sum_{i=1}^N s_{\text{ref}_i}(t).$$

Because the sounding signal is harmonic, the total signal is described by the expression:

$$(11) \quad s_{\text{ref}}(t) = \sum_{i=1}^N a \cdot k_i \cdot \cos(\omega t - \phi_i).$$

By writing the cosine of the difference by the corresponding trigonometric expression, we get:

$$(12) \quad s_{\text{ref}}(t) = \sum_{i=1}^N a \cdot k_i \cdot \cos(\omega t) \cdot \cos(\phi_i) + \sum_{i=1}^N a \cdot k_i \cdot \sin(\omega t) \cdot \sin(\phi_i).$$

If we consider the sum of the cosine functions with different amplitudes and phases, but of the same frequency, the result will be a cosine function with a certain amplitude and phase:

$$(13) \quad s_{\text{ref}}(t) = a_{\Sigma} \cdot \cos(\omega t - \phi_{\Sigma}).$$

When we write out the right-hand side of (12), we get:

$$(14) \quad s_{\text{ref}}(t) = a_{\Sigma} \cdot \cos(\omega t) \cdot \cos(\phi_{\Sigma}) + a_{\Sigma} \cdot \sin(\omega t) \cdot \sin(\phi_{\Sigma}).$$

Selecting the cosine and sine components in expressions (12) and (14) and combining them, we get:

$$(15) \quad \begin{cases} a_{\Sigma} \cdot \cos(\omega t) \cdot \cos(\phi_{\Sigma}) = \sum_{i=1}^N a \cdot k_i \cdot \cos(\omega t) \cdot \cos(\phi_i), \\ a_{\Sigma} \cdot \sin(\omega t) \cdot \sin(\phi_{\Sigma}) = \sum_{i=1}^N a \cdot k_i \cdot \sin(\omega t) \cdot \sin(\phi_i). \end{cases}$$

Cutting the factors and on the right and left sides, we get:

$$(16) \quad \begin{cases} a_{\Sigma} \cdot \cos(\phi_{\Sigma}) = \sum_{i=1}^N a \cdot k_i \cdot \cos(\phi_i), \\ a_{\Sigma} \cdot \sin(\phi_{\Sigma}) = \sum_{i=1}^N a \cdot k_i \cdot \sin(\phi_i). \end{cases}$$

Applying the Euler formula, it is not difficult to go over to recording the signal in a complex form:

$$(17) \quad a_{\Sigma} \cdot e^{j\omega t - j\phi_{\Sigma}} = \sum_{i=1}^N a \cdot k_i \cdot e^{j\omega t - j\phi_i}.$$

Having reduced the multiplier, we get:

$$(18) \quad a_{\Sigma} \cdot e^{-j\phi_{\Sigma}} = \sum_{i=1}^N a \cdot k_i \cdot e^{-j\phi_i}.$$

Expression (18) shows that the complex amplitude of the total signal is equal to the sum of the complex amplitudes of the signals reflected from all objects. When the value of the sounding signal frequency changes, the phase shifts of signals reflected from each object proportionally change. Thus, the expressions (16) and (18) are transformed in the case of the n-th frequency [11-13]:

$$(19) \quad \begin{cases} a_{\Sigma n} \cdot \cos(\phi_{\Sigma n}) = \sum_{i=1}^N a \cdot k_i \cdot \cos(n\phi_i), \\ a_{\Sigma n} \cdot \sin(\phi_{\Sigma n}) = \sum_{i=1}^N a \cdot k_i \cdot \sin(n\phi_i), \end{cases}$$

$$(20) \quad a_{\Sigma n} \cdot e^{-j\phi_{\Sigma n}} = \sum_{i=1}^N a \cdot k_i \cdot e^{-jn\phi_i}.$$

### Results and discussion

For the measurement, the following procedure is proposed. The first sounding frequency is selected from the condition of single-valued range measurement [1]. The following sounding frequencies with uniform pitch. Thus, the amplitude-frequency and phase-frequency characteristics of the total signal are obtained. Examples of such characteristics are shown in Figures 3 and 4.

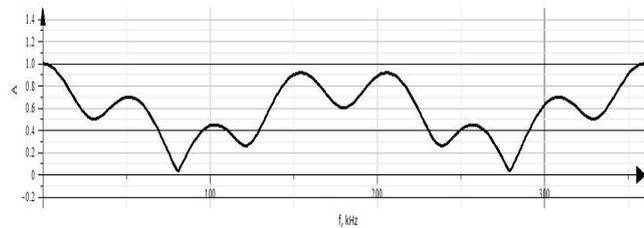


Fig.3. Amplitude-frequency characteristic of the total signal for three measurement objects with different reflection coefficients

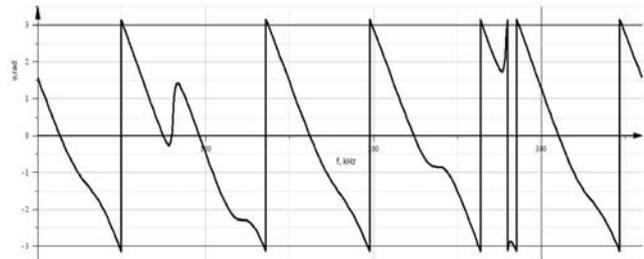


Fig.4. Phase-frequency characteristic of the total signal for three measurement objects with different reflection coefficients

$$(21) \quad \begin{cases} a_{\Sigma 1} \cdot e^{-j\phi_{\Sigma 1}} = \sum_{i=1}^N a \cdot k_i \cdot e^{-j1\phi_i} = b_1; \\ a_{\Sigma 2} \cdot e^{-j\phi_{\Sigma 2}} = \sum_{i=1}^N a \cdot k_i \cdot e^{-j2\phi_i} = b_2; \\ \dots \\ a_{\Sigma n} \cdot e^{-j\phi_{\Sigma n}} = \sum_{i=1}^N a \cdot k_i \cdot e^{-jn\phi_i} = b_n. \end{cases}$$

In the resulting system of equations, known are the complex amplitudes of the sum signals obtained as a result of measurements of phase shifts and amplitudes of the total reflected signals. Unknown - phase shifts and amplitudes of signals reflected from each object. Thus, the solution of the system (21) should allow us to determine the parameters of the signals reflected from each object. But the resulting system of equations is nonlinear. To solve it, it is necessary to find a certain method of mathematical transformation, which would allow us to obtain a system of equations having a simple solution. The following is suggested. In the system of equations (21), we will assume that the amplitudes of the signals reflected from each object are

unknown. In this case, the system is linear and can be solved with respect to the amplitudes  $a \cdot k_i$ . Then we find the value of the amplitude of the first signal [14]:

$$(22) \quad a_1 = \frac{b_1 \cdot (-1)^N \cdot \prod_{i=2}^N c_i - b_{N-3} \cdot \sum_{\substack{i,j,k=2 \\ i < j < k}}^N c_i \cdot c_j \cdot c_k + b_{N-2} \cdot \sum_{\substack{i,j=2 \\ i < j}}^N c_i \cdot c_j - b_{N-1} \cdot \sum_{i=2}^N c_i + b_N}{c_1 \cdot (c_1 - c_2) \cdot (c_1 - c_3) \cdot \dots \cdot (c_1 - c_N)}$$

where  $\dot{c}_i = e^{-j\phi_i}$  is the value of the unit vector reflected from the i-th object.

This value of the amplitude is obtained by transforming the system (21). It uses the values of the vectors of the sum signals obtained at frequencies from 1 to N. In order to obtain the second expression, which allows us to solve the system of equations (21), we do the following. To write the system of equations (20), we use the values of the total signals obtained at frequencies from 2 to N + 1 and find the amplitude value of the signal reflected from the first object [14,15]:

$$(23) \quad a_1 = \frac{b_2 \cdot (-1)^N \cdot \prod_{i=2}^N c_i - b_{N-2} \cdot \sum_{\substack{i,j,k=2 \\ i < j < k}}^N c_i \cdot c_j \cdot c_k + b_{N-1} \cdot \sum_{\substack{i,j=2 \\ i < j}}^N c_i \cdot c_j - b_N \cdot \sum_{i=2}^N c_i + b_{N+1}}{c_1^2 \cdot (c_1 - c_2) \cdot (c_1 - c_3) \cdot \dots \cdot (c_1 - c_N)}$$

Both expressions are similar to each other. The difference consists in the degree of the factor  $\dot{c}_1$  in the denominator.

Another difference is the difference in the coefficients  $b_1$  in the numerator.

In the next stage of the transformations, we find the difference in the values of the amplitudes obtained in expressions (21) and (23). The result is zero, but on the other hand, you can do the following. We reduce the difference to the common denominator and give such terms. Taking into account that the fraction is zero when the numerator is equal to zero, we obtain the equation:

$$(24) \quad \begin{aligned} & b_1 \cdot \prod_{i=1}^N c_i - b_{N-3} \cdot \sum_{\substack{i,j,k=1 \\ i < j < k}}^N c_i \cdot c_j \cdot c_k + \\ & + b_{N-2} \cdot \sum_{\substack{i,j=1 \\ i < j}}^N c_i \cdot c_j - b_{N-1} \cdot \sum_{i=1}^N c_i = -b_{N+1} \end{aligned}$$

In the resulting equation, the unknowns are the value of the unit vectors. In this case, the equation obtained is also nonlinear. But if you pay attention to the products, the sums of products and the sum of the values, then they are nothing but the coefficients of the polynomial equation. If we assume in the expression (23) to be unknown, then this equation is linear with N unknowns. If they are all found, then we can write the polynomial equation. The solution of this equation will give the values of all signal vectors reflected from each object. To find all the coefficients, it is necessary to compile a system of equations. To this end, we use the proposed approach, namely, we will write systems of equations analogous to expressions (24) in pairs with an offset by one. As a result of the transformations, we obtain an expression for the analogous expression (23). As a result, one can obtain a system of linear equations with respect to the coefficients of the polynomial equations:

$$(25) \quad \begin{cases} \sum_{i=1}^N (-1)^{i+1} r_i b_i = -b_{n+1}; \\ \sum_{i=1}^N (-1)^{i+1} r_i b_{i+1} = -b_{n+2}; \\ \dots \\ \sum_{i=1}^N (-1)^{i+1} r_i b_{i+n-1} = -b_{2n}, \end{cases}$$

where  $\dot{r}_i$  is the coefficients of the polynomial equation.

## Conclusion

Thus, having written the system of equations (25) and solving it, we obtain the values of the polynomial equation. The solution of this equation gives the values of  $\alpha$ . Finding the arguments of these values gives the values of the phase shifts of signals reflected from each object. This allows us to use the expression of the phase method for measuring the range of objects. To find the amplitudes of the signals reflected from each object, it is necessary to substitute the values of  $\alpha$  into the system of equations (21) and solve with respect to the amplitudes.

The developed method makes it possible, based on the results of measurements of phase shifts and amplitudes of harmonic summation signals at frequencies whose number should be twice as large as the number of objects, to find the ranges and reflection coefficients of each object.

The above studies develop the theory of phase measurements of distances in the case of the presence of many objects. In order to solve the problem, it is necessary to know exactly the number of objects for recording the system of equation (25). Unambiguously, this question can be used for example the impulse method. But it is possible to probe objects at frequencies the number of which allows you to determine the obviously greater number of objects. Moreover, additional frequencies, after determining the range of objects, additional measurements can be used to improve the accuracy of measuring ranges.

**Authors:** Prof. Vitalii Liubchik, Khmelnytskyi National University, Ukraine, Khmelnytskyi, e-mail: [vitaly1612@gmail.com](mailto:vitaly1612@gmail.com), Prof., Dr. Techn. Sc., Sergii M. Zlepko, Vinnytsia National Technical University, Khmelnytske shose, 95, 21021, Vinnytsia, Ukraine, e-mail: [smzlepko@ukr.net](mailto:smzlepko@ukr.net), Ph.D., docent Oleksandr Yanovickiy, Khmelnytskyi National University, Ukraine, Khmelnytskyi, e-mail: [vitaly1612@gmail.com](mailto:vitaly1612@gmail.com), M.Sc. Yuliya Senchshyna, Odessa National Academy of Communication named O.S. Popov, Ukraine, e-mail: [vitaly1612@gmail.com](mailto:vitaly1612@gmail.com); Prof. Waldemar Wójcik, Lublin University of Technology, Institute of Electronics and Information Technology, Nadbystrzycka 38A, 20-618 Lublin, Poland, e-mail: [waldemar.wojcik@pollub.pl](mailto:waldemar.wojcik@pollub.pl); M.Sc. Olga Ussatova, Al-Farabi Kazakh National University, Almaty, Kazakhstan, email: [uoa\\_olga@mail.ru](mailto:uoa_olga@mail.ru); M.Sc. Aliya Tergeusizova, Al-Farabi Kazakh National University, Almaty, Kazakhstan, email: [aliya55@mail.ru](mailto:aliya55@mail.ru).

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