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doi:10.15199/48.2019.04.46

The algorithm of multi-objective optimization of PM synchronous motors

***Abstract**. This paper presents multi-objective algorithm for optimal designing of permanent magnet synchronous motors. The special attention is paid on the formulation the optimization problem, especially on the correct selection of the partial criteria which constitute multi-objective function and constraints. It is pointed out that connection of multimodal parameter (cogging torque) and unimodal parameter (electromagnetic torque) in one multi-objective compromise function can lead to erroneous operation of optimization algorithm. Therefore, decomposition of the optimization task into two-level is proposed. The optimization calculation has been executed for permanent magnet synchronous motor structure with hybrid excitation system.*

Streszczenie.** W artykule przedstawiono algorytm do optymalizacji magnetoelektrycznych silników synchronicznych. Przedstawiono rozważania dotyczące poprawnego formułowania kompromisowych funkcji celu, w szczególności odpowiedniego doboru kryteriów cząstkowych. Wykazano, że włączenie do kompromisowej funkcji jednocześnie członu reprezentującego elektromagnetyczny moment użyteczny i moment zaczepowy może prowadzić do błędnego działania algorytmu optymalizacji. Zaproponowano dekompozycję zadania optymalizacji na dwa etapy. Przedstawiono i omówiono wybrane wyniki obliczeń optymalizacyjnych dla magnetoelektrycznego silnika synchronicznego z hybrydowym układem wzbudzenia **Algorytm optymalizacji magnetoelektrycznych silników synchronicznych.

Keywords: multi-objective optimization, genetic algorithm, finite element method, permanent magnet synchronous motors.
Słowa kluczowe: optymalizacja, algorytmy genetyczne, metoda elementów skończonych, magnetoelektryczne silniki synchroniczne.

Introduction

In comparison to high efficiency induction motors, the permanent magnet synchronous motors (PMSM) have many advantages: large torque-to-mass ratio, high efficiency, constant speed operation, and long operational life [1, 2, 3]. At present, the manufacturers of the permanent magnets produce high density magnets with better magnetic, mechanics and thermal properties. On the other hand, there are increasing requirements regarding energy saving and natural environmental protection. Therefore, the rapid development of the constructions of permanent magnet (PM) motors, in particular PM synchronous motors is observed.

The PMSM machines are usually small and medium power motors [4]. The stator winding is distributed in the slots and often are wye connected. The PMSM motor winding are similar to the winding used in induction motors. Depending on the application of the motor and properties of used magnetic material, the PMSM structures may differ considerably. The way of mounting, types and dimensions of the permanent magnets in the rotor have essential impact on motor parameters [5]. The magnets may be mounted on the surface or embedded inside the rotor [6, 7, 8]. Typical structures are shown in Fig. 1.

In recent years, the technology of magnetic powder has develops rapidly. This concerns both: hard and soft magnetic materials. Using this technology, it is possible to easy formation of the magnetic circuit geometry and also allows shaping the magnetic properties which depend on the used admixtures. More and more often designers and constructors of the permanent magnet machines make use of advantage of this technology [9]. Electric motors with hybrid excitation systems are constructed. In such constructions hybrid magnets excitation system compound from two or even several magnetic materials (with different properties) is often applied. Two chosen new constructions with hybrid rotors are presented in Fig. 2.

New constructions of permanent magnets need the development new algorithms of optimal design. Nowadays, the precise field-circuit model of the phenomena is used in designing process [10, 11, 12]. These models allow receiving good precision. Unfortunately, these methods of calculations are very time consuming. Therefore researchers are looking for new, more and more effective optimization algorithm. Different non-deterministic methods are used for optimization of permanent magnet motors. In order to increase quality and convergence of solution some modifications of optimization method have been introduced [13].

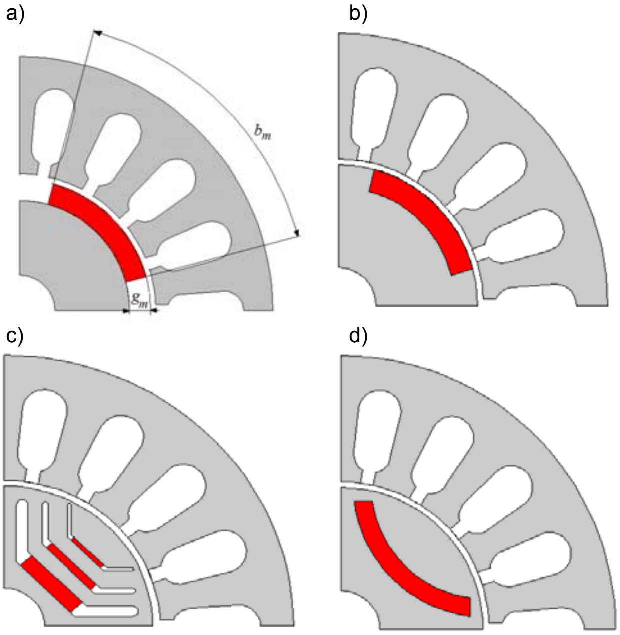

Fig. 1. Selected structures of PM synchronous motor

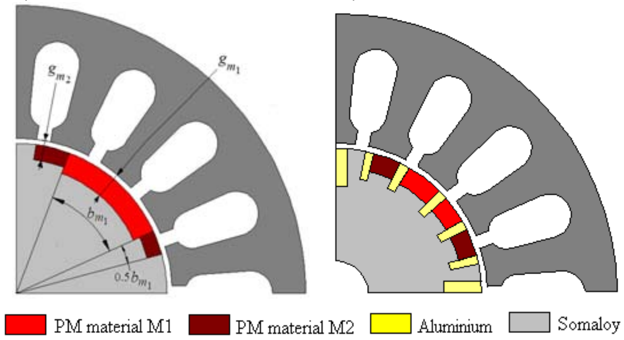

Fig. 2. New structures of PM synchronous motors with hybrid rotors

During formulation the optimization task, the important problem is selection of the functional parameters which constitute the compromise objective function and constraints. It has been proved, that incorrectly formulation of multi-objective compromise function may lead to irrational variants of the designed devices. Including cogging torque and electromagnetic torque into common compromise objective function may cause erroneous operation of the optimization procedure and often also lead to irrational results.

The aim of our research is to elaborate methodology and effect optimization procedure for permanent magnets synchronous motors.

Optimization strategies in designing PM Motors

Typical problem of the permanent magnet machines optimization is the constrained optimization. Constrained optimization is the process of optimizing electromagnetic devices with constraints imposed on selected functional parameters or main dimensions of the devices. Relations between PM motors actual parameters $p_j(\mathbf{x})$ and required values p_{jr} can be defined as:

$$(1)\quad p_j(\mathbf{x}) \geq p_{jr} \text{ or } p_j(\mathbf{x}) \leq p_{jr}, \quad j = 1,2,3,...,m$$

where $\mathbf{x}=[x_1, x_2, x_3,\ldots, x_n]^T$ is the design variables vector, m is the constraints number.

The maximization task consists in determining such vector $\hat{\mathbf{x}}$ that:

$$(2)\quad \bigwedge_{\mathbf{x}\in D} f(\mathbf{x}) \leq f(\hat{\mathbf{x}})$$

where D is the permissible search area defined by constraints $D = \{\mathbf{x} : g_j(\mathbf{x}) \leq 0\}$.

The constraints are usually described in dimensionless form:

$$(3)\quad \zeta_j\left(\frac{p_j(\mathbf{x})}{p_{jr}} - 1\right) = g_j(\mathbf{x}) < 0, \quad \zeta_j = \pm 1$$

In case of optimization tasks of permanent magnet machines, the constraints can be taken into account: (a) by penalty component or (b) by attaching them, as an additional component to the compromise function.

A. The Penalty Function Method

In such strategy the optimality criteria form one or more functional parameters. The other functional parameters are included as constraints and are attached to the objective function and representing the penalty term. According this method, the modified objective function $h_k(\mathbf{x})$ is created. The penalty term representing the penalty for overstepping the constraints. The penalty term $s_k(\mathbf{x})$ is increasing in subsequent iteration of the genetic algorithm. The $s_k(\mathbf{x})$ depends on value of constrains and is determined as follows:

$$(4)\quad s_k(\mathbf{x}) = b^k \sum_{j=1}^{m} \lambda_j g_j(\mathbf{x})$$

where k is the external penalty step number, b^k is the penalty ratio, b is the real number, λ_j are weighting factor defining the weight of selected constraints. In classical external penalty formulation, the value of number b is greater than 1.

In case of maximizing the primary objective function $f(\mathbf{x})$, the modified function has a form:

$$(5)\quad h_k(\mathbf{x}) = \begin{cases} f(\mathbf{x}) & \text{for } \mathbf{x} \in D \\ f(\mathbf{x}) - s_k(\mathbf{x}) = f(\mathbf{x})(1 - \chi_k(\mathbf{x})) & \text{for } \mathbf{x} \notin D \end{cases}$$

where $\chi_k(\mathbf{x})=s_k(\mathbf{x})/f(\mathbf{x})$ is normalized penalty component.

B. Formulation of Multi-objective problem

In this approach the main functional parameters $p_n(\mathbf{x})$ of the constructions are included into multi-objective function. The compromise function can be represented as an additive or multiplicative:

$$(6)\quad f(\mathbf{x}) = \sum_n \lambda_n p_n(\mathbf{x}) \quad \text{or} \quad f(\mathbf{x}) = \prod_n p_n^{q_n}(\mathbf{x})$$

where λ_n, q_n are the weighting factors.

In the case of permanent magnet motor optimization tasks, the objective function can constitute by one of the functional parameters, then the other parameters constitute a set of constraints. At the pre-optimization stage of new PMSM constructions (e.g. hybrid constructions), first the general relationships between the designing parameters describing the motor structure and obtained functional parameters should be recognized. For pre-designing process of new types of permanent magnet motors penalty strategy is more useful. In this paper multi-objective approach has been applied.

Decomposition of the optimization task

The problem of optimizing the rotor of PMSM has been discussed. The stator from commercially produced induction motor Sg100L-4B type with not skewed slots was applied. The rated values of voltage U_N and current I_N are the same as in induction motor. The other structural parameters of the applied stator are: the number of poles p=2, number of slots s=36, the stack length L_S=125 mm, the outer diameter D_S=154 mm, the inner diameter d_S=94 mm, and the air gap length δ=0.9 mm.

In order to explain the problem, the simple structure of PMSM rotor with homogeneous not skewed arc magnets has been considered (Fig. 1a). The two design parameters have been taken into account: the relative magnet span $\alpha_m=b_m/\tau$, where τ is the pole pitch and the magnet thickness g_m. In order to recognize the relationship between design variables (α_m and g_m) and main electromagnetic parameters of the PMSM, the series of test field calculations have been performed. The useful torque T, harmonic distortion coefficient THD of the back electromotive force and the cogging torque T_c have been determined. As an additional, economical criterion, "torque density" t_v, i.e. ratio torque-to-magnet volume has been included.

The electromagnetic torque is a monotonically increasing function of both analyzed design variables – Fig. 3. Whereas the parameter t_v decreases with both variables. Thus, these two criteria are opposing.

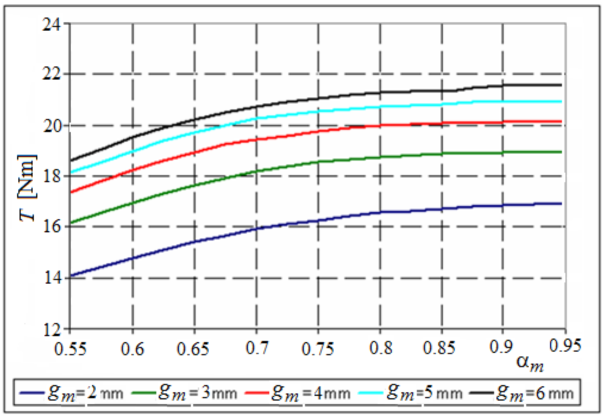

Fig. 3. Impact of the design variables on the electromagnetic torque

Very interesting and important is the relationship of THD coefficient in relation to these both design variables. It can be notice (Fig. 4) that there is an optimum of magnet span at which the THD has the smallest value.

The THD coefficient has it minimum for α_m≈0.7÷0.8. For other values of α_m, the back electromotive force is highly distorted and may have a waveform similar to a trapezoid or even rectangular.

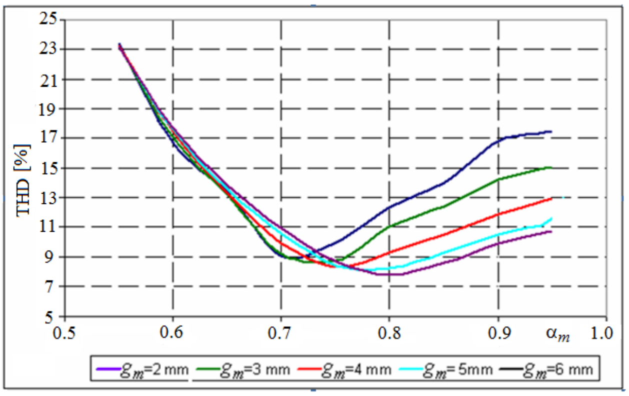

Fig. 4. Impact of the design variables on THD coefficient

The cogging torque T_c is the multimodal function of variable α_m and it has a several extremes – see Fig. 5. It can be noticed that change of spam α_m of order 0.05 causes very small change of the electromagnetic torque (about 4%), while it causes enormous changes of cogging torque (even 380%). The cogging torque has an enormous impact on value of compromise objective function. The optimization procedure is too sensitive to the changes of this torque.

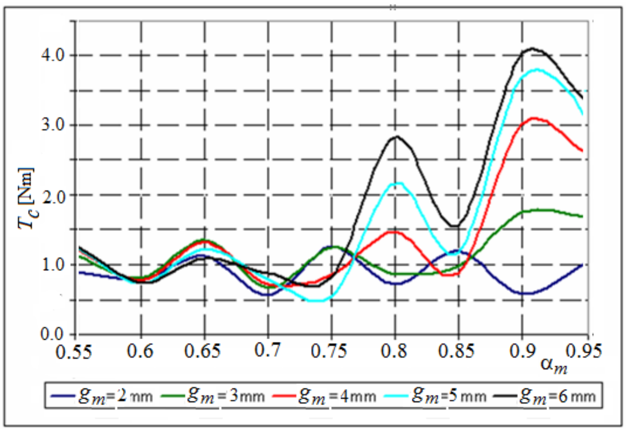

Fig. 5. Impact of the design variables on the cogging torque

On the basis of many trial calculations, it was pointed out that combining in one multi-objective compromise function the cogging torque (multimodal parameter) and electromagnetic torque (unimodal parameter) can leads to improper operation of the optimization procedure. Therefore, decomposition of the optimization task has been proposed. Two-level approach has been applied.

In order to explain the idea of decomposition, the PMSM structure with hybrid excitation system (Fig. 2a) has been discussed. The excitation system of the rotor is built of two magnets ($M1$ and $M2$) with different magnetic properties. The $M1$ magnet is formed from sintered NdFeB material with properties: B_r=1.20 T and H_c=890 kA/m, whereas magnet $M2$ is made of powder magnetic material with properties: B_r=0.646 T and H_c=404.98 kA/m. The rotor core is produced from soft powder material (Somaloy 500) [14].

The structure of the motor has been determined by 4 design variables (Fig. 2a): $x_1=g_{m1}$ – height of the $M1$ magnet, $x_2=\xi=g_{m2}/g_{m1}$ – relative height of the $M2$ magnet, $x_3=\alpha=b_{m1}/(b_{m1}+b_{m2})$ – relative angular width of area $M1$, $x_4=\alpha_1=(b_{m1}+b_{m2})/\tau$ – total angular width of the magnet area related to pole pitch.

A. The First Level of Decomposition

In the first level, the compromise objective function has been composed of three components: electromagnetic torque T, total harmonic coefficient THD and magnet volume $V_m = T/t_v$. Based on many test calculation, the additive multi-objective compromise function for l-th individual has been proposed:

$$(7)\quad f\left(\mathbf{x}^l\right)=\lambda_1 t\left(\mathbf{x}^l\right)+\lambda_2\left[2-h\left(\mathbf{x}^l\right)\right]+\lambda_3\left[2-v\left(\mathbf{x}^l\right)\right]$$

Here λ_1, λ_2, λ_3 is the set of weighting factors, $t=T(\mathbf{x}^l)/T_0$, $h(\mathbf{x}^l)$=THD$(\mathbf{x}^l)$/THD$_0$, $v_m(\mathbf{x}^l)=V_m^l(\mathbf{x})/V_0$ are the parameters related to the average values gained in the genetic algorithm initiation.

In order to determine the values of weighting factors in the pre-designing process, the optimal parameters obtained during the optimization calculations has been compared with rated parameters of mass produced permanent magnet motors. It has been assumed that the offered for sale motors are optimally designed.

The genetic algorithm with roulette wheel, one cut-point in chromosome crossover procedures has been applied [15]. The crossover operation was carried out in two stages. All individuals in the current population were crossover. The new generation of the genetic algorithm was created by half of the parents and half of the children. Also, the simple elitism procedure has been used. The parameters of genetic algorithm procedure have been assumed: number of individual L=80, the mutation probability p_m=0.004 and the maximum number of a genetic populations as a stop criteria equal $(N_g)_{max}$=30. The calculation were executed out for λ_1=1.0, λ_2=0.75, λ_3=0.25. The 2D FEM model of the PM motor has been elaborated in Maxwell environment. All functional parameters of the motor have been determined on the basis of field model of electromagnetic phenomena. The comparison of optimization results in the successive genetic populations is listed in table 1. In the table the results for the best individual in every generation are presented.

Table 1. The first level of the optimization process

N_g	g_{m1} [mm]	ξ [-]	α [-]	α_1 [-]	T [N·m]	THD [%]	V_m [cm^3]	T_c [N·m]	f(x) [-]
1	4.022	1.019	0.545	0.799	14.40	8.98	57.39	0.58	1.2468
3	3.197	1.195	0.629	0.809	16.19	6.82	46.39	1.04	1.4255
5	3.144	1.072	0.704	0.782	16.31	7.74	44.12	0.65	1.4701
10	3.144	0.900	0.704	0.791	16.29	7.49	44.63	0.25	1.4735
16	3.144	0.824	0.704	0.821	16.56	7.19	46.32	0.92	1.4746
20	3.019	0.881	0.704	0.813	16.29	7.28	44.07	0.69	1.4900
25	3.019	0.806	0.704	0.812	16.17	7.36	44.02	0.57	1.4959
30	3.019	0.749	0.704	0.817	16.24	7.06	44.29	0.75	1.4968
35	3.019	0.749	0.704	0.817	16.24	7.06	44.29	0.75	1.4968

Due to the relatively large number of individuals in an each generation, the results close to optimal solution are usually reached after about 15 populations. Thus, the optimization process can be finished earlier. This allows shortening calculation time even more than two times. Increasing the numbers of individuals to 120 is not affect on total computation time, because, the optimal solution is reached much faster – even after 8-10 generations. It is advantageous because larger number of individuals increases the probability of global minimum finding.

B. The Second Level of Decomposition

In the second level of optimization, the cogging torque has been additionally included into compromise function. This parameter strongly depends on the span α (Fig. 5). In the second level, the calculation has been executed for the following range of $\alpha \subset \left\langle \hat{\alpha}-0.52\tau_S/\tau, \hat{\alpha}+0.52\tau_S/\tau \right\rangle$, where $\hat{\alpha}$=0.817 is the optimum value of α gained during the first level of optimization process.

The optimum values: $\hat{g}_{m1}$ =3.019 and $\hat{\xi}$ =0.749 from the first level have been taken into consideration. In the second level, due to specific of problem being solved, especially due to high diversity and sensitivity of the partial criteria, the following multiplicative compromise objective function [16] has been proposed:

$$f\left(\mathbf{x}^l\right) = t^{q_1}\left(\mathbf{x}^l\right) h^{q_2}\left(\mathbf{x}^l\right) t_c^{q_3}\left(\mathbf{x}^l\right) \quad (8)$$

The course of optimization process in selected genetic population is presented in Table 2.

Table 2. Results o the second level of optimization

N_g	α [-]	α_1 [-]	T [N·m]	THD [%]	V_m [cm^3]	T_c [N·m]	$f(\mathbf{x})$ [-]
1	0.774	0.868	16.875	8.57	41.960	0.1850	2.631355
2	0.888	0.769	17.323	12.02	48.140	0.1005	2.708466
5	0.775	0.868	16.884	8.61	42.014	0.1827	2.710094
7	0.749	0.868	16.643	7.69	40.605	0.3015	2.710094
10	0.749	0.769	15.963	7.96	40.605	0.1648	2.710594
12	0.749	0.769	15.963	7.96	40.605	0.1648	2.710594

As a result of the second level calculation, the motor with significantly decreased cogging torque (about 80 %) has been obtained. The others parameters changed in very small degree. The value THD coefficient is slightly deteriorated. This confirms the validity of a two-level approach.

Using the elaborated algorithm, the optimization of the classical motor (with homogeneous excitation system) presented in Fig. 1a has been also performed. The both structures have been compared in Table 3.

Table 3. The value of parameters for optimal PMSM

Type of the motor	T [N·m]	THD [%]	V_m [cm^3]	T_c [N·m]
The motor with hybrid magnets	15.963	7.964	40.60	0.1648
The motor with arc homogenous magnets	19.439	12.789	46.94	0.5383

It can be observed that the motor with hybrid excitation system has much better properties, especially it has 4 times lower cogging torque with comparison to the classical excitation system. Although the value of useful torque is lower in comparison with motor with arc homogeneous magnets but the values of "torque density" $t_v = T/V_m$ of the both structures are similar.

Conclusions

The paper presents the effective multi-objective algorithm for designing of permanent magnet machines with not skewed magnets and not skewed stator slots.

It has been proved that forming multi-objective compromise function consisting of two different components: (a) the unimodal (useful torque) and (b) the multimodal (cogging torque), can lead to erroneous operation of optimization algorithm. Therefore, the two-level approach has been proposed.

Additionally, it has been proved, that application of hybrid excitation system in permanent magnet synchronous motor generates better distribution of the magnetic field in the air gap than in case of motor with homogenous magnets. Therefore, the hybrid machines have lower value of THD and significantly lower cogging torque in comparison to classical arc homogeneous magnets machine. Therefore the motors with hybrid excitation system are being used for the drive in which the minimization of torque ripple is required.

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