

## Mathematical model of XLPE insulated cable power line with underground installation

**Abstract.** This paper presents mathematical modeling of a stationary thermal field in the cross section of a single-conductor cable with XLPE insulation. The equivalent circuit of thermal processes is made using the method of homogeneous bodies, and it includes dielectric losses, takes into account the ambient temperature, as well as the temperature dependence of the active resistance of the cable core. The assessment of the mathematical model adequacy is performed by comparing the obtained results with the calculation of thermal and electrical processes using the finite element method implemented in the software «ANSYS Workbench». The resulting mathematical model can be used to control the capacity of cable lines with XLPE insulation and limit their service life due to temperature aging of the insulation.

**Streszczeni.** Przedstawiono matematyczne modelowanie stacjonarnego pola temperatur w przekroju kabla z izolacją xlpe. Schemat procesów termicznych wykonano metodą jednorodnych ciał i obejmuje on straty dielektryczne biorąc pod uwagę temperaturę otoczenia, a także zależność od temperatury rezystancji żyły kablowej. Ocena adekwatności modelu matematycznego odbywa się poprzez porównanie uzyskanych wyników z obliczeń cieplnych i elektrycznych procesów z wykorzystaniem metody elementów skończonych, realizowanego w programie „ANSYS Workbench”. Otrzymany model matematyczny może być używany do kontroli przepustowości linii kablowych z izolacją xlpe i ograniczenia ich żywotności poprzez temperatury starzenia się izolacji. **Model matematyczny kabla w izolacji XLPE przy podziemnej energetycznej linii kablowej**

**Keywords:** XLPE insulation cable, equivalent circuit, thermal resistance.

**Słowa kluczowe:** Kabel o izolacji XLPE, ekwiwalent schemat, opór cieplny.

### Introduction

Underground medium-voltage cable lines are used for distribution and transmission of electric energy. Single-core cables with cross-linked polyethylene insulation are widely used. Power consumption due to urban and industrial park development increases. Therefore, the issue of analyzing the capacity of cable lines becomes urgent. The capacity of electric power lines depends on the conductor temperature [1-4]. The temperature is significantly affected by the process of cable heat dissipation into the environment. Specific nature of thermal processes plays an important role in assessing the rational use of cable systems [5-7]. Various aspects in the research of power cables with allowance for thermal processes are presented in [8-17].

A popular approach is to determine the temperature of cable elements using the numerical method of the finite elements. However, significant computational resources that are required, the complexity of the initial data preparation, and analysis of the calculation results in some cases limit the application of this approach. Below it is proposed to determine the temperature of the cable based on the thermal equivalent circuit. The ambient temperature and losses in the cable elements, being taken into account, are considered to be the advantages of this model, as well as the ability to determine the effect of adjoining cables on the processes.

### Mathematical model of cable power line with cross-linked polyethylene insulation

The mathematical model of the cable line with cross-linked polyethylene insulation is formed for the cable cross-section shown in Fig. 1, where  $r_0 - r_n$  are outer radii of cable layers,  $\theta_i$  is the temperature of the medial line of the  $i$ -th cable layer, that is the temperature of the  $i$ -th layer on the radius  $r_{medi} = r_i + 0.5 \cdot (r_{i+1} - r_i)$ , °C.

For a computational model of the single-core cable, each layer (Fig.2) of the cable cross-section ( $r_i - r_n$ ) is represented as the thermal resistance ( $R_{m1} - R_{mn}$ ) depending on the material and layer thickness. The cable equivalent circuit in a short form is shown in Fig. 3, where  $P_0 - P_n$  is the heat release of the core and layers, which are analogues of current sources;  $\theta_0$  is the temperature of the interface of air-ground regions, which is analogue to the voltage source.

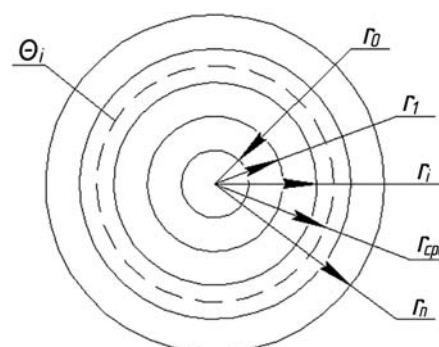


Fig.1. Schematic of cable separation into rings for the numerical calculation of temperature



Fig.2. AXLPEP cable formation: 0 is the aluminium core; 1 is the core shield; 2 is the insulation; 3 is the insulation shield; 4 and 6 are cable paper; 5 is the copper shield; 7 is the cable sheath.

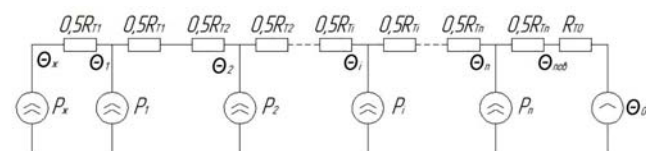


Fig.3. Equivalent circuit of the thermal circuit for numerical calculation of temperature

The following assumptions are made in generating the equivalent circuit:

- the cable is of a perfect cylindrical shape;
- the parameters of the cable and its environment remain constant along the axis.

It appears that under these assumptions the heat will spread evenly from the cable axis to its surface, and from the surface the heat is absorbed by the atmosphere. Isotherms being in the form of concentric circles will be formed in the process of heat transfer in the cable section.

The temperature value  $\theta_i$  of the medial line corresponds to each uniform layer  $i$  (Fig. 1). The temperature  $\theta_0$  is assumed to be pre-determined (obtained during measurements). The temperature  $\theta_{surface}$  corresponds to the surface temperature of the cable.

The thermal resistance of each layer and the thermal resistance of the earth are determined according to the following expressions [18]:

$$(1) \quad R_T = \frac{1}{2 \cdot \pi} \cdot \frac{1}{L \cdot \lambda} \cdot \ln \left( \frac{r_{outer}}{r_{inner}} \right)$$

$$(2) \quad R_{TE} = \frac{1}{2 \cdot \pi} \cdot \frac{1}{\lambda_E} \cdot \ln \left( \frac{h}{R_{outer}} + \sqrt{\left( \frac{h}{R_{outer}} \right)^2 - 1} \right)$$

where  $R_T$  is the thermal resistance of the layer, °C/W;  $\lambda$  is the specific thermal conductivity of the environment, W/(m·°C);  $L$  is the length of the cable, m;  $r_{outer}$  is the outer radius of the layer, mm;  $r_{inner}$  is the inner radius of the layer, mm;  $R_{TE}$  is the thermal resistance of the earth, °C/W;  $\lambda_E$  is the specific thermal conductivity of the earth, W/(m·°C);  $R_{outer}$  is the outer radius of the cable, mm;  $h$  is the depth of cable laying, mm.

When determining the power released in the cable core, as well as in the shielded layer, the resistance of the conductor with allowance for heating is represented by the equation:

$$(3) \quad R_t = R_{20} \cdot [1 + \alpha \cdot (t - 20)]$$

where  $R_t$  is the conductor resistance at the temperature  $t$  °C, Ohm;  $R_{20}$  is the conductor resistance at the temperature of 20 °C,  $\alpha$  is the temperature coefficient, 1/°C.

In high voltage cables, heat release in insulation due to dielectric losses is quite significant. With the assumption that the dielectric loss tangent and the dielectric permittivity do not depend on the radius, it is possible to calculate the power loss in the insulation by the expression [18-20].

$$(4) \quad P_{ins} = U^2 \cdot \omega \cdot C \cdot tg \delta$$

where  $P_{ins}$  is the dielectric power loss, W;  $U$  is the voltage applied to the insulation, V;  $\omega$  is the angular signal frequency,  $c^{-1}$ ;  $C$  is the capacity of the single-core cable, F;  $tg \delta$  is the dielectric loss tangent.

The equation [18-20] is recommended for the capacity of a single-core cable.

$$(5) \quad C = \frac{2\pi \cdot \varepsilon \cdot \varepsilon_0 \cdot L}{\ln \frac{R}{r_0}}$$

where  $\varepsilon$  is the dielectric permittivity of the cable insulation;  $\varepsilon_0 \cong 8.85 \cdot 10^{-12}$  F/m is the electrical constant;  $R$  is the outer radius of the cable, mm;  $r_0$  is the cable core radius, mm.

Let us compile the system of equations to determine the temperature of the cable line without shield grounding. The equivalent circuit for the AXLPEP cable 1x50/16 – 10 kV (Fig. 2) is shown in Fig. 4.

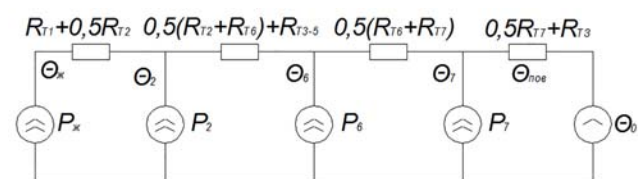


Fig.4. Equivalent circuit for a single AXLPEP cable without shield grounding

Mathematical modeling is performed under the following conditions:

- the temperature  $\theta_0 = 20$  °C;
- the current flowing in the cable core  $I = 150$  A.

The geometric and thermal parameters of the AXLPEP cable are presented in Table 1.

Table 1. Parameters of the layers and the cable environment (soil)

No of layer	Name of layer	1/λ, °C·m/W	Outer radius of layer r <sub>i</sub> , mm
0	Conductive core (aluminum)		3.95
1	Insulation shield	3.5	4.55
2	Insulation	3.5	7.95
3	Insulation shield	3.5	8.55
4	Conductive paper	6	8.75
5.1	Twist of copper wires	$2.7 \cdot 10^{-3}$	10.75
5.2	Copper strip	$2.7 \cdot 10^{-3}$	10.85
6	Cable paper	6	10.98
7	Polyethylene sheath	3.5	12.75
Earth			h=700 mm R <sub>outer</sub> =12,75 mm

The values of thermal resistance of each layer, obtained by the formula (1), are represented in Table 2.

Table 2. Cable layer thermal resistance

Layers	R <sub>t</sub> , °C/W
Insulation shield	0.0788
Insulation	0.3109
Insulation shield	0.0405
Conductive paper	0.0221
Copper shield	$9.2437 \cdot 10^{-5}$
Cable paper	0.0131
Polyethylene sheath	0.0822
Soil	0.8974

In Tables 3, 4 dielectric permittivity and dielectric loss tangent for layers 2, 6, 7 are proposed, respectively, as well as the calculated values of dielectric losses.

Table 3. Dielectric parameters

No of layer	Layer material	ε	tgδ
2, 7	Polyethylene	2.25	$3.5 \cdot 10^{-4}$
6	Cable paper	3	$23 \cdot 10^{-4}$

Table 4. Dielectric cable loss

Layers	P <sub>i</sub> , W
Insulation	$7.0075 \cdot 10^{-4}$
Cable paper	$8.9904 \cdot 10^{-3}$
Polyethylene sheath	$1.174 \cdot 10^{-3}$

To compile the system of equations of the thermal processes in the cable, we use the method of nodal potentials. According to Fig. 4 the system of equations for calculating the cable temperature takes the form

$$(6) \quad \begin{cases} \frac{\theta_{x_i} - \theta_{x_2}}{R_{T1} + 0,5R_{T2}} - P_0 = 0 \\ \frac{\theta_{x_2} - \theta_{x_6}}{R_{T1} + 0,5R_{T2}} + \frac{\theta_{x_2} - \theta_{x_6}}{0,5(R_{T2} + R_{T6}) + R_{T3} + R_{T4} + R_{T5}} - P_2 = 0 \\ \frac{\theta_{x_6} - \theta_{x_7}}{0,5(R_{T2} + R_{T6}) + R_{T3} + R_{T4} + R_{T5}} + 2 \frac{\theta_{x_6} - \theta_{x_7}}{R_{T6} + R_{T7}} - P_6 = 0 \\ 2 \frac{\theta_{x_7} - \theta_{x_0}}{R_{T6} + R_{T7}} + \frac{\theta_{x_7} - \theta_{x_0}}{0,5R_{T7} + R_{T8}} - P_7 = 0 \end{cases}$$

where  $P_0 = I^2 \cdot \frac{\rho_{Al}}{F_0} \cdot [1 + \alpha_{Al} \cdot (\Theta_{Al} - 20)]$  is the cable

core heat release;  $\rho_{Al}$  is the specific resistance of aluminum to a direct current, Ohm·m;  $F_0$  is the cross-sectional area of the core, mm<sup>2</sup>;  $\alpha_{Al}$  is the thermal coefficient of aluminum, 1/°C,  $\Theta_{Al}$  is the temperature of conductive core (aluminum), °C.

In Table. 5 the calculation data on temperature for a single cable are introduced.

Table 5. Temperature values for single cable

Layers	Core	2	7	8	Surface
$\Theta$ , °C	39.8133	36.6033	33.5241	32.8707	32.3071

### Confirmation of the mathematical model adequacy for a single cable with cross-linked polyethylene insulation

To confirm the adequacy of the developed mathematical model, its verification was made using numerical simulation in the software "ANSYS Workbench".

The temperature distribution in the radial direction from the center of the cable to its surface, and then to the adjacent soil was modeled. To take into account the temperature dependence of the cable active resistance, it is necessary to use the module "Electric" to set the cable electrical parameters, as well as the module "Steady-State Thermal" for accounting the temperature distribution over the cable and soil section.

The temperature distribution pattern obtained during modeling is shown in Fig. 5.

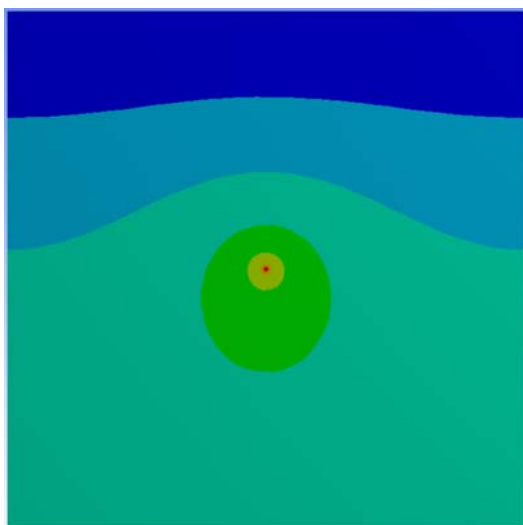


Fig.5. Temperature distribution pattern obtained in "ANSYS Workbench" software

### Results Discussion

Cable core heating was 40.507 °C (Fig. 6, 7), which differs from the value obtained on the basis of the created mathematical model by less than 1 °C (39.813 °C). The resulting discrepancies can be explained by a more accurate definition of dielectric losses and eddy currents in the shield using the finite element method.

The developed mathematical model of the cross-linked polyethylene cable allows one to calculate the temperature of the medial line of each cable layer. This model can be used as a basis for modeling the cable system of three parallel cables, when it is necessary to take into account the influence of cables on each other. This effect is manifested not only in the form of supplemental heating from adjacent

cables, but also in the form of changes in the inductive resistance of conductive materials.

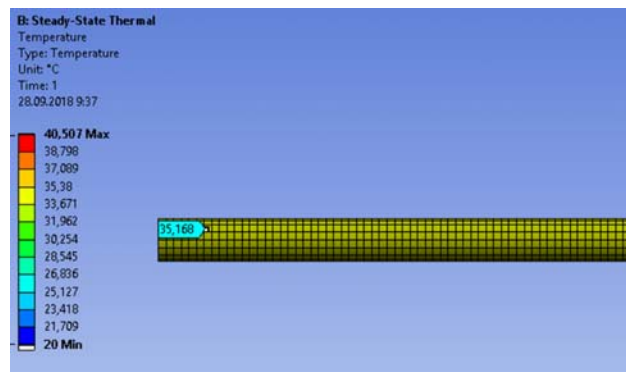


Fig.6. Cable surface temperature

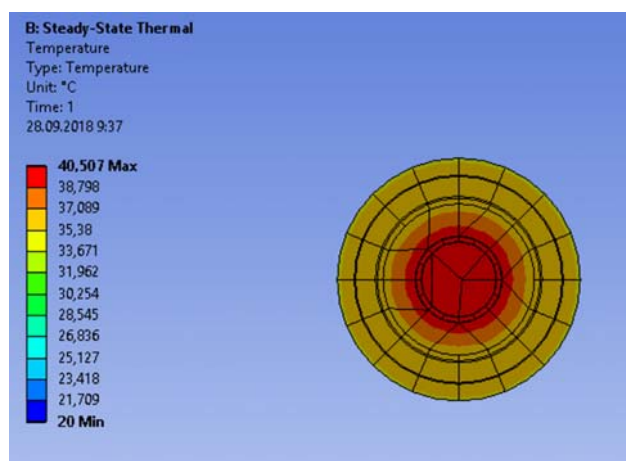


Fig.7. Temperature distribution in the layers of the cable

**Authors:** Oleg Kropotin, e-mail: [kropotin@mail.ru](mailto:kropotin@mail.ru); Vsevolod A. Tkachenko, e-mail: [sevaatmail@gmail.com](mailto:sevaatmail@gmail.com); Aleksandr O. Shepelev, e-mail: [alexshepelev93@gmail.com](mailto:alexshepelev93@gmail.com); Elena V. Petrova, e-mail: [kpk@esppedu.ru](mailto:kpk@esppedu.ru); Vladimir N. Goryunov, e-mail: [vladimirgoryunov2016@yandex.ru](mailto:vladimirgoryunov2016@yandex.ru); Aleksandr A.Y. Bigun, e-mail: [barsbigun@list.ru](mailto:barsbigun@list.ru).  
Correspondence author e-mail: [alexshepelev93@gmail.com](mailto:alexshepelev93@gmail.com)

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