Effect of Nanocontacts on Transient States in Electrical Circuits

Abstract. This paper proposes a model of mechanical switch with stretched nanocontacts based on an analysis of the mechanisms of electron transport within a nanocontact. We use the model proposed to derive equations describing the current in a circuit with an opening switch. The measurement data and the calculation results confirm that nanocontacts substantially modify transient states in the studied circuit and therefore their effect must be taken into account in theoretical analysis.

Introduction

Atomic-sized conductors have been intensively studied for more than ten years, resulting in the discovery of many interesting effects that do not occur in conductors larger in size [1]. A two-position switch [2] and a transistor [3] based on a single metal nanocontact placed on a substrate have been designed and verified to operate. Nanocontacts have been proposed to be used in sensors for the detection of lead and copper ions [4], terahertz signals [5] and linear shift [6]. Studies have also shown that nanocontacts can form between the contacts of a mechanical relay [7, 8] or a MEMS switch [9]. Of special importance is the observation of nanocontacts created in MEMS switches, used in radar components [10] and reduced power consumption systems [11]. Intensive research is conducted to improve the reliability, lifetime and switching rate of micromechanical switches. This requires an in-depth insight into the temperature effects [12], adhesion [13], creep [14] and wear [15]. The resistivity and hardness of the contact material as well as the surface topography affect the contact resistance [16]. The observed creation of nanocontacts makes this effect relevant for the analysis of the properties of MEMS switches and circuits including MEMS switches.

Using switches in an electrical circuit results in the occurrence of transient states in the circuit when a switch is being opened or closed. A switch in a circuit is modeled as a two-state device characterized by two parameters, the closed-state resistance and the open-state resistance. However, if a nanocontact forms between its terminals in the opening or closing process, a switch cannot be modeled as a two-state element. This must be taken into consideration in the analysis of transient states in an electrical circuit including such a switch, as discussed in Ref. [17] for the final stage of the opening process. In this paper we present measurements and a theoretical analysis of the whole process of opening a mechanical switch and derive equations that fully describe the current-time dependence in transient states.

Model of mechanical switch with stretched nanocontacts

When a mechanical switch is being closed its electrodes collide and a contact is made between the compressed electrodes. The compressive force acting on the electrodes is referred to as the contact force. The conductance of the contact increases with the contact force [18]. An adhesive connection is created as a result of the collision and compression of the electrodes. In an opening mechanical switch the electrodes move apart; the contact force decreases at first and then reverses direction, which results in stretching the adhesive connection area. Due to surface irregularities, a number of contact spots occur between the separating electrodes [19, 20]; these nanocontacts break successively as the electrodes continue to move apart. At the final stage of the stretching process a single nanocontact remains between the electrodes. This nanosized neck between the macroscopic electrodes narrows to the size of a single atom to break in the end [1]. Figure 1a shows the conductance trace (i.e., the measured time dependence of the conductance) of a contact created in stretching the adhesive connection area.

Fig.1. a) Conductance trace of the contact between the gold electrodes of an opening mechanical switch (see text for details). In the inset (top left), photograph of the electrodes of the mechanical switch used in the measurements. b) Calculated Sharvin, Maxwell and Wexler conductances vs. Knudsen number, compared with the experimental dependence resulting from the measurement data presented in a) (see text for details)
In this experiment we used a homemade switch with copper electrodes coated with a 5 μm thick nickel layer and a 0.2 μm thick gold layer on top. The switch offered a choice of electrode separation rates, which are proportional to the contact stretching rates. The photograph in Fig. 1a, inset, shows the electrodes (with diameter $D = 1$ mm) with hemispherical ends (of radius $D/2$). In the three ranges indicated in Fig. 1a the shape of the characteristic results from different mechanisms of contact stretching and electron transport in the contact. Range I, indicated with a red double-ended arrow line, corresponds to the initial phase of electrode separation. Immediately before this phase the electrodes were compressed. As the electrodes move apart, the nanocontacts between them break successively, resulting in mechanical instabilities, which can lead to a short-time increase, observed in range I, in the conductance-time dependence. Also surface contaminants have a substantial effect on the conductance of the contact in this range [22]. Indicated with a green double-ended arrow line, range II corresponds to the next phase of contact stretching. The shape of the conductance-time dependence in this range is due to the mechanism of electron transport, which changes from diffusive to ballistic with decreasing contact radius [1, 18]. In range III (blue double-ended arrow line) the conductance changes in a stepwise fashion. The conductance plateaus observed in this region correspond to metastable atom configurations in a single stretched nanocontact. In a metastable atom configuration the conductance is roughly constant and the nanocontact undergoes elastic deformation until the stress reaches a critical value and the system relaxes abruptly to another metastable configuration [23, 24]. Atom reconfigurations occur in the narrowest part of the nanocontact/neck and result in plastic deformation. Sudden reconfigurations of atoms in a stretched nanocontact following an elastic stretching phase occur also in region II. However, in this region atom reconfigurations are more frequent and the conductance plateaus corresponding to elastic stretching are poorly visible [23].

For the comparison of the measurement data obtained for regions I and II with the theoretical dependences based on the Sharvin conductance formula [25]:

$$G_s = G_0 \left( \frac{\pi}{\lambda_F} - 0.5 \frac{P}{\lambda_F} \right),$$

(1)

assuming that the minimal cross section $\lambda$ of a nanocontact is a circle with radius $r$ ($\lambda = \pi r^2$, circumference $P = 2\pi r$), we determine $r$ for a single nanocontact:

$$r = \left( 1 + \frac{4G_s}{G_0} \right)^{\frac{1}{2}},$$

(2)

where $G$ is the conductance of the nanocontact, $G_0 = 2e^2/h$ the conductance quantum, and $\lambda_F$ the electron Fermi wavelength ($= 0.52$ nm in gold). Next, from equation (2) we determine $r$ for the last experimental value of $G/G_0$ in range II; the obtained radius is $r = 1.17$ nm. In the characteristic presented in Fig. 1a this point is just before the appearance of conductance steps. Thus, we can assume ballistic electron transport and use the Sharvin formula. We can also assume that there is only one nanocontact between the electrodes, as indicated by the conductance steps following region II. In many studies such conductance steps have been observed in a single stretched nanocontact [1]. Equation (2) can be rewritten as [26]:

$$G_s = \frac{3\pi r^2}{4\rho l},$$

(3)

where $\rho$ is the resistivity and $l$ the electron mean free path. From equation (3) we calculate the resistivity for the value of $r$ determined above and $l = 3.8$ nm (Ref. 24). We obtain $\rho = 2.26 \times 10^{-7} \Omega m$, a value an order of magnitude larger than the resistivities of macroscopic conductors. The Sharvin equation can be used in the case of ballistic electron transport; in a conductor with a cross-section radius $r$ this corresponds to $r < l$. If $r \gg l$, the electron transport is diffusive, and the resistance of the conductor should be determined from the classical Maxwell formula (also known as Holm’s classical relation) [26, 1, 18]:

$$G_M = \frac{2r}{\rho}.$$  

(4)

The transition region between the diffusive and ballistic transport regimes (with $r \sim l$) is referred to as quasiballistic. The conductance in this regime can be determined from the interpolation Wexler formula [27] modified by Mikrajuddin et al. (Ref. 28):

$$G_W = \frac{G_S}{1 + \frac{G_S}{G_M} \Gamma_M(K)},$$

(5)

where $K$ is the Knudsen number ($K = l/r$). The original Wexler formula contains a slowly varying function $\Gamma(K)$, which is replaced by $\Gamma_M(K)$ in equation (5) (Ref. [28, 29]):

$$\Gamma_M(K) = \frac{2}{\pi} \int_0^\infty e^{-x} \sin(x) dx,$$

(6)

For the comparison of the measurement data for ranges I and II with theoretical curves based on equations (3), (4) and (5) we assume a linear decrease of the effective contact radius from $r_{eff} = r_{max}$ to the value $r_{eff} = r_{min}$ = 1.17 nm determined above; the effective contact radius $r_{eff}$ allows for the possibility of multi-nanocontact connection between the electrodes. For the determination of $r_{max}$ in the contact stretching phase with more than one contact spot between the electrodes the nanocontacts can be regarded as resistors connected in parallel [30]. This, under the assumption that the cross section of each nanocontact in its narrowest point is circular, leads to the relation:

$$r_{eff} = \frac{1}{\sum_i r_i^2},$$

(7)

where $r_i$ is the radius of nanocontact $i$. Figure 1b compares the Sharvin (blue solid line), Maxwell (black solid line) and Wexler (green dashed line) conductance values determined from equations (3), (4) and (5), respectively, with the measurement data obtained for ranges I and II. The conductance values are plotted versus the Knudsen number $K$ in the range $K_1 = l/r_{max}$ to $K_2 = l/r_{min}$ ($r_{min} = 1.17$ nm, $l = 3.8$ nm). The assumed value of $r_{max} = 7.14$ nm provides the best fit of the experimental data to the Wexler conductance formula. Arrows in the plot indicate characteristic points A, B, C, D and E of the experimental dependence. To a good approximation, the experimental data fit the theoretical curve resulting from the Wexler formula between points C and D. Between D and E, as predicted, the experimental conductance characteristic draws near to the Sharvin dependence. Also confirming predictions, the experimental characteristic approaches the Maxwell dependence between C and B. In the segment between points B and A the experimental characteristic
When $T_n$ is the channel number in order of closing ($n = 1, 2, 3, ..., N$), $N$ is the total number of conductance channels considered, $t_n$ the time of closing of channel $n$, $t_0$ a reference time (for $n = 1$, $t = t_0$, which corresponds to the beginning of the analysis), and $R_k$ denotes the resistance of channel $k$. The resistance $R_{\text{nc}}$ of the nanocontact after the closing of the last conductance channel fulfills the equation:

$$R_{\text{nc}} = \sum_{k=1}^{N} \frac{1}{R_k}$$

where $n$ is the channel number in order of closing ($n = 1, 2, 3, ..., N$). $N$ is the total number of conductance channels considered, $t_n$ the time of closing of channel $n$, $t_0$ a reference time (for $n = 1$, $t = t_0$, which corresponds to the beginning of the analysis), and $R_k$ denotes the resistance of channel $k$. The resistance $R_{\text{nc}}$ of the nanocontact after the closing of the last conductance channel fulfills the equation:

$$R_{\text{nc}} = \sum_{k=1}^{N} \frac{1}{R_k}$$

where $n < N$ and $R_n = R_{n+1}$ for $n = N$; resistances $R_n$ and $R_{n+1}$ are the plateau values in the conductance trace in the stepped conductance range.

Figure 2 presents the proposed model of stretched nanocontact for range III in Fig. 1(a).

Fig. 2. Model of stretched nanocontact for range III in Fig. 1(a)

From equations (9) and (10) it follows that for $n = 2, 3, ..., N$:

$$R_{n+1} = \frac{R_n}{R_n + R_{n+1}}$$

Once the resistance values before and after the closing of successive conductance channels are known, the resistances $R_1$, $R_2$, $R_3$, ..., $R_N$ of the conductance channels can be determined from the following relation, resulting from equations (9) and (10):

$$R_n = \frac{R_{n-1} R_n - R_n R_{n+1}}{R_{n-1} - R_{n+1}}$$

where $n < N$ and $R_n = R_{n+1}$ for $n = N$; resistances $R_n$ and $R_{n+1}$ are the plateau values in the conductance trace in the stepped conductance range.

Figure 3 presents the conducted model of stretched nanocontact for range III, in which successive conductance channels close as the nanocontact is being stretched. In the diagram shown in Fig. 2 conductance channels are modeled as resistances connected in parallel. The closing of conductance channel $n$ in the nanocontact corresponds to the opening of disconnector $n$. The resistance values before and after the closing of conductance channel $n$ (the opening of switch $n$), denoted below by $R_n$ and $R_{n+1}$, respectively, obey the equations:
resistance is shown in Fig. 3b. The number of conductance channels considered is \( N = 5 \). Successive channels close at times \( t_1, t_2, t_3, t_4 \) and \( t_5 \). The resistance plateau values \( R_1, R_2, R_3, R_4 \) and \( R_5 \) are indicated by arrows in the plot. The resistance values \( R_1, R_2, R_3, R_4 \) and \( R_5 \) corresponding to the closing of successive conductance channels in the adopted model are calculated from equation (13). Figure 3c presents the model of the considered nanocontact, the conductance trace of which is shown in Fig. 3a.

Comparison of theoretical and experimental results. Continuous nonocontacts conductance changes

The nanocontacts that can form between the electrodes of an opening switch have an influence on the transient states of the current in the circuit containing the switch. Because of the changing character of the conductance-time dependence in the opening switch, the current in the circuit should be analyzed first in the range preceding the conductance steps as the nanocontact created between the contacts of the switch is being stretched. Figure 4a shows an electrical circuit represented by an impedance stretching of the contact between the electrodes. Figure 4b compares the instantaneous current at the moment of disconnecting of a voltage \( V' \) by a switch \( Sw \). A resistance \( R \) is included in the circuit for current measurement.

The current in the circuit in a transient state (during the opening of the switch) before the occurrence of conductance steps can be calculated from the following equation:

\[
\begin{align*}
i(t)&=\left(\frac{1}{G_m(K)} + Re(Z) + R\right)^{-1},
\end{align*}
\]

where \( G_m(K) \) can be calculated from equation (5) for \( K = V/(r(t)) \). The time dependence of the contact radius in the model described above can be calculated from the following equation:

\[
\begin{align*}
r(t)&=r_{\min} + a(t_{\max} - t),
\end{align*}
\]

where \( r_{\min} \) is the nanocontact radius calculated from equation (2) for the measurement point directly before the conductance steps (last point in range II in Fig. 1(a)), for which \( t = t_{\max}; a \) is the rate of change of \( r \) per unit of time resulting from the fitting of the experimental data (red solid line in Fig. 1b) to the Wexler conductance formula (green dashed line in Fig. 1b) for a given rate of electrode separation (0.847 \( \times \) 10\(^{-6} \) m/s for the results presented in Fig. 1b). Figure 4b compares the instantaneous current measured in the range preceding the conductance steps (red solid line) with theoretical values calculated from equation (14) (blue dashed line). The good agreement between the theoretical dependence and the experimental data validates the model and the theoretical current-time dependence used.

Comparison of theoretical and experimental results. Stepped nonocontacts conductance changes

A number of successive transient states may occur in a circuit containing a mechanical switch when conductance decreases in steps as a result of nanocontact stretching. Each of these transient states is caused by the closing of a conductance channel in the stretched nanocontact. Figure 5a shows an electrical circuit represented by an impedance \( Z \) at the moment of removing a voltage \( V' \) by a mechanical switch \( Sw \) in the phase where conductance decreases in steps as the nanocontact created between the contacts of the switch is being stretched.

Based on the model of the nanocontact, the channel resistances \( R_1, R_2, ..., R_{p} \) can be determined as described in second section. Nanocontact resistances \( R_1 \) and \( R_p \) before and after the opening of switch \( n = p \) (closing of channel \( n = p \)), respectively, can be determined from equations (9) and (10) with \( n = p \). The effect of the stepwise changes in the nanocontact conductance on the transient states of the current \( i(t) \) will be the most apparent if the impedance \( Z \) of the circuit connected to the switch \( Sw \) can be replaced by
the equivalent circuit, shown in Fig. 5a, with inductance $L$, resistance $R_L$, and capacitance $C$. Also shown in the diagram is a resistor $R_c$ included for the measurement of the current $i(t)$ in the circuit. Using this schematic we are going to determine the current $i(t)$ in the circuit after closing of channel $p$, which is represented by the open switch $p$ in the diagram.

Figure 5b shows the Laplace model of the circuit presented in Fig. 5a transformed for the determination of the current $I(s)$ after the opening of switch $p$ (where $s$ is the Laplace operator). In this diagram the instantaneous current flowing through the inductor $L$ immediately before the opening of switch $p$ (at time $t_p$) is denoted by $I(t_p)$, and the instantaneous voltage across the capacitor $C$ immediately before the opening of switch $p$ (at time $t_p$) is denoted by $u_C(t_p)$. Using the loop current method we determine the current $I(s)$ in the circuit in the Laplace domain:

$$I(s) = \frac{A_1 + A_2s + A_3}{s^2 + A_1s + A_2}.$$  \hspace{1cm} (16)

Formulas for the determination of the constants $A_1$ to $A_3$ are provided in Appendix. Next, from equation (16) we determine the inverse Laplace transform for aperiodic and periodic circuits. In an aperiodic circuit the values of $R_p$, $R$, $L$, $R_L$, and $C$ must fulfill the condition:

$$A_1^2 - 4A_2 > 0.$$  \hspace{1cm} (17)

In this case the current $i(t)$ in the circuit has the following time dependence:

$$i(t) = B_1 + B_2e^{B_1t} + B_3e^{B_2t},$$  \hspace{1cm} (18)

where $t = t - t_p$, and $B_1$, $B_2$, $B_3$ are constants defined in the Appendix. In a periodic circuit the condition to be fulfilled by the values of $R_p$, $R$, $L$, $R_L$, and $C$ is:

$$A_1^2 - 4A_2 < 0.$$  \hspace{1cm} (19)

In this case the current in the circuit has the time dependence:

$$i(t) = B_1 + B_2e^{B_1t} \sin(B_3t) + B_4e^{B_2t} \cos(B_5t).$$  \hspace{1cm} (20)

where $t = t - t_p$, and $B_1$, $B_2$, $B_3$, $B_4$, and $B_5$ are constants specified in the Appendix. Under the assumption that immediately before the opening of switch $p$ (at time $t_p$) the circuit was in a steady state, the instantaneous values of the current $i(t_p)$ flowing through the inductor $L$ and the voltage $v_C(t_p)$ across the capacitor $C$ can be calculated from the respective equations:

$$i(t_p) = \frac{V}{R + R_L + R_\Delta - \frac{1}{C}},$$  \hspace{1cm} (21)

$$v_C(t_p) = i(t_p)R_L.$$  \hspace{1cm} (22)

If at time $t_p$ the circuit was not in a steady state, the time dependences of the current $i(t)$ and voltage $v_C(t)$ before opening disconnector $p$ must be determined to provide a basis for the calculation of the instantaneous values of $i(t)$ and $v_C(t)$ at time $t_p$.

To verify equations (18) and (20) we simulated the conductance trace measured during the opening of the switch $Sw$. For better readability, the graph shows every twentieth signal sample, plotted with red diamonds. The current steps visible in the graph correspond to the last five steps in the conductance trace of the nanocontact created between the terminals of the switch. The respective times are denoted by $t_1$ to $t_5$. Next, using the method described in second section, we constructed a model of the switch based on the current-time dependence presented in Fig. 6a. The constructed model is shown in Fig. 6b.

To decide which of the above-derived equations describing the time dependence of the current in the circuit should be used for the determination of the theoretical dependence we checked the character of the circuit after the opening of each disconnector in the switch model at times $t_1$, $t_2$, $t_3$ and $t_4$. After the opening of the model disconnectors at times $t_1$, $t_2$ and $t_3$ the circuit parameters fulfill the inequality (17), which indicates that the circuit is aperiodic and the instantaneous current values can be calculated from equation (18). The opening of the model disconnector at time $t_4$ results in circuit parameters fulfilling the inequality (19); thus, the circuit has a periodic character and the instantaneous current can be calculated from equation (20). The circuit parameter values adopted for inequality testing are:

- $R_p = 330 \Omega$, $L = 94 \mu H$, $R_\Delta = 409 \Omega$, $R = 49 \Omega$, and $C = 330 \mu F$.
- $R_\Delta = 49 \Omega$, $L = 94 \mu H$, $R = 409 \Omega$, $R_p = 330 \mu F$, and $C = 94 \mu F$.
- $R_\Delta = 409 \Omega$, $L = 94 \mu H$, $R = 49 \Omega$, and $C = 330 \mu F$.

In the next step we determined analytically the instantaneous current, using equation (21) for the range $0 < t < t_1$. Equation (18) for $t_1 < t < t_2$, $t_2 < t < t_3$, $t_3 < t < t_4$, and equation (20) for $t_4 < t < t_5$. For times $t < t_5$, the current in the circuit is 0. The analytical current-time dependence is represented by the blue solid line in the plot shown in Fig. 6a.

Since the resistance of a switch with a nanocontact increases in time as successive disconnectors open in the model circuit, there are three possible scenarios of changes in character of the circuit as a result of the stretching of the nanocontact: the circuit is aperiodic after each conductance step, the circuit changes from aperiodic to periodic (case illustrated by Fig. 6), or the circuit is periodic after each conductance step. The latter case is illustrated by Fig. 7a, showing four current steps at times $t_1$, $t_2$, $t_3$ and $t_4$ measured in the circuit presented in Fig. 5a with the following parameters: $C = 948 \mu F$, $L = 7.05 \mu H$, $R_\Delta = 168 \Omega$, and $R = 49 \Omega$. For better readability, the plot only includes every twentieth sample of the measured characteristic (red diamonds). The experimental data are compared with the characteristic determined analytically (blue solid line).
The instantaneous current is determined from equation (21) for the range \(0 \leq t \leq t_1\), and from equation (20) for \(t_1 \leq t \leq t_2\), \(t_2 \leq t \leq t_3\), and \(t_3 \leq t \leq t_4\). After the opening of the model disconnectors at times \(t_1\), \(t_2\), and \(t_3\) the circuit parameters fulfill the inequality (19), which means that the circuit is periodic. Besides the parameters specified above, we put \(R_{\text{s}} = 4.758 \, \text{k\Omega}\), \(R_{N} = 8.689 \, \text{k\Omega}\) and \(R_{\text{b}} = 14.123 \, \text{k\Omega}\), for inequality testing. For times \(t \geq t_5\), the current in the circuit is 0.

Figures 6a and 7a indicate that the current traces determined analytically are in good agreement with the experimental data, which validates the model proposed and the theoretical current-time dependencies derived.

**Summary**

Our study and the analysis of its results demonstrate that when nanocontacts form between the terminals of an opening switch, its contact resistance results from the conductance of the stretched nanocontacts. The experimentally determined time dependence of the conductance of the switch used in the study has two segments that differ in the character of conductance changes, continuous in one segment and stepwise in the other. In the first segment the conductance of the opening switch can be determined based on the Wexler model, as described in the second section. In the other segment the conductance steps have a stochastic character and the conductance can be described by the Landauer formula. This equation provides a basis for the proposed model, with conductance channels represented by resistors connected in parallel. The resistances of the model resistors can be determined from the measured stepped conductance trace.

Using the adopted models, we have determined the theoretical time dependences of the current in the circuit during the opening of the switch. For verification of the theoretical dependence we have measured the current at the time of opening of a mechanical switch removing voltage from a circuit represented by impedance \(Z\). The measurement data, in good agreement with the calculation results, show that nanocontacts created between the terminals of an opening mechanical switch modify the transient states of the current in a circuit connected with the switch. This effect must be taken into account in the analysis of transient states in electrical circuits.

In the initial phase of switch opening the current-time dependence can be determined analytically from an equation derived from the Wexler conductance formula. An especially interesting phenomenon is observed at the time of closing of conductance channels just before the last nanocontact breaks. Short-lived transient states occur in this phase as a result of the closing of successive conductance channels. For the determination of the current-time dependence in this final phase of switch opening the character of the circuit, which can be aperiodic or periodic, should be tested for an appropriate choice of theoretical equation.

**Appendix**

The constants in formulas (16), (17) and (19) can be determined from equations (23) to (27).

\[
A_1 = b_4 - b_0 \\
A_2 = (b_4 - b_0)b_1 - b_6 \\
A_3 = b_3b_4 \\
A_4 = b_2 \\
A_5 = 0.25b_3
\]

The constants in equations (18) and (20) can be determined from equations (28) to (36).

\[
B_1 = b_5 \\
B_2 = \frac{b_4 - b_0 - b_5}{2} + \frac{(b_4 - b_0)b_1 + b_6 - 0.5(b_4 - b_0 + b_2)b_2}{b_7} \\
B_3 = -0.5(b_2 - b_1) \\
B_4 = \frac{b_4 - b_0 - b_5}{2} + \frac{0.5(b_5 + b_4 - b_0)b_2 - (b_4 - b_0)b_3 - b_0}{b_7} \\
B_5 = -0.5(b_2 + b_7) \\
B_6 = \frac{b_5 + (b_4 - b_0)b_1 - b_6[b_3 + 0.5(b_4 - b_0 - b_5)]}{0.5b_5}
\]

The auxiliary constants in equations (23) to (36) can be determined from equations (37) to (46).

\[
b_1 = \frac{R_L}{L} \\
b_2 = b_1 + \frac{1}{R + R_{p+} + C} \\
b_3 = 4\frac{R_L + R + R_{p+}}{R + R_{p+} + L/C} \\
b_4 = \frac{V}{R + R_{p+}} \\
b_5 = \frac{V}{R_L + R + R_{p+}} \\
b_6 = \frac{V}{R + R_{p+}} \\
b_7 = \frac{V}{R + R_{p+}} \\
b_8 = \frac{\sqrt{b_3^2 - b_5^2}}{b_3} \\
b_9 = \frac{1}{b_3 - b_5/C}
\]
\[ h_{10} = \frac{v_c f_{p-}}{R + R_p^+} \]

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